Assigned: Wednesday, September 10, 1997
Due: Wednesday, September 17, 1997
Reading: Ross, Chapter 2.1-2.5 and 2.7, Chapter 3
Noncredit Exercises: (Do not turn these in) Ross pp. 54-61: 3-5, 8-14, 23, 27-29, 36, 38, 39, 41, 43, 45; pp. 61-64: 4-7, 11, 18
Those with the **4th** edition should try the following problems. Ross pp. 56-57: 4-7, 10, 12; p. 59: 3, 4; pp. 60-61: 1-6, 8, 9, 12, 15-17, 23, 24, 26, 27, 29, 31, 33

Problems:

1. Consider an experiment with a finite sample space containing n equally likely outcomes. Thus, there are 2^n different events defined on this sample space.
If you cannot solve the problem for general n, solve it (for 50% credit) for the case n = 3 and Ω = {a,b,c} discussed in class. In (d), assume outcome b occurs.

(a) Show that 2^{n-1} events are comprised of an odd number of outcomes while 2^{n-1} events are comprised of an even number of outcomes. (Zero is an even number)

(b) Find the “average probability” of an event by adding up the probabilities of all 2^n events and dividing the resulting sum by 2^n.

(c) How many of the 2^n different events have probability equal to the average probability that you found in part (b)?

(d) It was shown in class that when a trial of the experiment is performed, exactly 2^{n-1} events occur while the other 2^{n-1} events do not occur. What is the average probability of the 2^{n-1} events that do occur on a given trial of the experiment?

2. Five basketball teams play in a round-robin tournament, that is, each team plays the other four teams exactly once.

(a) What is the total number of games played in the tournament?

(b) In each game, one team wears a dark uniform while the other wears a light uniform. Is it possible to arrange matters such that each team wears dark uniforms for two of its games and light uniforms for the other two?

(c) No game ends in a tie; one team wins and the other loses. If n denotes your answer to part (a), then there are 2^n different results that might occur, and we assume that all of 2^n results are equally likely. With this assumption, find the probability that each team wins at least one game and loses at least one game (that is, no team has 4-0 or 0-4 record in the tournament.)

3. The experiment consists of picking a letter at random from the word CHATTANOOGA.

(a) Define a sample space with 11 equally likely outcomes. What is the probability that the letter picked is a vowel?

(b) Another way of setting up a probability space is to take Ω = {A,C,G,H,N,O,T}. Are the seven outcomes equally likely? Do any of the outcomes have the same probability as some other outcomes?
Now consider picking three letters at random (choosing a subset of size 3) from the letters in the word CHATTANOOGA.

(c) What is the probability that the letters chosen can be arranged to form one of the common words CAT, HAT, OAT, TAN and ANT? (Doesn’t matter which word is formed)

(d) Repeat part (c) assuming that **sampling with replacement** is being used

(e) Both for sampling with replacement and for sampling without replacement, find the probability that the letters, **as they are picked**, form one of the 5 words of part (c) (doesn’t matter which one) **without having to be re-arranged.**

4. An experiment consists of tossing a fair coin ten times

(a) State TRUE or FALSE: The sample space Ω comprises 10 equally likely outcomes.

(b) What is the probability that the third toss results in Head? Call this event A

(c) What is the probability that Head turns up exactly 5 times? Call this event B

(d) What is P(A   B) ? What is $P(A \cap B)$? What is the probability that exactly one of the events A and B occurs, i.e. what is P(A   B)?

5. The manufacturer of a cereal tests samples from the production line to see if the samples snap, crackle, and pop as advertised. Let A, B, and C denote respectively the events that the sample **does not** snap, **does not** crackle, and **does not** pop. The manufacturer's tests show that P(A) = 0.2, P(B) = P(C) = 0.3, P(AB   BC   AC) = 0.3, P(ABC) = 0.05, P(AB) = 0.1, and P(BC) = 2P(AC).

(a) Sketch the sample space and indicate on it the events A, B, and C.

(b) What is the probability that the cereal snaps, crackles, and pops ?

(c) Cereal that fails exactly one test is sold to supermarket chains at discount prices as Soggies, Bursties, and Mushies. What is the probability of the sample failing the snap test **only**? the crackle test **only**? the pop test **only**?

6. The experiment consists of picking a student from the set of all UIUC students registered this semester. It is **not** necessary to assume that all students are equally likely to be picked, but you may make this assumption if it makes you feel happier and more confident.

(a) Let A and B denote the events that the student picked has had respectively four years of science (FYS) and calculus in high school. Let P(A) = 0.45 and P(B) = 0.35. If the probability that the student had neither FYS nor calculus is 0.3, what is the probability that the student had both FYS **and** calculus? What is the probability that the student had FYS but **not** calculus ?

(b) Let C denote the event that the student is registered in ECE 313, and let A and B be as in part (a). Suppose that $P(A \cap B \cap C) = 0.002$. What is the probability that the student picked is not registered in ECE 313, but did have both FYS **and** calculus ? If the probability that the student picked is registered in ECE 313, and has had either FYS or calculus (but not both) is 0.004, and if students who had neither FYS nor calculus did not register in ECE 313, what is P(C)?

(c) Using the data given in parts (a) and (b), which of the following probabilities can you compute? It is not necessary to actually compute each probability.

P(A   C), P(A B C), P(A   B C^c), P($A^cB^cC^c$), P(A^cBC^c), P(ABC^c)