Problem 1.

A/
True or false.
Consider these two statements:
S1: "A CDF can have a negative slope"
S2: "A pdf can have a negative slope"

Mark only one of the boxes below:

- Only S1 is true
- Only S2 is true
- Both S1 and S2 are true
- Neither S1 nor S2 is true

B/

Let f_X be the pdf of the continuous random variable X and F_X be its associated CDF. Consider these two statements:

S3: " $\int_{-\infty}^{+\infty} f_X(x)dx = 1$ "

S4: " $\int_{-\infty}^{+\infty} F_X(x)dx = 1$ "

Mark only one of the boxes below:

S3 is always true; S4 is always false

S4 is always true; S3 is always false

Both S3 and S4 are true

S3 is always true but S4 is only true for certain distributions of X.

C/ Let f_X be the pdf of the continuous random variable X and F_X be its associated CDF.
Consider these two statements:
S5: "$f_X(u) > 1$ for some u"
S6: "$F_X(u) > 1$ for some u"
Mark only one of the boxes below:

S5 and S6 may both be true.
S5 may be true but S6 is never true.
S6 may be true but S5 is never true.
Neither S5 nor S6 is ever true.

Problem 2.

Consider a Poisson random process with arrival rate λ.

a) What is the mean number of arrivals from time 0 to 4?

mean number of arrivals from time 0 to 4 =

b) What is the probability of having three arrivals from time 0 to time 3 AND no arrivals from time 2 to time 6?

P(three arrivals from time 0 to time 3 AND no arrivals from time 2 to time 6) =

c) Suppose we observe 5 arrivals from time 0 to time 6. What is the maximum likelihood (ML) estimate of λ?

Maximum likelihood (ML) estimate of λ =

d) Suppose now that $\lambda = \ln(2)$. What is the probability that at least one arrival occurs from time 0 to 3? Give a numerical answer.

Probability that at least one arrival occurs from time 0 to 3 =

Problem 3.

Let X be a continuous random variable with mean 0. Let $f_X(u)$ be the pdf of the random variable X and $F_X(u)$ be its associated CDF. We are told that $f_X(u)$ is even.

Recall that

Function g(u) is even if and only $g(u) = g(-u)$ for all u
Function g(u) is odd if and only $g(u) = -g(-u)$ for all u.

For each of the following parts, say whether the statement is true, false, or cannot be determined from the information given. Circle the correct answer.

A/ $F_X(u)$ is even.

True **False** **Cannot be determined**

B/ $F_X(u) - ½$ is odd.

True **False** **Cannot be determined**

C/ $P(|X|>a) = 2F_X(-a)$ for all $a>0$.

True **False** **Cannot be determined**

Problem 4.

Consider the normal random variables, $X \sim N(1,2^2)$.

a) Find the pdf of $Y = 2X + 3$.

The pdf of $Y = 2X + 3$ is

b) Calculate $E[Y^2]$.

$E[Y^2] =$

Problem 5.

Each box of Soggies cereal contains either a Pikachu card with probability 1/3 or a Charmander card with probability 2/3. It is not known which card is in the box until it is opened. Mr. T buys cereal boxes for his son E until the child has acquired at least one copy of one card and two of the other. Let X be the number of Pikachu cards and Y the number of Charmander cards.

A/ What is P(X=1)?

P(X=1) =

B/ We define Z = X + Y. Fill in the boxes below:

P(Z = 0) =

P(Z = 1) =

P(Z = 2) =

P(Z = 3) =

.
.
.
.

P(Z = k) =