ECE 307

Spring 2019
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Room 4052 ECEB

Homework 3

Date due: Tuesday, February 19, 2019

Some of the problems in this assignment are rather challenging. Indeed, these may very well be the most challenging problems in the entire course. While each of you may solve the problems individually, this particular assignment is a good vehicle to introduce a team approach to problem solving. Please form groups of three or four students to work on these problems. A single submission by each group suffices. On the front page, please state the names of the group members and have each member sign the sheet in the same line as the name.

Each problem is marked out of* 40 *points and so the HW maximum grade is* 200 *points.

1. The plant manager, Rollin K. Old, of the *Jericho Steel Company* must decide how many pounds of pure steel x_1 and how many pounds of scrap metal x_2 to use in manufacturing an alloy casting for one of its customers. Assume that the cost per pound of pure steel is 3 and the cost per pound of scrap metal is 6 (which is larger because the impurities must be skimmed off). The customer's order is expressed as a demand for at least five pounds and the customer is willing to accept a greater amount if *Jericho* requires a larger production run. Assume that the supply of pure steel is limited to four pounds and of scrap metal to seven pounds. The ratio of scrap to pure steel cannot exceed 7/8. The manufacturing facility has only 18 hours of melting and casting time available; a pound of pure steel requires three hours whereas a pound of scrap requires only two hours to process through the facility.

   a) **Express** the entire problem as a linear programming model.

   b) **Draw** a diagram displaying the feasible mixes of pure steel and scrap metal, and indicate an optimal solution.

2. The *Turned-On Transistor Radio Company* manufactures models A, B, and C which have profit contributions of 16, 30, and 50, respectively. The weekly minimum production requirements are 20 for model A, 120 for model B, and 60 for model C.

   Each type of radio requires a certain amount of time for the manufacturing of component parts, for assembling, and for packaging. Specifically, a dozen units of model A require three hours for manufacturing, four hours for assembling, and one hour for packaging. The corresponding figures for a dozen units of model B are 3.5, 5, and 1.5, and for a dozen units of model C are 5, 8, and 3. During the forthcoming week, the company has available 120 hours of manufacturing, 160 hours of assembling, and 48 hours of packaging time.

   Formulate the production scheduling problem as a linear programming problem.

3. The *Clone Manufacturing Company* must contract for the purchase of boards from each of N manufacturing outfits to produce its devices. These boards are shipped to the M assembly plants to be assembled into the devices sold by *Clone*. There are D classes of circuit boards. Plant i requires a quantity R_{ik} of boards of class k where $i = 1, 2, \ldots, M$ and $k = 1, 2, \ldots, D$.

   Let x_j be the number of boards purchased from manufacturer j, and let U_j be the maximum number of boards available from semiconductor manufacturer j. Let p_{jk} be the fraction of boards purchased from semiconductor manufacturer j that are of type k. (e.g., if $p_{34} = 0.5$, then one-half of all the boards purchased from semiconductor manufacturer 3 falls into the class size 4.) Thus,

$$p_{jk} \geq 0 \quad \text{and} \quad \sum_{k=1}^{D} p_{jk} = 1.$$

The purchasing cost per board from manufacturer j is c_j and the shipping cost per board from manufacturer j to plant i is c_{ji}. Note that, typically, a purchase plan causes the company to have more boards of a certain class than is actually required. Assume that after purchase, the boards are sorted by class at the plant, and boards of any class that are found to be in excess supply relative to requirements are stored without additional cost at the plant.

Formulate an appropriate optimization model that indicates how many boards to purchase from each semiconductor manufacturer, and the number of boards to ship to each plant.

4. The *Faye Stout Rayon Company* has introduced a new synthetic acetate fiber that it expects will replace much of its current sales of staple rayon. The resultant large demand for the synthetic coupled with production difficulties at the plant make it necessary for the Stout Co. to ship substitutions, when permitted, to some of its customers. Thus, a customer ordering acetate fiber 3 may actually be shipped some of fiber 5 whenever fiber 3 is in short supply and the substitution is allowed by the customer. Even with such substitutions, not all customer demand for certain fiber types can be satisfied. For each of the acetate types in short supply, the company wants to fill the same proportion of every customer's order. (In other words, for the case of total orders for fiber j are 150,000 pounds with only 100,000 pounds of fiber j available, these orders cannot be supplied in full with the fiber j and some of the supply is replaced by a substitute product – each customer who ordered fiber j receives a shipment of 2/3 of the quantity ordered of fiber j and 1/3 of its ordered quantity may be satisfied with permitted substitute product shipped by Stout Co.

Let F be the total number of fibers manufactured and C be the total number of customers. Let x_{ijk} be the pounds of fiber k that are shipped to customer i to satisfy its order q_{ij} for fiber j, where $i = 1, 2, ..., C$, and $j = 1, 2, ... , F$, and $k = 1, 2, ... , F$. When $j \neq k$, a substitute product is being shipped. The amount of fiber j available for shipment is A_j pounds. The cost per pound of shipping fiber k to customer i who ordered fiber j is c_{ijk} (this cost may include a penalty for substitution; if substitution of fiber k for fiber j in customer i's order is not permitted, this cost can be set arbitrarily large). Let x_j be the fraction of all customers' order for fiber j that is actually filled with fiber j and permitted substitutes. For each customer i, let d_{ij} be the penalty cost per pound of fiber j ordered but not filled (with fiber j and permitted substitutes). It is required that every customer receives at least 95% but no more than 105% of its fair share of each fiber ordered that is in short supply. **Formulate** an appropriate optimization model.

5. The *Monty Zooma Company* has announced the introduction of an exciting new perfume *Revenge*. The product manager, Tuffon de Butts, wants to draw up a production and employment plan for the next 18 months. Assume that each worker can produce 300 bottles of perfume per month. Perfume can be stored from one month to the next, but due to spoilage and pilferage, there is a 5% loss (in other words, for each 100 bottles stored in month t, only 95 are available in month t + 1). The initial level of employment is 50 workers; each month, additional workers may be hired if needed, released if not needed, or kept on the payroll but left idle. Workers left idle have a tendency to quit, and the attrition rate is 10%. Hence for each 10 workers left idle in month t, only 9 choose to remain on the

work force at month $t + 1$. There is only a 1% monthly attrition rate for workers engaged in production.

For each month t, let e_t be the number of workers engaged in perfume production, s_t the inventory of bottles at the end of the month, x_t and y_t the increase and decrease, respectively, in the work force at the beginning of the month, and d_t the number of workers left idle. Let the corresponding unit costs be c_t, i_t, h_t, f_t, and n_t, respectively. In month t, de Butts expects to sell S_t bottles of perfume, and hence wants to find a minimum cost employment and production plan that is capable of meeting these sales requirements. Assume that at the start of month 19, inventory of bottles must be at least I, and the work force must be at least W. **Formulate** an appropriate optimization model.