HOMEWORK_3 SOLUTIONS

Problem_1

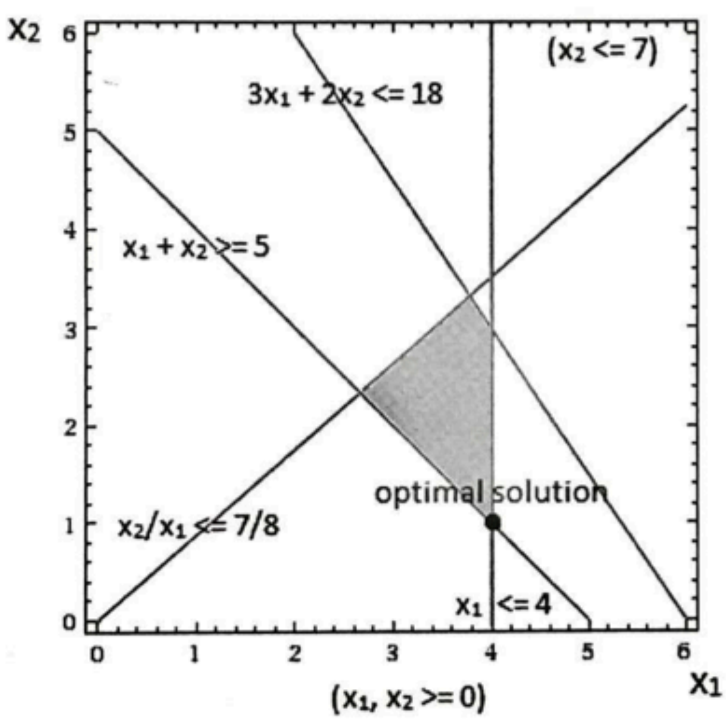
X1 : pounds of pure steel
X2 : pounds of scrap metal

a)
Objective: $min Z = 3X_1 + 6X_2$

Constraints:

$$3X_1 + 2X_2 \le 18$$
$$X_1 + X_2 \ge 5$$
$$8X_2 - 7X_1 \le 0$$
$$X_1 \le 4, X_2 \le 7$$

b)



The optimal solution is obtained at $(x_1, x_2) = (4,1)$
3(4)+6(1) = 18
4 pounds of pure steel and 1 pound of scrap metal should be used.

Problem_2

Decision variables: X_A, X_B, X_C

Objective:

$$min\, Z = 16X_A + 30X_B + 50X_C$$

Constraints:

$$X_A \geq 20, X_B \geq 120, X_C \geq 60$$

$$\frac{1}{12}(3X_A + 3.5X_B + 5X_C) \leq 120$$

$$\frac{1}{12}(4X_A + 5X_B + 8X_C) \leq 160$$

$$\frac{1}{12}(X_A + 1.5X_B + 3X_C) \leq 48$$

CLONE MANUFACTURING COMPANY

- **Notation:**

N **manufacturers** : $j = 1, 2, \ldots, N$

M **plants** : $i = 1, 2, \ldots, M$

D **classes** : $k = 1, 2, \ldots, D$

plant i **requires** R_{ik} **boards** $\quad i = 1, \ldots, M$; $k = 1, \ldots, D$

CLONE MANUFACTURING COMPANY

x_j = number of boards from manufacturer j

c_j = costs per board from manufacturer j

U_j = maximum number of boards from manufacturer j

p_{jk} = fraction of class k boards from manufacturer j

c_{ji} = costs of shipping per board from manufacturer j to plant i

$j = 1, \ldots, N \qquad i = 1, \ldots, M \qquad k = 1, \ldots, D$

CLONE MANUFACTURING COMPANY

- **Observations:**

$$p_{jk} \geq 0 \quad \text{and} \quad \sum_{k=1}^{D} p_{jk} = 1 \qquad j = 1, \ldots, N$$

- **Decision variables:**

x_j = number of boards from manufacturer j

x_{ji} = number of boards shipped from manufacturer j to plant i

- **Objective:**

$$\min \sum_{j=1}^{N} c_j \, x_j + \sum_{j=1}^{N} \sum_{i=1}^{M} c_{ji} \, x_{ji}$$

CLONE MANUFACTURING COMPANY

- Constraints:

$$\sum_{j=1}^{N} p_{jk}\, x_{ji} \;=\; R_{ik} \qquad k = 1, 2, \ldots, D\,,\; i = 1, \ldots, M$$

$$x_j \;\le\; U_j \qquad j = 1, 2, \ldots, N$$

$$\sum_{i=1}^{M} x_{ji} \;\le\; x_j \qquad j = 1, 2, \ldots, N$$

$$x_j \;\ge\; 0 \qquad j = 1, 2, \ldots, N$$

$$x_{ji} \;\ge\; 0 \qquad j = 1, 2, \ldots, N \quad i = 1, 2, \ldots, M$$

FAYE STOUT COMPANY : NOTATION

x_{ijk} = quantity of fiber k shipped to customer i to satisfy the order q_{ij} for fiber j

customer i

fiber j requested

fiber k shipped

$k = j$

product demanded is the product shipped

$k \neq j$

a substitute product is shipped

$j = 1, \ldots, F$

$k = 1, \ldots, F$

$i = 1, \ldots, C$

F = number of fiber types

C = number of customers

FAYE STOUT COMPANY : NOTATION

q_{ij} = quantity of fiber j demanded by customer i

A_j = quantity of fiber j available for shipment

c_{jk} = costs per unit of shipping fiber j to customer i who ordered fiber j and the term may include a penalty for substitution

Note : whenever substitution is not allowed, such a penalty is made very large

FAYE STOUT COMPANY : NOTATION

x_j = fraction of every customer's order for fiber j that is met with fiber j and permitted substitutes

x_j is uniform for each customer i

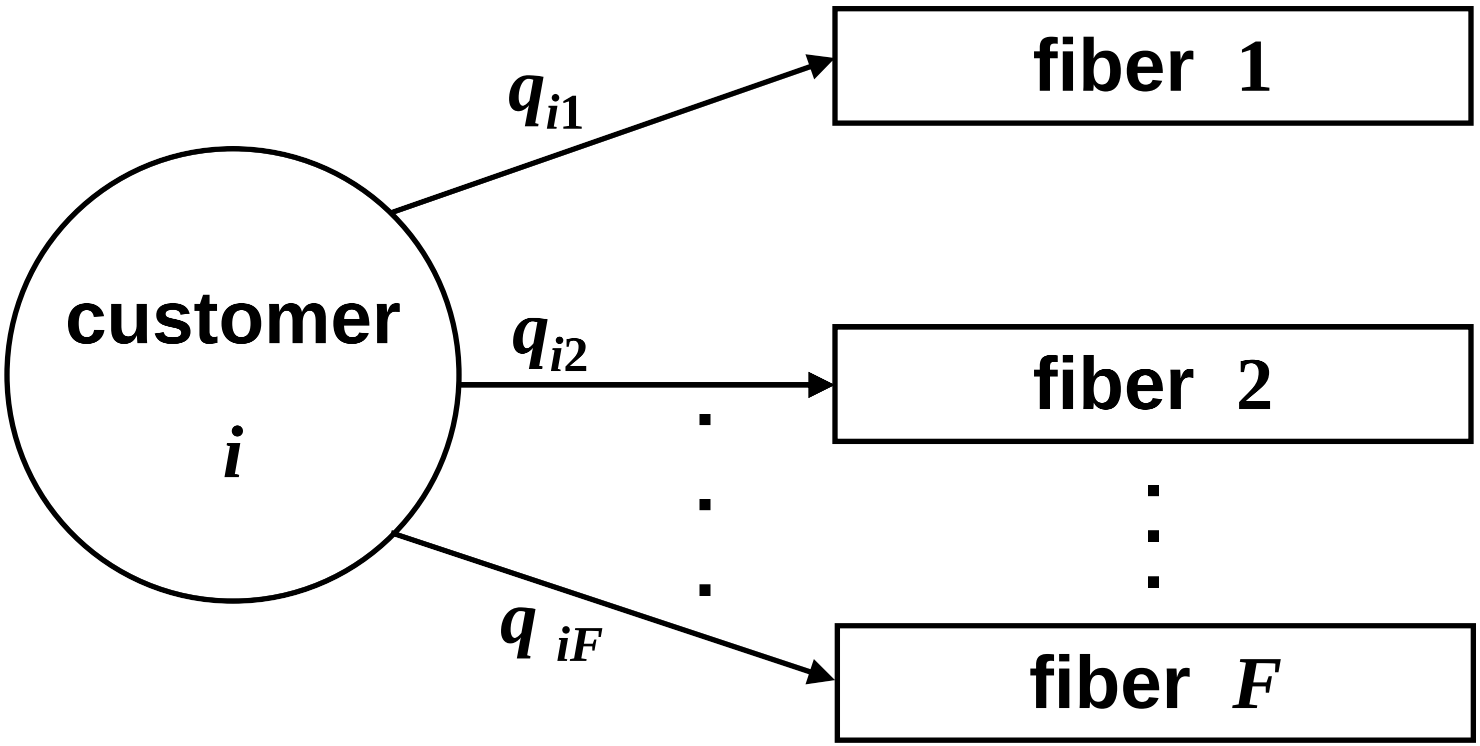
d_{ij} = penalty per unit of fiber j ordered by customer i but not filled with fiber j and permitted substitutes

FAYE STOUT COMPANY : INFORMATION PROVIDED

Φ_j = fair share for fiber j

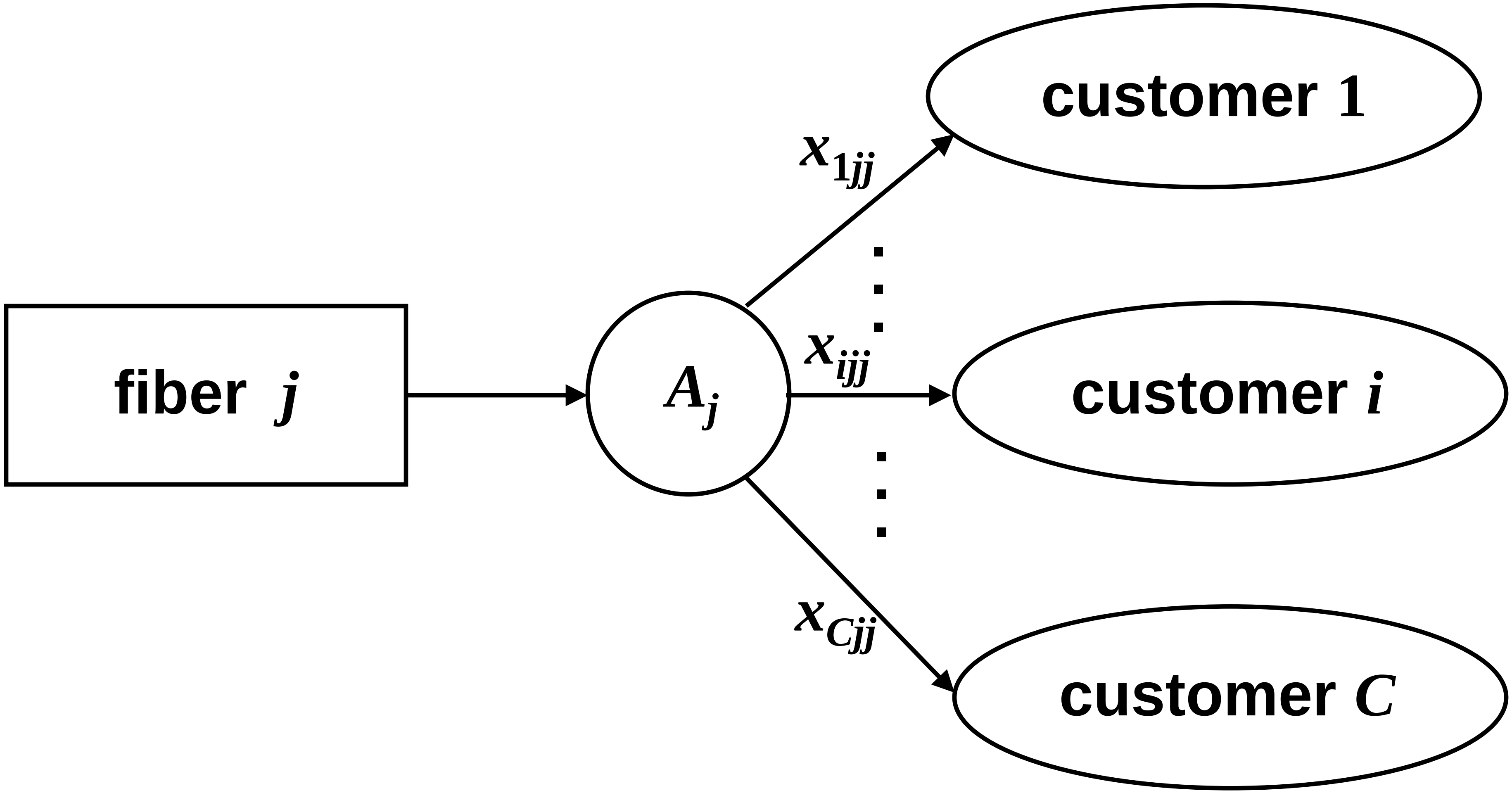
$$.95\,\Phi_j \leq \begin{array}{c}\text{quantity of fiber received}\\ \text{by customer } i \text{ of fiber in}\\ \text{short supply}\end{array} \leq 1.05\,\Phi_j$$

FAYE STOUT COMPANY : FLOWS



q_{ij} are fixed and known data

FAYE STOUT COMPANY : FLOWS



fiber *j* delivery to customers

FAYE STOUT COMPANY : FLOWS

availability of fiber j is A_j ;however demand is

$$\sum_{i=1}^{C} q_{ij} = Q_j \leftarrow \text{total demand for fiber } j \left[\text{ fixed} \right]$$

fair share is defined by

$$\Phi_j \triangleq \frac{A_j}{Q_j} \leftarrow \text{ fixed parameter for } j = 1, 2, \ldots, F$$

fiber j is in short supply if and only if

$$\Phi_j < 1$$

FAYE STOUT COMPANY : DECISION VARIABLES

x_{ijk} = **amount of fiber sent to meet customer *i's* demand for fiber *j***

y_{ij} = **amount of fiber *j* not supplied to customer *i* , or more precisely, amount of fiber *j* ordered by customer *i* but not filled with either fiber *j* or permitted substitutes**

FAYE STOUT COMPANY : OBJECTIVE

$$\min \quad \underbrace{\sum_{i=1}^{C} \sum_{j=1}^{F} \sum_{k=1}^{F} c_{ijk}\, x_{ijk}}_{\text{costs of items supplied}} \quad + \quad \underbrace{\sum_{i=1}^{C} \sum_{j=1}^{F} d_{ij}\, y_{ij}}_{\text{penalties incurred for items not supplied}}$$

costs of items supplied

penalties incurred for items not supplied

FAYE STOUT COMPANY : CONSTRAINTS

- **balance**

$$\sum_{k=1}^{F} x_{ijk} + y_{ij} = q_{ij} \qquad i = 1, \ldots, C \quad j = 1, \ldots, F$$

- **availability**

$$\sum_{i=1}^{C} x_{ijj} \leq A_j \qquad j = 1, \ldots, F$$

- **uniform fraction of order filled for fiber** j

$$\frac{1}{q_{ij}} \sum_{k=1}^{F} x_{ijk} = x_j \qquad i = 1, 2, \ldots, C$$

FAYE STOUT COMPANY : CONSTRAINTS

- **fair share constraints**

$$j = 1, 2, \ldots, F$$

$$0.95\,\Phi_j \;\leq\; x_j \;\leq\; 1.05\,\Phi_j$$

such that $\Phi_j < 1$

- **nonnegativity**

$$x_{ijk} \geq 0 \qquad \forall\, i, \forall\, j, \forall\, k$$

$$y_{ij} \geq 0 \qquad \forall\, i, \forall\, j$$

THE MONTY ZOOMA COMPANY

- **Problem data:**
  - 18 - month production schedule
  - each worker produces 300 bottles per month
  - storage from month t to month $t + 1$ incurs a 5% loss
  - $n_0 = 50$ workers and for each month t
  - each month t $\begin{cases} \text{new workers hired} \\ \text{old workers released} \\ \text{workers kept idle} \end{cases}$

THE MONTY ZOOMA COMPANY

- attrition rates for workers are

  10% for idle

  1% for productive

- Decision variables are associated with costs

$c_t \leftrightarrow e_t$ = number of workers in production

$h_t \leftrightarrow x_t$ = number of workers hired

$f_t \leftrightarrow y_t$ = number of workers released

$n_t \leftrightarrow d_t$ = number of workers idle

decisions at the beginning of each month t

THE MONTY ZOOMA COMPANY

month $t = 1, 2, \ldots, 18$

$i_t \leftrightarrow s_t$ = bottles in storage at the end of the month t

S_t = number of bottles sold in month t

- Terminal constraints are given by

$$s_{18} \geq I / 0.95$$

work force at $t = 19 \geq W$

THE MONTY ZOOMA COMPANY

- The objective is to minimize the costs of production
  - we ignore costs of resources other than labor for period t and so costs are employment plus storage for each month t

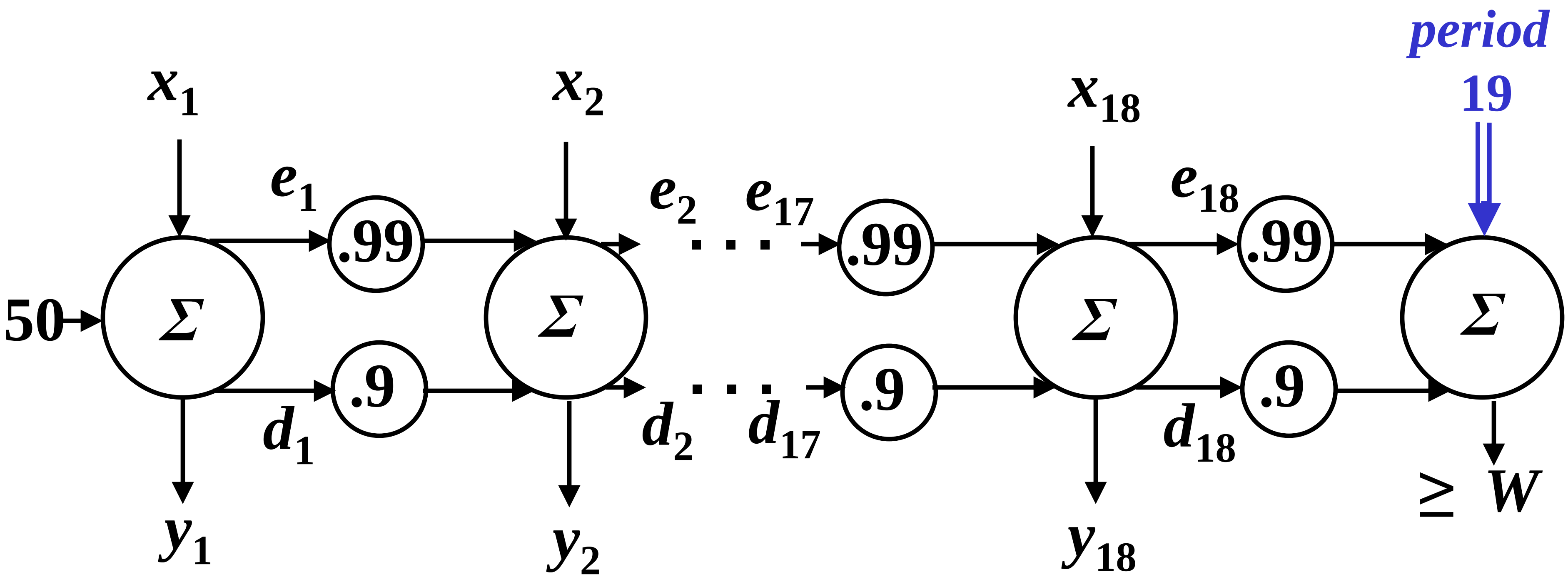
$$c_t e_t + h_t x_t + f_t y_t + n_t d_t + i_t s_t$$

  - the objective is

$$min \sum_{t=1}^{18} \left[c_t e_t + h_t x_t + f_t y_t + n_t d_t + i_t s_t \right]$$

THE MONTY ZOOMA COMPANY : CONSTRAINTS

- work-force constraints:



period 1

$$50 + x_1 - y_1 = e_1 + d_1$$

$$.99e_1 + .9d_1 + x_2 - y_2 = e_2 + d_2$$

THE MONTY ZOOMA CORPORATION

general relationship

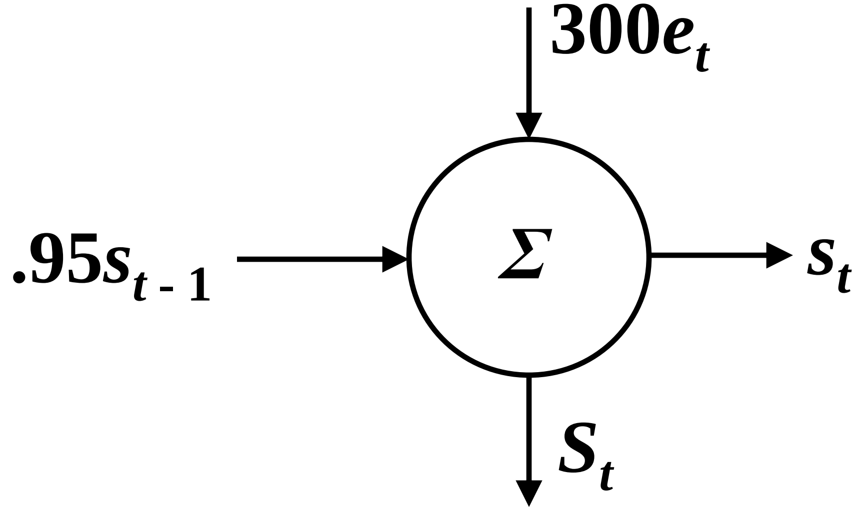
$$.99e_{t-1} + .9d_{t-1} + x_t - y_t = e_t + d_t \qquad t = 2, \ldots, 18$$

terminal requirement

$$.99e_{18} + .9d_{18} \geq W$$

THE MONTY ZOOMA CORPORATION : CONSTRAINTS

- production levels



general relationship

$$300\ e_t = S_t + s_t - .95\ s_{t-1} \qquad t = 1, \ldots, 18$$

terminal requirements

$$s_0 = 0 \qquad\qquad .95\ s_{18} \geq I$$

THE MONTY ZOOMA CORPORATION : PROBLEM STATEMENT

$$\min \sum_{t=1}^{18} \left\{ c_t e_t + h_t x_t + f_t y_t + n_t d_t + i_t s_t \right\}$$

$$e_1 + d_1 - x_1 + y_1 = 50$$

$$.99e_{t-1} + .9d_{t-1} + x_t - y_t - e_t - d_t = 0 \quad t = 2, \ldots, 18$$

$$.99e_{18} + .9d_{18} \geq W$$

$$300e_1 - s_1 = S_1$$

$$300e_t - s_t + 0.95s_{t-1} = S_t \quad t = 2, \ldots, 18$$

$$0.95s_{18} \geq I$$

$$e_t, x_t, y_t, d_t, s_t, \geq 0$$