ECE313 Problem Set 8

Issued: Wednesday October 20. Due: Wednesday October 27.

Reading assignment: Ross, Chapter 5.

Exercises: PP. 232-237 Exercises 6, 8, 10, 21, 33, 39. PP. 237-241: Theoretical Exercises 1, 7, 12, 13.

PROBLEMS

Problem 1 In two communications systems, we receive bit B corrupted by Gaussian noise N $\mathcal{N}(0, 4)$. B is a Bernoulli variable which takes value 1 with probability 1/2 and value -1 with probability 1/2. The received signal for the first system is R_1, the random variable given by $R_1 = B + N$. The received signal for the second system is R_2, the random variable given by $R_2 = B \times N$.

a. Give the pdf of R_1.

b. Suppose we observe R_1 to be 0.5. What is the ML estimate of B? What is the MAP estimate of B?

c. Repeat part b when R_1 is -0.25.

d. Repeat parts b and c substituting R_2 for R_1

e. We now suppose that B is a Bernoulli variable which takes value 1 with probability 3/4 and value 0 with probability 1/4. Repeat parts a-d.

Problem 2

The night before tickets go on sale, people line up at the ticket sales office to get tickets to concerts. The first line is for tickets to go see Nine Inch Whales. The customers arriving to this first line at this form a Poisson process with rate λ_1. The second line is for tickets to go see Boos Traveler. The customers arriving to this first line at this form a Poisson process with rate λ_2. The two processes are independent.

a. What are the mean and variance of the time between the arrival of the first customer and the arrival of the fifth customer at the first line?

b. What is the probability that, from time t to time $t + \tau$, no new customer arrives to either line?

c. Suppose that, from time t to time $t + \tau$, three customers come to the ticket sales office. What is the probability that the first customer goes to the second line and that the second and third customers go to the first line? What is the probability that the second customer goes to the first line and that the first and third customers go to the second line?

d. What is the probability that, from time t to time $t+\tau$, more customers arrive to the second line than to the first line? (Hint: try conditioning on the number of total arriving customers and look at part c).

e. We observe the two lines from time t to time $t + \tau$. Let the random variable D be defined as: number of customers that arrive to the first line minus number of customers that arrive to the second line. Give the mean and variance of D.

f. What is the distribution of the time between two arrivals to the ticket office (i.e. arrival to either line)? Give the mean and variance.

g. You do not know λ_2, but you observe that k customers arrived from time t to time $t + \tau$ at the second line. What is the ML estimate of λ_2?

The sales office finally opens. The first line has 100 people in queue and the second line has 148 people in queue. The lines are now closed (no new arrivals are allowed) Clerk 1 serves the first line. His service times of customers are IID with mean $\frac{1}{\mu_1}$. Clerk 2 serves the second line. Her service times of customers are IID with mean $\frac{1}{\mu_1}$.

h. What is the distribution of the service time for the first customer in the first line? For the second customer in the second line?

Problem 3

Do Theoretical Exercises 2, 3 (pg. 237), 8 (pg. 238) and 30 (pg. 240). Note: the lognormal distribution is very important in wireless applications.

Problem 4

Exercise 22 pg. 235 in Ross.

Problem 5

Let $Q(x) = \int_x^{-\infty} \frac{1}{\sqrt{2\pi}} exp\left(-\frac{u^2}{2}\right) du = 1 - \Phi(x)$ where $\Phi(x)$ denotes the CDF of a unit Gaussian random variable.

a. Some tables list the values of $Q(x)$ (instead of $\Phi(x)$) for large values of x. Why might the tabulator have chosen to specify $Q(x)$ instead of $\Phi(x)$? Explain briefly. On page 211 (p. 218 in 4th edition), Ross gives an upper and a lower bound on $Q(x)$ (Eq. (4.4)). The rest of this problem leads you through a derivation of Eq. (4.4) that does not use the "obvious inequality" invoked by Ross in his proof, and it also looks at another, simpler bound.

b. What is the derivative of $exp(-u^2/2)$ with respect to u?

c. Write the integrand for $Q(x)$ as $\frac{1}{\sqrt{2\pi}}u^{-1}\left(uexp\left(-\frac{u^2}{2}\right)\right)$ and integrate by parts to deduce the upper bound on $Q(x)$. Repeat the trick of rewriting and integrating by parts to deduce the lower bound on $Q(x)$. Are these bounds useful as $x \to 0$? Why or why not? What is the asymptotic value of the ratio of the bounds as $x \to \infty$?

d. A useful bound when x is small is $Q(x) \leq (1/2)exp(-x^2/2)$ for $x \geq 0$ in which equality holds only at $x = 0$. Derive this bound by first showing that $t^2 - x^2 > (t-x)^2$ for $t > x > 0$ and then applying this result to $exp(x^2/2)Q(x) = \int_x^{-\infty} \frac{1}{\sqrt{2\pi}} exp\left(-\frac{t^2-x^2}{2}\right) dt$.

e. For what values of x is this smaller than the upper bound of Eq.(4.4)?