University of Illinois | **Problem Set #6** | **ECE 313**

 | **Fall 1997**

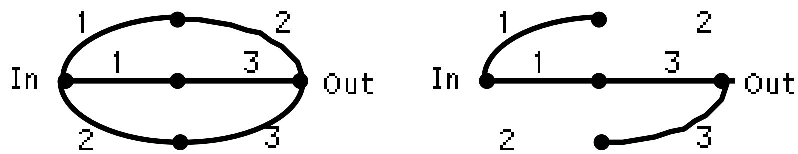
Assigned: Wednesday, October 8, 1997
Due: **Wednesday,** October 22, 1997
Reminder: **Hour Exam I is on Tuesday October 14, 7 pm to 8 pm, in Room 228 Natural History Building.**
Reading: Ross, Chapter 3
Noncredit Exercises: (Do not turn these in) Ross pp.104-117: 53, 58, 59, 62, 63, 70-74, 78, 81
Those with the **4th** edition should try: Ross, pp. 112-125: 50, 52, 53, 57, 58, 64-69, 73, 76.

Problems:

1. A QMR (quintuple modular redundancy) system is a fancier and more expensive version of the TMR system studied in class. It uses 5 identical circuits.

(a) If each circuit has probability p of failing, what is the probability that the majority gate output is incorrect? Ignore the possibility that the majority gate has failed. (Hint: condition on IV and V both failed, one of IV and V failed, and neither IV nor V failed; combine results using the theorem of total probability)

(b) A graph model for the TMR system is shown below in which we must replicate links, e.g. each circuit is represented by two links, and if the circuit fails, both links are removed from the graph. For example, if circuit 2 fails, the resulting graph is as shown on the right.



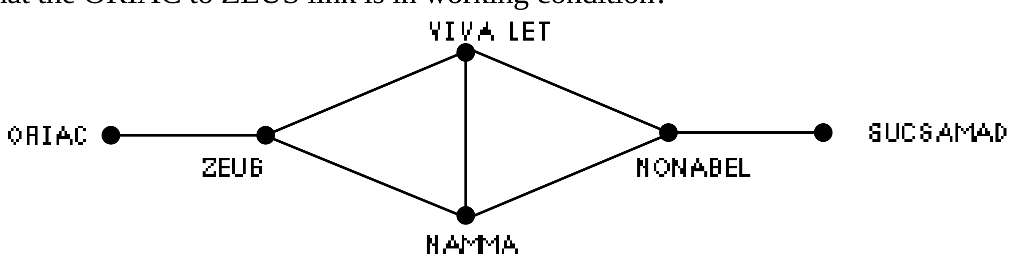
Consider the graph model of the QMR system. If there are no failures, how many paths are there from In to Out? How many links represent each of the circuits?

2. MiddleEast Bell, a division of NYAAHNEX Corp., has built a telephone network as shown below. Terrorists attack each of the seven links. The attacks may be considered to be independent events, and the attack on a link succeeds in severing the link with probability p. If a link is severed, switches automatically re-route calls so as to avoid the failed link (if possible).

(a) What is the probability of being able to call from ORIAC to SUCSAMAD?

(b) Given that it is possible to call from ORIAC to SUCSAMAD, what is the conditional probability that the ZEUS to NAMMA link is in working condition?

(c) If it is not possible to call from VIVA LET to NAMMA, what is the conditional probability that the ORIAC to ZEUS link is in working condition?



3. Let A denote an event with probability 1/2. Given that A occurred at least 4 times in 10 *independent* trials, what is the probability that it occurred no more than 5 times ?

4. Suppose that 105 passengers hold reservations for a 100–seat flight from Chicago to Champaign. Each passenger decides *independently* (with probability 0.9) whether to show up for the flight.

(a) Find the probability that all passengers who show up get seats.

(b) Suppose that 15 passengers are arriving in Chicago on a connecting flight which is late with probability 1/3. If the connecting flight is on time, all 15 show up for the flight to Champaign (nobody stops off at a bar and misses the flight!); else, obviously none of the 15 shows up. The remaining 90 passengers decide *independently* as before (and also *independently* of the fate of the connecting flight). What is the probability that all passengers who show up get a seat? Given that all passengers who showed up got a seat, find the (conditional) probability that the connecting flight was late.

5.(a) Your father will buy you a Porsche if you can win the following unusual three set tennis match. The three sets are to be played **alternately** against your father and against your country club's tennis instructor. You can choose whether your opponents will be (in order) father–instructor–father or instructor–father–instructor. The three sets may be considered to be **independent experiments**. The probability that you win a set against your father is 0.5 while the probability that you win a set against the instructor is 0.4. Which choice of opponents gives a greater probability of winning the tennis match? Note: You win the tennis match by winning **at least** two sets out of the three.

(b) Your mother feels that you are too young to have such an expensive car, and insists that you will be given the Porsche only if you can win **two consecutive sets** in the three set match described above. Which choice of opponents gives a greater probability of winning **at least two consecutive sets** ?

6. Consider the following simplified model for a game of tennis. On each serve, let p denote the probability that player A wins the point, and q = 1–p the probability that player B wins the point. Assume that the outcome of each serve is independent of all others. Player A wins the game if the score reaches 4–0, 4–1, or 4–2, while B wins the game if the score reaches 2–4, 1–4, or 0–4. Else, the score reaches 3–3 (called deuce) and from this point onwards, the game continues until one player is two points ahead of the other, and thereby wins the game.

(a) Find the probabilities that the score reaches 4–0, 4–1, or 4–2 and the probabilities that the score reaches 2–4, 1–4, or 0–4. I need 6 answers here!

(b) Find P(score reaches deuce). Show that the sum of the seven probabilities obtained in parts (a) and (b) is 1 regardless of the value of p.

(c) Given that the score is deuce, what is P(A wins the next two points)? (This means A wins the game). What is P(B wins the next two points)? (This means B wins the game). What is the probability that both players win one point each? In this case, the score is tied again, and is also called deuce.

(d) Once the score reaches deuce, there *may* be further deuces until ultimately, either A or B wins both points and thereby wins the game. What is the probability that A ultimately wins the game given that the score is deuce? What is the probability that B ultimately wins the game given that the score is deuce? (Hint: these answers are different from those of part (c)) What is the probability that the game goes on forever with the score continuing to reach deuce after every two points?

(e) Use the results of parts (a)–(d) to express the probability that A wins the game as a function f(p) of p. A little thought shows that B wins with probability f(q) = f(1–p). Now, if p = 0, A wins no points which makes it difficult for him to win any games. Does your function f(p) satisfy f(0) = 0? If not, what do you get as the probability that A wins a game while losing every point? Similarly, if A wins every point, he is sure to win the game. Does your function f(p) satisfy f(1) = 1? If not, what do you get as the probability that A loses a game while winning every point?
Other reasonable properties of f(p) are f(0.5) = 0.5, f(p) + f(1–p) =1. Which of these is satisfied by your function f(p)?

(f) Expand f(p) in a Taylor series in the neighborhood of p = 0.5 (only the first two terms are needed) What does this say about the probability of winning a game if $p = 0.5 + \varepsilon$ where ε is very small?

(g) Use your favorite graphing program to sketch f(p) as a function of p for $0 \leq p \leq 1$. Determine the minimum value of p for which $f(p) \geq 2/3$.