1. Check the appropriate box for each of the statements below. Answers need not be justified. However, in order to discourage guessing, you will be penalized for wrong answers.

(a) (+4 points for a correct answer, –4 points for a wrong answer, and 0 points for no answer) Which of the following statements are true for all events A and B such that $0 < P(A) < 1$ and $0 < P(B) < 1$?

TRUE	FALSE	
☐	☐	$P(A\|B) > P(A)$
☐	☐	$P(A\|B) + P(A\|B^c) = 1$
☐	☐	$P(A\|B) + P(A^c\|B^c) = 1$
☐	☐	$P(A\|B) + P(A^c\|B) = 1$
☐	☐	If $P(A) = P(B)$, then $P(A\|B) = P(B\|A)$
☐	☐	If $P(A\|B) = P(B\|A)$, then $P(A) = P(B)$
☐	☐	If $P(A\|B) = P(B\|A)$, then A and B are independent
☐	☐	$P(A \cap B) \le P(A)P(B)$ with equality if and only if A and B are independent
☐	☐	$P(B\|A)P(A) + P(A\|B^c)P(B^c) = P(A)$
☐	☐	$P(A\|B) = P(B\|A)P(B)/P(A)$

(b) (+8 points for a correct answer, –2 points for a wrong answer, and 0 points for no answer)

Which of the following four statements are true for all events A and B such that $0 < P(A) < 1$, $0 < P(B) < 1$?

‹ $P(A \cap B) \le \min\{P(A), P(B)\}$ ` $P(A \quad B) \ge [P(A) + P(B)]/2$

´ $P(A \cap B) \ge P(A) + P(B) - 1$. ^ $P(A \cap B) \le P(A|B)$

- ☐ **Only** ‹ and ´ are true statements.
- ☐ **Only** ` and ´ are true statements.
- ☐ **Only** ‹ , ´ , and ^ are true statements.
- ☐ **All four** are true statements.
- ☐ None of the above: **only** the following are true statements_____

(c) (+8 points for a correct answer, –2 points for a wrong answer, and 0 points for no answer) Which of the following four statements are **NOT** properties of **all** CDFs?

‹ $P\{\mathbf{X} > b\} = 1 - F_{\mathbf{X}}(b)$. ` If $F_{\mathbf{X}}(a) < F_{\mathbf{X}}(b)$, then $a < b$.

´ If $a < b$, then $F_{\mathbf{X}}(a) < F_{\mathbf{X}}(b)$. ^ $F_{\mathbf{X}}(u) = 1/2$ for some u, $-\infty < u < \infty$.

- ☐ **Only** ` is **not** a property of all CDFs.
- ☐ **Only** ` and ´ are **not** properties of all CDFs.
- ☐ **Only** ´ is **not** a property of all CDFs.
- ☐ **Only** ´ and ^ are **not** properties of all CDFs.
- ☐ You blew it, Professor! All four **are** properties of all CDFs.

(d) (+8 points for a correct answer, –2 points for a wrong answer, and 0 points for no answer)
Which of the following four statements are true for **all** random variables **X** and **Y** with identical finite variance σ^2 ?

‹ $E[\mathbf{X}^2] = E[\mathbf{Y}^2]$ ` $\text{var}(4\mathbf{X} - 5\mathbf{Y}) = \text{var}(4\mathbf{Y} - 5\mathbf{X})$
´ $|\text{cov}(\mathbf{X}, \mathbf{Y})| \le \sigma^2$ ^ **X** + **Y** and **X** – **Y** are uncorrelated RVs

- **Only** ‹ , ` , and ^ are true statements.
- **Only** ` and ^ are true statements.
- **Only** ` , ´ , and ^ are true statements.
- **All four** are true statements.
- None of the above. **Only** the following are true statements:_____

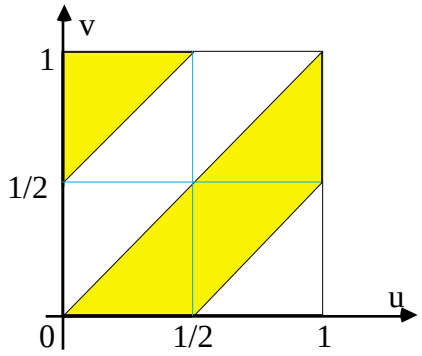
2. (10 points) Consider a Poisson process with arrival rate λ, and define the random variables **X**, **Y**, and **Z** as the number of arrivals in the intervals (0,4], (3,4] and (3,6] respectively. Find the conditional probability $P\{\mathbf{X} = 5, \mathbf{Z} = 4 \mid \mathbf{Y} = 2\}$.

3. (20 points) A continuous random variable **X** has pdf $f_{\mathbf{X}}(u) = \exp(-\pi u^2)$, $-\infty < u < \infty$.

(a) (10 points) Find $E[\mathbf{X}^2 + 2\mathbf{X} + 3]$.

(b) (10 points) Find $P\left\{ |\mathbf{X}| > \sqrt{\frac{2}{\pi}} \right\}$

4. (56 points) The joint probability density function $f_{\mathbf{X},\mathbf{Y}}(u,v)$ for the continuous random variables **X** and **Y** has constant value 2 on the shaded region in the figure below.



(a) (12 points) Find $f_{\mathbf{X}}(u)$, the marginal probability density function for **X**.
In order to obtain full credit, you must specify the value of $f_{\mathbf{X}}(u)$ for all real numbers u.

(b) (10 points) Compute $P\{\mathbf{X} > \mathbf{Y}\}$.

(c) (4 points) Are **X** and **Y** independent ? (Note: No credit will be given if the correct box is checked but an incorrect explanation (or no explanation) is provided)

- Yes, **X** and **Y** are independent.
- No, **X** and **Y** are not independent.

(d) (20 points) Find $P\{\mathbf{X} + \mathbf{Y} < 1/3\}$, $P\{\mathbf{X} + \mathbf{Y} < 2/3\}$, $P\{\mathbf{X} + \mathbf{Y} > 4/3\}$ and $P\{\mathbf{X} + \mathbf{Y} > 5/3\}$.

(e) (10 points) Find the pdf of the random variable $\mathbf{Z} = \mathbf{X} + \mathbf{Y}$.
In order to obtain full credit, you must specify the value of $f_{\mathbf{Z}}(\alpha)$ for all real numbers α.

5. (30 points) The weight **W** in pounds and the height **H** in inches of a randomly chosen high-school football player can be modeled as continuous random variables uniformly distributed on the intervals [185, 215] and [67, 73] respectively. It is also known that the correlation coefficient $\rho_{\mathbf{W},\mathbf{H}}$ for these random variables has value 1/3.

(a) (10 points) Compute E[**WH**].

(b) (16 points) Let $\mathbf{M} = \mathbf{W} - \mathbf{H}$. Find the mean and variance of **M**.

(c) (+4 points for a correct answer, –1 point for a wrong answer, and 0 points for no answer)
Let $\mathbf{S} = \mathbf{W} + \mathbf{H}$. The pdf of **S**

- is the sum of the pdfs of **W** and **H**
- is the product of the pdfs of **W** and **H**
- is Gaussian according to the Central Limit Theorem
- is not any of the above choices but *can* be found from the given information
- is not any of the above choices and *cannot* be found from the given information

6. (30 points) Consider a biased coin for which P(Heads) = 0.8 and P(Tails) = 0.2, and define the random variable **X** to have value 1 if the coin turns up Heads and to have value 0 if the coin turns up Tails.

(a) (10 points) What is the mean and variance of **X**?

(b) (10 points) 10,000 independent coin tosses give results $\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3, \ldots, \mathbf{X}_{10,000}$. Let $\mathbf{Y} = \mathbf{X}_1 + \mathbf{X}_2 + \mathbf{X}_3 + \ldots + \mathbf{X}_{10,000}$ denote the number of Heads observed on the 10,000 trials. Use the Chebyshev inequality to find a *lower bound* on $P\{7920 < \mathbf{Y} < 8080\}$.

(c) (10 points) Recall that **Y** is the number of Heads observed on 10,000 tosses of a biased coin with P(Heads) = 0.8. Use the Central Limit Theorem to *estimate* $P\{7920 < \mathbf{Y} < 8080\}$. A numerical answer is required.

7. (15 points) The jointly Gaussian random variables **X** and **Y** have means 0 and 7 respectively, variances 4 and 16 respectively, and correlation coefficient 1/16. Find the numerical value of $P\{\mathbf{Y} > 3\mathbf{X}\}$.