University of Illinois **Problem Set # 2** **ECE 313**
Fall 1999

Assigned: Wednesday, September 1
Due: Wednesday, September 8
Reading: Ross, Chapters 1 and 2, Section 4.1.
Reminder: have a good Labor Day weekend - **no class on Monday September 6.**
Non-credit exercises (do not turn in): Ross, pp. 18-19, exercises 19, 29, 33, pp.19-24 exercises 10, 12, 20, pp. 54-61 exercises 26, 54, pp. 61-64 exercises 4, 16, 19, 21.

1. A tub at an apple bobbing contest contains 8 red apples and 7 green apples. First Tommy bobs for apples and gets an apple. He leaves with his apple. No new apples are placed in the tub. Next, Linda bobs for apples and gets an apple. She leaves with her apple. Five more children after her bob for apples. Each child gets an apple and leaves with it. No apples are replaced in the tub. Assume that, when the children bob for apples, all apples in the tub are equally likely to be caught by a child.
(a) What is the probability that Tommy got a green apple?
(b) What is the probability that Linda got a green apple?
(c) What is the probability that there is at least one green apple in the tub after the last child is done bobbing for apples?
(d) What is the probability that there is at least one red apple in the tub after the last child is done bobbing for apples?
Suppose now that each child replaces his apple after he or she bobs for it (this is very unhygienic).
(e) Redo parts (a) through (d).
The Board of Health gets wind of the unhygienic practices at the apple bobbing contest and fines the organizers. The organizers then decide to outfit the next child, Ed, with a little net to catch apples. There are still 8 red apples and 7 green apples in the tub. Ed gets two apples.
(f) What is the probability that Ed catches two red apples?
(g) What is the probability that Ed catches at least one green apple?
(h) What is the probability that Ed catches no red apples?

2. For each of the following random variables, give the size of the sample space, Ω, give an example of an elementary event and an example of an event which is not an elementary event.
(a) The contents of a 16-bit register.
(b) A test is graded out of 100. No fractions of points are awarded. We consider the random variable which is the record of grades for that test for a class of 9 students (no student missed the test!).

3. Four events, A, B, C, D are defined on a sample space. For each of the following events, write a set-theoretic expression. For each event, draw a Venn diagram or a Karnaugh map.
(a) Either A or C occurred.
(b) A and B both occurred but D did not occur.
(c) B and D did not occur together.
(c) If A occurred, then B did not occur.
(d) If C occurred, then either A or B occurred.
(e) Neither A nor C occurred.

(f) Either A and B both occurred, or C and D both occurred.

4. The probability that a student plays intramural soccer is 1/10. The probability that a student plays intramural softball is 1/17. Give the tightest upper and lower bounds you can find on the following events (you must justify your answer):

(a) A student plays soccer and/or softball.
(b) A student plays neither soccer nor softball.
(c) A student **either** plays soccer and not softball **or** plays softball and not soccer.

5. Events A and B are **disjoint** events defined on a sample space. What is the probability that at least one of the two events A and B did not occur?

6. Your residence hall food service has on hand three servings of beans and two servings of carrots for next week. The food service manager chooses at random the vegetables to be served each day (Monday, Tuesday, Wednesday, Thursday, and Friday). Thus, all matchings of foods to days are equally likely.

(a) How many elements are there in the sample space Ω ?
(b) What is the probability of having beans on Monday ?
(c) What is the probability of having beans on Monday and Friday ?
(d) What is the probability of having respectively beans, beans, carrots, beans, and carrots on Monday, Tuesday, Wednesday, Thursday, and Friday ?

7. Let A, B, C, and D denote the events that Amy, Betty, Cathy, and Dorothy attend a conference. Suppose that $P(A) = P(B) = P(C) = P(D) = 0.6$,
$P(AB) = P(AC) = P(AD) = P(BC) = P(BD) = P(CD) = 0.36$,
$P(ABC) = P(ABD) = P(ACD) = P(BCD) = 0.216$,
and $P(ABCD) = 0.1296$.
For $k = 0, 1, 2, 3$, and 4, find the probability that exactly k of the four women attend the conference.