University of Illinois | **Problem Set #4** | **ECE 313 Fall 1999**

Assigned: Wednesday, September 15, 1999
Due: Wednesday, September 29, 1999
Reading: Ross, Chapter 4.1, 4.3–4.5, 4.7- 4.8, 4.9.1-4.9.3 and Chapter 3
Noncredit Exercises: (Do not turn these in) Ross pp. 104-117: 1, 2, 5, 10, 12, 16, 31, 38, 39, 44; pp. 118-122: 4, 23; p. 123: 2,3; pp.173-184: 4, 12, 14, 21, 24, 33, 35, 38, 39, 47, 57, 60, 69; pp. 184-188: 4.
Reminder: there is no class on September 22 and 24.
Problems:

1. You are in charge of collecting radar information for an important military flight mission. You want to identify military targets. There are three readings that come out of your system: Low, Medium and High. Moreover, there are three possibilities for targets: absent, non-military and military. P(Low|military) = 0.1, P(Low|non-military) = 0.2, P(Low|absent|) = 0.8. Moreover, you know that P(High|military) = 0.7, P(High|non-military) = 0.6, P(Medium|absent) = 0.2. Intelligence sources inform you that P(absent) = 0.3, P(military) = 0.1.

(a) Find P(Low and absent) and P(military and medium).

(b) First, you use Maximum Likelihood (ML) Hypothesis testing. How do you interpret a reading of Low, Medium and High?

(c) What is the probability that you misidentify a non-military target as a military target?

(d) Compute your total probability of error.

(e) You decide to change you hypothesis testing to maximum a posteriori criterion (Bayesian). Redo parts b and c for a Bayesin decision rule.

(c) Using the Bayesian test, you misidentified a skating rink as military, causing embarassment to your superiors. Now, you are told that if you identify a military correctly, you'll get paid an extra $100 dollars, but if you misidentify it, they will dock your pay for $200. If you misidentify an absent or non-military target as a military target, they will dock your pay for $100. They do not care if you get absent and non-military targets confused. How would you tailor your test to maximize the money you get (or at least minimize the expected amount of money that gets docked)?

2. Suppose that 105 passengers hold reservations for a 100–seat flight from Chicago to Champaign. The number of passengers showing up for the flight can be modeled as a binomial random variable **X** with parameters (105,0.9).

(a) Find the probability that all passengers who show up get seats, i.e. and P{**X** = 100}.

(b) Explain why the number of no-shows can be modeled as a Poisson random variable **Y**, and compute the value of the parameter λ.

(c) Compute the probability that all passengers who show up get seats based on this Poisson model, i.e find P{**Y** > 4}, and compare to the "more exact" answer of part (a).

3. This problem on conditional probability has three ***unrelated*** parts.

(a) If P(A|B) = 0.3, P(A^c|B^c) = 0.4, and P(B) = 0.7, find P(A|B^c), P(A), and P(B|A).

(b) If P(E) = 1/4, P(F|E) =1/2, and P(E|F) = 1/3, find P(F).

(c) If P(G) = P(H) = 2/3, show that P(G|H) ε 1/2.

4. Monty Hall, the host of the TV game show "Let's Make A Deal™", shows you three curtains. One curtain conceals a valuable prize, while the other two conceal junk. All three curtains are equally likely to conceal the prize. He offers you the following "deal": pick a curtain, and you can have whatever is behind it. When you pick a curtain, instead of giving you your just deserts, Monty (who knows where the prize is) opens one of the remaining curtains to show you that there is junk behind it, and offers the following "new, improved deal™" : you can either stick with your original choice, or switch to the remaining (unopened) curtain. Amidst the deafening roars of "Stand pat" and "Switch, you idiot" from the crowd, Monty points out that previously your chances of winning were 1/3. Now, since you know that the prize is behind one of the two unopened curtains, your chances of winning have increased to 1/2, and thus the new improved deal is indeed better. Use the theorem of total probability (Ross, Equation (3.1), Chapter 3) to determine

(a) the probability of winning if you always switch.

(b) the probability of winning if you would rather fight than switch.

(c) whether Monty is correct in asserting that if you choose randomly betweeen the two unopened curtains, you have a probability of winning of 1/2.

8. **Ross,** exercises 13 and 18 on page 186.