Assigned: Wednesday, September 3, 1997 (or Friday September 5, 1997)
Due: Wednesday, September 10, 1997
Reading: Ross, Chapter 1.1–1.5, Chapter 2.1–2.5 and 2.7
Noncredit Exercises: (Do not turn these in) Ross, p. 16: 1–5, 7, 9; p. 59-60: 3, 4, 9, 10, 11-14; pp. 61-64: 1–3, 6, 7, 10, 11, 12, 16. Note: page numbers refer to the 5th edition.
Problems: These problems are based entirely on material covered in the *prerequisites* (calculus and ECE 210/ECE 309) to this course. You should have mastered this stuff already, but may need to review the material one more time before starting the course. The problems below are assigned to help your review, and also as a self–diagnosis aid. If you cannot solve *all* these problems correctly, you will have difficulty in comprehending the material in the latter half of this course. Discovering *after* the drop date that you don't understand ECE 210/ECE 309 or calculus as well as you thought you did, and that consequently you are in some danger of failing this course is probably not in your best interest.

Do not use Mathematica or Matlab or a calculator etc. to do these problems except when you are specifically asked to do so.

1. (a) Does the commutative law of addition: a + b = b + a imply that – 4 + 1 equals 1 – 4?

(b) Determine whether -2^2+1 equals $1-2^2$ and -2^3+2 equals $2-2^3$ using

(i) ordinary grade–school arithmetic.

(ii) your calculator.
On an algebraic-entry calculator (e.g. TI, Casio, Sharp, most "desk accessory" calculators on PCs and Macintoshes etc), what happens when you press the keys marked $\boxed{-}$ $\boxed{2}$ $\boxed{x^2}$ $\boxed{+}$ $\boxed{1}$ $\boxed{=}$?

(iii) the Microsoft spreadsheet program Excel. (Enter the four formulas = –2^2+1, = 1–2^2, = –2^3+2, and = 2–2^3 into different cells in the spreadsheet)

(c) If you agree with Excel that $-2^2+1 \neq 1-2^2$, while $-2^3+2 = 2-2^3$, find the error in the following "proof" via the distributive and cancellation laws that -2^2+1 does in fact equal $1-2^2$**:** $-2^3+2 = 2-2^3$ $\quad (-2^2+1)\times 2 = (1-2^2)\times 2$ $\quad -2^2+1 = 1-2^2$ (since $2 \neq 0$)

2. The angles in this problem are expressed in **degrees** and **not in** the **radians** more commonly used in mathematical circles.

(a) Use your calculator to evaluate cot(10˚)cot(30˚)cot(50˚)cot(70˚) *without writing down intermediate results such as the values of* cot(10˚), cot(30˚), *etc and re-entering the numbers into your calculator.* If your calculator cannot be used in this fashion, you are urged to replace it with a more sophisticated machine.

(b) If your calculator's arithmetic unit is designed in accordance with the IEEE Standard for floating-point arithmetic, you should have obtained exactly 3 as the answer to part (a). Does it surprise you that cot(10˚)cot(30˚)cot(50˚)cot(70˚) = 3? If so, *prove* that the answer is exactly 3. No, just because your calculator says so does not *prove* the result.
If it does *not* surprise you that cot(10˚)cot(30˚)cot(50˚)cot(70˚) = 3, find four other integers a, b, c, and d such that 0 < a < b < c < d < 90 and cot(a˚)cot(b˚)cot(c˚)cot(d˚) is an integer. If you cannot, find (for reduced credit) three integers a, b, c such that 0 < a < b < c < 90 and cot(a˚)cot(b˚)cot(c˚) is an integer.

3. In this problem, all **angles** are expressed in **radians**.

(a) Use your calculator to evaluate $\sqrt{52}\cos(3^{-1}\arctan(18\sqrt{3}/35))$.

(b) In this part, you have to find the limit of $\dfrac{1}{[\sin x]^2} - \dfrac{1}{x^2}$ as x approaches 0. Use your calculator to evaluate this function for *small* values of x say, $x = 10^{-1}$, $x = 10^{-2}$, $x = 10^{-3}$, etc. Does the function seem to be approaching a limit, and if so, what do you think is the limit? Now, use what you have learned about limits in calculus to find $\lim_{x \to 0} \frac{1}{[\sin x]^2} - \frac{1}{x^2}$ analytically. (Hint: the answer is not 0, or 1, or ∞)

(c) Find the maxima of $f(x) = x^{25}(1.0001)^{-x}$ for $x > 0$. (If you have a graphing calculator, try it on this problem; otherwise just use standard calculus methods)

4. (a) What is the value of $\int_{-2}^{1} |x|\, dx$? the value of $\int_{0}^{1} x(1-x)^{19}\, dx$?

(b) Prove or disprove: there exists a function f(x) satisfying both of the following two conditions: **(i)** $0 \le f(x) \le 10$ for all real numbers x in the range $a \le x \le b$, and **(ii)** $\int_a^b f(x)\, dx < 0$. (Hint: Does either function of part (a) satisfy both conditions?)

(c) Let $\frac{d}{dx}f(x) = g(x)$ for – < x < . Which of the following statements are true for all x, – < x < ? C denotes an arbitrary constant.

(i) $\frac{d}{dx}f(-x) = g(-x)$. (ii) $\frac{d}{dx}f(x^2) = 2x\, g(x^2)$. (iii) $\frac{d}{dx}\exp(f(x^2)) = \exp(f(x^2))\, g(x^2)$.

(iv) $\int g(-x)dx = -f(-x) + C$. (v) $\int g(x^2)dx = f(x^2)/(2x) + C$. (vi) $\int \frac{g(x)}{f(x)}dx = \ln(f(x)) + C$

5. (a) Integrate $f(x, y) = \begin{cases} 6, & 0 < y < x,\ 0 < x < 1, \\ 0, & \text{elsewhere,} \end{cases}$ over the region $\{(x,y) : y > x^2\}$.

(b) Compute the integral of $(x^2 + y^2)^{-2}$ over the region $\{(x,y) : x^2+y^2 > 2\}$.

6. Let f and g denote two finite–energy signals, and let h denote their convolution f * g, that is, for all t, $-\infty < t < \infty$,

$$h(t) = \int_{-\infty}^{\infty} f(t-\tau)g(\tau)\, d\tau.$$

Let $\hat{f}(t) = f(-t)$, $\hat{g}(t) = g(-t)$, and $\hat{h}(t) = h(-t)$.

(a) Can $\hat{h}$ be expressed as the convolution of any (two or more) of f, g, $\hat{f}$, and $\hat{g}$? If so, which ones ?

(b) TRUE OR FALSE ? $h(0) = \int_{-\infty}^{\infty} f(t)\hat{g}(t)\, dt$.

(c) Let $R = f * \hat{f}$. Show that $R(t) = R(-t)$ and that $|R(t)| \le R(0)$ for all t.