Homework 8 Solutions

Problem 1:

P(offer) = 0.50

P(good interview | offer) = 0.95

P(good interview | no offer) = 0.75

P(offer | good interview) = P(offer | good)

$$= \frac{P(\text{good} \mid \text{offer})\, P(\text{offer})}{P(\text{good} \mid \text{offer})\, P(\text{offer}) + P(\text{good} \mid \text{no offer})\, P(\text{no offer})}$$

$$= \frac{0.95\,(0.50)}{0.95\,(0.50) + 0.75\,(0.50)}$$

$$= 0.5588$$

Problem 2:

a)

E(Revenue from A)

= \$3.50 E(Unit sales)

= \$3.50 (2000)

= \$7000

Var(Revenue from A)

= 3.50^2 Var(Unit sales)

= 3.50^2 (1000)

= 12,250 "dollars squared"

b)

E(Total revenue)

$$= \$3.50\ (2000) + \$2.00\ (10{,}000) + \$1.87\ (8500)$$
$$= \$42{,}895$$

Var(Total revenue)

$$= 3.50^2\ (1000) + 2.00^2\ (6400) + 1.87^2\ (1150)$$
$$= 41{,}871 \text{ "dollars squared"}$$

Problem 3

7.20. Let X_1 = random number of breakdowns for Computer 1, and X_2 = random number of breakdowns for Computer 2.

$$\text{Cost} = \$200\ (X_1) + \$165\ (X_2)$$

$$E(\text{Cost}) = \$200\ E(X_1) + \$165\ E(X_2) = \$200\ (5) + \$165\ (3.6) = \$1594$$

If X_1 and X_2 are independent, then

$$\begin{aligned} \text{Var(Cost)} &= 200^2\ \text{Var}(X_1) + 165^2\ \text{Var}(X_2) = 200^2\ (6) + 165^2\ (7) \\ &= 430{,}575 \text{ "dollars squared"} \end{aligned}$$

$$\sigma_{\text{Cost}} = \sqrt{430{,}575 \text{ "dollars squared"}} = \$656.18$$

The assumption made for the variance computation is that the computers break down independently of one another. Given that they are in separate buildings and operated separately, this seems like a reasonable assumption.

Problem 4:

$$P(\text{TR+ and FR+} \mid \text{CP high}) = P(\text{TR+} \mid \text{FR+ and CP high})\ P(\text{FR+} \mid \text{CP high})$$

$$= P(\text{TR+} \mid \text{CP high})\ P(\text{FR+} \mid \text{CP high})$$

The second equality follows because FR and TR are conditionally independent given CP. In other words, we just multiply the probabilities together. This is true for all four of the probabilities required:

P(TR+ and FR+ | CP high) = 0.82 (0.95) = 0.779

P(TR+ and FR- | CP high) = 0.82 (0.05) = 0.041

P(TR- and FR- | CP low) = 0.79 (0.83) = 0.6557

P(TR- and FR+ | CP low) = 0.79 (0.17) = 0.1343

Problem 5:

To start, we need some labels. Let us say that we have chosen Door A. The host has opened Door B, revealing the goat, and Door C remains closed. The question is whether we should switch to C. The decision rule is simple: switch if the probability of the car being behind Door C is greater than the probability that5 it is behind A. Let "Car C" denote the outcome that the

car is behind C, and likewise with the goats and the other doors. We want to calculate P(Car C | Goat B). Use Bayes theorem:

$$P(\text{Car C} \mid \text{Goat B}) = \frac{P(\text{Goat B} \mid \text{Car C})\, P(\text{Car C})}{P(\text{Goat B} \mid \text{Car A})\, P(\text{Car A}) + P(\text{Goat B} \mid \text{Car B})\, P(\text{Car B}) + P(\text{Goat B} \mid \text{Car C})\, P(\text{Car C})}$$

✓

The prior probabilities P(Car A), P(Car B), and P(Car C) are all equal to 1/3. For the conditional probabilities, the key is to think about the host's behavior. The host would never open a door to reveal the car. Thus, P(Goat B | Car C) = 1 and P(Goat B | Car B) = 0. Finally, what if the car is behind A? What is P(Goat B | Car A)? In this case, we assume that the host would randomly choose B or C, so
P(Goat B | Car A) = 0.5. Plug these numbers into the formula to get:

$$P(\text{Car C} \mid \text{Goat B}) = \frac{1\,(1/3)}{0.5\,(1/3) + 0\,(1/3) + 1\,(1/3)} = \frac{2}{3}$$

Thus, you should always switch when the host reveals the goat!

Here's another way to think about it: You had a one-third chance of getting the correct door in the first place. Thus, there is a two-thirds chance that the goat is behind B or C. By showing the goat behind B, the host has effectively shifted the entire two-thirds probability over to Door C.

Still another way to think about the problem: If you played this game over and over, one-third of the time the car would be behind A, and two-thirds of the time it would be behind one of the other doors. Thus, two-thirds of the time, the host shows you which door the car is *not* behind. If you always switch to the door that the host did not open, you will find the car 2/3 of the time. The other 1/3 the car is behind the door you chose in the first place.

This question was asked of Marilyn Vos Savant, the person with the highest recorded I.Q. Her published answer was correct, but it created quite a stir because many people (including PhDs) did not understand how to solve the problem.

Problem 6:

Let "+" indicate positive results, and "-" indicate negative results.

$$
\begin{aligned}
P(+) &= P(+ \mid \text{Dome})\, P(\text{Dome}) + P(+ \mid \text{No dome})\, P(\text{No Dome}) \\
&= 0.99\,(0.6) + 0.15\,(0.4) \\
&= 0.654
\end{aligned}
$$

$$
P(\text{Dome} \mid +) = \frac{P(+ \mid \text{Dome})\, P(\text{Dome})}{P(+ \mid \text{Dome})\, P(\text{Dome}) + P(+ \mid \text{No dome})\, P(\text{No Dome})}
$$

$$
= \frac{0.99\,(0.6)}{0.99\,(0.6) + 0.15\,(0.4)}
$$

$$
= 0.908
$$

$$
P(\text{No Dome} \mid +) = 1 - 0.908 = 0.092
$$

We can now calculate the EMV for Site 1, given test results are positive:

$$
\begin{aligned}
\text{EMV(Site 1} \mid +) &= (\text{EMV} \mid \text{Dome})\, P(\text{Dome} \mid +) + (\text{EMV} \mid \text{No dome})\, P(\text{No dome} \mid +) \\
&= (\$52.50\text{ K})\, 0.908 + (-\$53.75\text{ K})\, 0.092 \\
&= \$42.725\text{ K}
\end{aligned}
$$

[EMV | Dome and EMV | No dome have been calculated and appear in Figure 7.15.]
EMV(Site 1 | +) is greater than EMV(Site 2 | +). If the test gives a positive result, choose Site 1.

If the results are negative:

$$
\begin{aligned}
P(-) &= 1 - P(+) \\
&= 1 - 0.654 \\
&= 0.346
\end{aligned}
$$

$$
P(\text{Dome} \mid -) = \frac{P(- \mid \text{Dome})\, P(\text{Dome})}{P(- \mid \text{Dome})\, P(\text{Dome}) + P(- \mid \text{No dome})\, P(\text{No Dome})}
$$

$$
= \frac{0.01\,(0.6)}{0.01\,(0.6) + 0.85\,(0.4)}
$$

$$
= 0.017
$$

$$
P(\text{No Dome} \mid -) = 1 - 0.017 = 0.983
$$

We can now calculate the EMV for Site 1, given test results are negative:

$$
\begin{aligned}
\text{EMV(Site 1} \mid -) &= (\text{EMV} \mid \text{Dome})\, P(\text{Dome} \mid -) + (\text{EMV} \mid \text{No dome})\, P(\text{No dome} \mid -) \\
&= (\$52.50\text{ K})\, 0.017 + (-\$53.75\text{ K})\, 0.983 \\
&= -\$51.944\text{ K}
\end{aligned}
$$

EMV(Site 1 | -) is less than the EMV(Site 2 | -). If the test gives a negative result, choose Site 2.

Problem 7:

$$P(+ \text{ and Dome}) = P(+ \mid \text{Dome})\, P(\text{Dome}) = 0.99\,(0.60) = 0.594.$$

$$P(+ \text{ and Dome and Dry}) = P(\text{Dry} \mid + \text{ and Dome})\, P(+ \text{ and Dome})$$

But P(Dry | + and Dome) = P(Dry | Dome) = 0.60. That is, the presence or absence of the dome is what matters, not the test results themselves. Therefore:

$$P(+ \text{ and Dome and Dry}) = 0.60\,(0.594) = 0.356$$

Finally,

$$P(\text{Dome} \mid + \text{ and Dry}) = \frac{P(\text{Dome and } + \text{ and Dry})}{P(+ \text{ and Dry})}$$

But

$$P(+ \text{ and Dry}) = P(+ \text{ and Dry} \mid \text{Dome})\, P(\text{Dome}) + P(+ \text{ and Dry} \mid \text{No dome})\, P(\text{No dome})$$

and

$$\begin{aligned} P(+ \text{ and Dry} \mid \text{Dome}) &= P(\text{Dry} \mid + \text{ and Dome})\, P(+ \mid \text{Dome}) \\ &= P(\text{Dry} \mid \text{Dome})\, P(+ \mid \text{Dome}) \\ &= 0.6\,(0.99) \end{aligned}$$

$$\begin{aligned} P(+ \text{ and Dry} \mid \text{No dome}) &= P(\text{Dry} \mid + \text{ and No dome})\, P(+ \mid \text{No dome}) \\ &= P(\text{Dry} \mid \text{No dome})\, P(+ \mid \text{No dome}) \\ &= 0.85\,(0.15) \end{aligned}$$

Now we can substitute back in:

$$P(+ \text{ and Dry}) = 0.6\,(0.99)\,(0.6) + 0.85\,(0.15)\,(0.4) = 0.407$$

and

$$\begin{aligned} P(\text{Dome} \mid + \text{ and Dry}) &= \frac{0.356}{0.6\,(0.99)\,(0.6) + 0.85\,(0.15)\,(0.4)} \\ &= 0.874. \end{aligned}$$