Problem 1

X1 : pounds of pure steel
X2 : pounds of scrap metal

a)
Objective: \( \text{min } Z = 3X_1 + 6X_2 \)

Constraints:

\[
egin{align*}
3X_1 + 2X_2 &\leq 18 \\
X_1 + X_2 &\geq 5 \\
8X_2 - 7X_1 &\leq 0 \\
X_1 &\leq 4, X_2 \leq 7
\end{align*}
\]

b)

The optimal solution is obtained at \((x_1, x_2) = (4,1)\)

\(3(4) + 6(1) = 18\)

4 pounds of pure steel and 1 pound of scrap metal should be used.
Problem_2

Decision variables: $X_A, X_B, X_C$

Objective: 
\[ \min Z = 16X_A + 30X_B + 50X_C \]

Constraints:
\[ X_A \geq 20, X_B \geq 120, X_C \geq 60 \]
\[ \frac{1}{12} (3X_A + 3.5X_B + 5X_C) \leq 120 \]
\[ \frac{1}{12} (4X_A + 5X_B + 8X_C) \leq 160 \]
\[ \frac{1}{12} (X_A + 1.5X_B + 3X_C) \leq 48 \]
Notation:

\( N \) manufacturers : \( j = 1, 2, \ldots, N \)

\( M \) plants : \( i = 1, 2, \ldots, M \)

\( D \) classes : \( k = 1, 2, \ldots, D \)

plant \( i \) requires \( R_{ik} \) boards

\( i = 1, \ldots, M \)

\( k = 1, \ldots, D \)
CLONE MANUFACTURING COMPANY

\[ x_j = \text{number of boards from manufacturer } j \]
\[ c_j = \text{costs per board from manufacturer } j \]
\[ U_j = \text{maximum number of boards from manufacturer } j \]
\[ p_{jk} = \text{fraction of class } k \text{ boards from manufacturer } j \]
\[ c_{ji} = \text{costs of shipping per board from manufacturer } j \text{ to plant } i \]

\[ j = 1, \ldots, N \quad i = 1, \ldots, M \quad k = 1, \ldots, D \]
CLONE MANUFACTURING COMPANY

- **Observations:**

  \[ p_{jk} \geq 0 \quad \text{and} \quad \sum_{k=1}^{D} p_{jk} = 1 \quad j = 1, \ldots, N \]

- **Decision variables:**

  \[ x_j = \text{number of boards from manufacturer } j \]

  \[ x_{ji} = \text{number of boards shipped from manufacturer } j \text{ to plant } i \]

- **Objective:**

  \[
  \min \sum_{j=1}^{N} c_j x_j + \sum_{j=1}^{N} \sum_{i=1}^{M} c_{ji} x_{ji}
  \]
Constraints:

\[ \sum_{j=1}^{N} p_{jk} x_{ji} = R_{ik} \quad k = 1, 2, \ldots, D, \quad i = 1, \ldots, M \]

\[ x_j \leq U_j \quad j = 1, 2, \ldots, N \]

\[ \sum_{i=1}^{M} x_{ji} \leq x_j \quad j = 1, 2, \ldots, N \]

\[ x_j \geq 0 \quad j = 1, 2, \ldots, N \]

\[ x_{ji} \geq 0 \quad j = 1, 2, \ldots, N \quad i = 1, 2, \ldots, M \]
\[ x_{ijk} = \text{quantity of fiber } k \text{ shipped to customer } i \]

to satisfy the order \( q_{ij} \) for fiber \( j \)

- if \( k = j \), the product demanded is the product shipped
- if \( k \neq j \), a substitute product is shipped

\[ j = 1, \ldots, F \]
\[ k = 1, \ldots, F \]
\[ i = 1, \ldots, C \]

\( F = \text{number of fiber types} \)
\( C = \text{number of customers} \)
\[ q_{ij} = \text{quantity of fiber } j \text{ demanded by customer } i \]

\[ A_j = \text{quantity of fiber } j \text{ available for shipment} \]

\[ c_{jk} = \text{costs per unit of shipping fiber } j \text{ to customer } i \text{ who ordered fiber } j \text{ and the term may include a penalty for substitution} \]

**Note:** whenever substitution is not allowed, such a penalty is made very large
\[ x_j = \text{fraction of every customer’s order for fiber } j \text{ that is met with fiber } j \text{ and permitted substitutes} \]

\[ d_{ij} = \text{penalty per unit of fiber } j \text{ ordered by customer } i \text{ but not filled with fiber } j \text{ and permitted substitutes} \]
\[ \Phi_j = \text{fair share for fiber } j \]

quantity of fiber received by customer \( i \) of fiber in short supply

\[ .95 \Phi_j \leq \text{by customer } i \text{ of fiber in short supply} \leq 1.05 \Phi_j \]
FAYE STOUT COMPANY : FLOWS

$q_{ij}$ are fixed and known data
FAYE STOUT COMPANY : FLOWS

fiber \( j \) delivery to customers

Customer 1

Customer \( i \)

Customer \( C \)

\[ x_{1jj} \]

\[ x_{ijj} \]

\[ x_{Cjj} \]
availability of fiber $j$ is $A_j$; however demand is

$$\sum_{i=1}^{c} q_{ij} = Q_j \leftarrow \text{total demand for fiber } j \left[ \text{fixed} \right]$$

fair share is defined by

$$\Phi_j \triangleq \frac{A_j}{Q_j} \leftarrow \text{fixed parameter for } j=1,2,\ldots,F$$

fiber $j$ is in short supply if and only if

$$\Phi_j < 1$$
FAYE STOUT COMPANY: DECISION VARIABLES

\[ x_{ijk} = \text{amount of fiber sent to meet customer } i's \text{ demand for fiber } j \]

\[ y_{ij} = \text{amount of fiber } j \text{ not supplied to customer } i, \text{ or more precisely, amount of fiber } j \text{ ordered by customer } i \text{ but not filled with either fiber } j \text{ or permitted substitutes} \]
\[ \text{min} \quad \sum_{i=1}^{C} \sum_{j=1}^{F} \sum_{k=1}^{F} c_{ijk} \ x_{ijk} \quad + \quad \sum_{i=1}^{C} \sum_{j=1}^{F} d_{ij} \ y_{ij} \]

- costs of items supplied
- penalties incurred for items not supplied
FAYE STOUT COMPANY: CONSTRAINTS

- balance

\[ \sum_{k=1}^{F} \sum_{i} x_{ijk} + y_{ij} = q_{ij} \quad i=1, \ldots, C \]

- availability

\[ \sum_{i=1}^{C} x_{ijj} \leq A_{j} \quad j=1, \ldots, F \]

- uniform fraction of order filled for fiber \( j \)

\[ \frac{1}{q_{ij}} \sum_{k=1}^{F} x_{ijk} = x_{j} \quad i=1, 2, \ldots, C \]
**FAYE STOUT COMPANY: CONSTRAINTS**

- **fair share constraints**

  \[ 0.95 \Phi_j \leq x_j \leq 1.05 \Phi_j \]

  such that \( \Phi_j < 1 \)

- **nonnegativity**

  \[ x_{ijk} \geq 0 \quad \forall i, \forall j, \forall k \]
  \[ y_{ij} \geq 0 \quad \forall i, \forall j \]
Problem data:

- 18-month production schedule
- Each worker produces 300 bottles per month
- Storage from month $t$ to month $t+1$ incurs a 5% loss
- $n_0 = 50$ workers and for each month $t$
  - New workers hired
  - Old workers released
  - Workers kept idle

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○ attrition rates for workers are

  10% for idle

  1% for productive

□ Decision variables are associated with costs

\[
\begin{align*}
c_t & \leftrightarrow e_t = \text{number of workers in production} \\
h_t & \leftrightarrow x_t = \text{number of workers hired} \\
f_t & \leftrightarrow y_t = \text{number of workers released} \\
n_t & \leftrightarrow d_t = \text{number of workers idle}
\end{align*}
\]

decisions at the beginning of each month \( t \)
Terminal constraints are given by

\[ S_{18} \geq \frac{I}{0.95} \]

work force at \( t = 19 \geq W \)
The objective is to minimize the costs of production

- we ignore costs of resources other than labor for period $t$ and so costs are employment plus storage for each month $t$

$$c_t e_t + h_t x_t + f_t y_t + n_t d_t + i_t s_t$$

- the objective is

$$\min \sum_{t=1}^{18} \left[ c_t e_t + h_t x_t + f_t y_t + n_t d_t + i_t s_t \right]$$
work-force constraints:

\[
\begin{align*}
50 + x_1 - y_1 &= e_1 + d_1 \\
.99e_1 + .9d_1 + x_2 - y_2 &= e_2 + d_2
\end{align*}
\]
general relationship

\[ .99e_{t-1} + .9d_{t-1} + x_t - y_t = e_t + d_t \quad t = 2, \ldots, 18 \]

terminal requirement

\[ .99e_{18} + .9d_{18} \geq W \]
THE MONTY ZOOMA CORPORATION:
CONSTRAINTS

○ production levels

![Diagram showing the general relationship and terminal requirements]

**general relationship**

\[ 300e_t = S_t + s_t - 0.95s_{t-1} \quad t = 1, \ldots, 18 \]

**terminal requirements**

\[ s_0 = 0 \quad \text{and} \quad 0.95s_{18} \geq I \]
THE MONTY ZOOMA CORPORATION:
PROBLEM STATEMENT

\[
\min \sum_{t=1}^{18} \left\{ c_t e_t + h_t x_t + f_t y_t + n_t d_t + i_t s_t \right\} \\
\]

\[
e_1 + d_1 - x_1 + y_1 = 50
\]

\[
.99e_{t-1} + .9d_{t-1} + x_t - y_t - e_t - d_t = 0 \quad t = 2, \ldots, 18
\]

\[
.99e_{18} + .9d_{18} \geq W
\]

\[
300e_1 - s_1 = S_1
\]

\[
300e_t - s_t + 0.95s_{t-1} = S_t \quad t = 2, \ldots, 18
\]

\[
0.95s_{18} \geq I
\]

\[
e_t, x_t, y_t, d_t, s_t, \geq 0
\]