**Assigned:** Wednesday, November 3, 1999
**Due:** Wednesday, November 10, 1999
**Reading:** Ross, Chapter 6 (except Sections 6.6 and 6.8)
**Noncredit Exercises:** pp. 293-300: 8-12, 15, 20-23, 26, 28-30; pp. 300-304: 8, 14, 22, 23; pp. 305-308: 3-6, 11, 12, 14.

**Problems:**

**1.** [Read Example 3d on pp. 203-204 first.] Let the (straight) line segment ACB be a diameter of a circle of unit radius and center C. Consider an arc AD of the circle where the length **X** of the *arc* (measured clockwise around the circle) is a random variable uniformly distributed on $[0,2\pi)$. Now consider the "random chord" AD.

**(a)** Find the probability that the length **L** of the random chord is greater than the side of the equilateral triangle inscribed in the circle.

**(b)** Express **L** as a function of the random variable **X**, and find the probability density function for **L**.

**2.** Do **either** part (a) **or** part (b). Then do parts (c)–(e).

**(a)** Attach to your homework a **photocopy** of your calculator's manual page(s) that explains which **formula** your calculator computes Q(x). Reading the page might help too!
Note: I **do not want to know which buttons** you have to press in order to find Q(x); I **want to know what formula** your calculator uses internally to find Q(x).
The xerographically–challenged are permitted to just copy the relevant formulas to their homework.
**NEXT:** press the appropriate buttons to find Q(5).
*If your calculator cannot compute Q(x), or if the manual does not state what formula is used to calculate Q(x) but just tells you which buttons to press, or if you have lost the manual, do part (b) instead.*

**(b)** Read Chapter 26.2 of Abramowitz and Stegun (***reference*** book *(not a **reserve** book)* in Grainger Engineering Library), and use Equation 26.2.17 to calculate Q(5).

**(c)** The number found in part (a) or (b) is just an *approximation* to the value of Q(5). Use the maximum error specification to find the *range* in which the actual value of Q(5) must necessarily lie. What is the *maximum relative error* in the approximation to Q(5) that you found in part (a) or (b)?

Note: the relative error is defined as $\left|\frac{\text{true value}-\text{computed value}}{\text{true value}}\right|$ expressed as a percentage.

**(d)** On p. 972, Abramowitz and Stegun give the value of $-\log_{10}Q(5)$. Use your calculator to find Q(5) from this information, and compute the *actual relative error* in the approximation to Q(5) that you found in part (a) or (b). What would the actual relative error have been if you had simply used the upper bound of Eq. (4.4) as an approximation to Q(5) as suggested by Ross? What if you had ignored Ross's suggestion and used the lower bound as an approximation to Q(5) instead?

**(e)** Explain why the "much easier" Equation 26.2.18 of Abramowitz and Stegun is not particularly useful for computing Q(5).

**3.** The discrete random variables **X** and **Y** have joint pmf $p_{\mathbf{X},\mathbf{Y}}(u,v)$ given by

| | | | | |
|---|---|---|---|---|
| 4 | 0 | 1/12 | 1/6 | 1/12 |
| 3 | 1/6 | 1/12 | 0 | 1/12 |
| –1 | 1/12 | 1/6 | 1/12 | 0 |
| v↑ / u→ | 0 | 1 | 3 | 5 |

**(a)** Find the marginal pmfs $p_{\mathbf{X}}(u)$ and $p_{\mathbf{Y}}(v)$ of **X** and **Y**.
**(b)** Are the random variables **X** and **Y** independent ?
**(c)** Find $P\{\mathbf{X} \le \mathbf{Y}\}$ and $P\{\mathbf{X} + \mathbf{Y} \le 8\}$.

**4.** The joint pmf of **X** and **Y** is $p_{\mathbf{X},\mathbf{Y}}(i,j) = 2^{-(i-1)}3^{-j}$, $i = 1, 2, \ldots$ ; $j = 1, 2, \ldots$

**(a)** Find the marginal pmfs of **X** and **Y** and show that these are valid pmfs. What kinds of random variables are these?

**(b)** Find the conditional pmf of **X** given that **Y** = k.

**(c)** Are **X** and **Y** independent random variables?

**5.** The jointly continuous random variables **X** and **Y** have joint pdf given by

$$f_{\mathbf{X},\mathbf{Y}}(u,v) = \begin{cases} 2 \exp -(u+v), & 0 < u < v < \infty, \\ 0, & \text{elsewhere.} \end{cases}$$

**(a)** Sketch the u-v plane and indicate on it the region over which $f_{\mathbf{X},\mathbf{Y}}(u,v)$ is nonzero.

**(b)** Find the marginal pdfs of **X** and **Y**.

**(c)** Are the random variables **X** and **Y** independent ?

**(d)** Find P{**Y** > 3**X**}.

**(e)** For $\alpha > 0$, find P{**X** + **Y** ≤ $\alpha$}.

**(f)** Use the result in part (e) to determine the pdf of the random variable **Z** = **X** + **Y**.

**6.** The jointly continuous random variables **X** and **Y** have joint pdf

$$f_{\mathbf{X},\mathbf{Y}}(u,v) = \begin{cases} 1/2, & 0 \le u < 1,\ 0 \le v < 1, \text{ and } 0 \le u+v < 1, \\ 3/2, & 0 \le u < 1,\ 0 \le v < 1, \text{ and } 1 \le u+v < 2, \\ 0, & \text{otherwise.} \end{cases}$$

Find $f_{\mathbf{X}}(u)$, P{**X** + **Y** ≤ 3/2} and P{$\mathbf{X}^2 + \mathbf{Y}^2 \ge 1$}.