Problem 1

Variables:

x_{12} - No. of units shiped from city 1 to city 2.
x_{13} - No. of units shiped from city 1 to city 3.
x_{23} - No. of units shiped from city 2 to city 3.
x_{24} - No. of units shiped from city 2 to city 4.
x_{25} - No. of units shiped from city 2 to city 5.
x_{34} - No. of units shiped from city 3 to city 4.
x_{35} - No. of units shiped from city 3 to city 5.
x_{46} - No. of units shiped from city 4 to city 6.
x_{54} - No. of units shiped from city 5 to city 4.
x_{56} - No. of units shiped from city 5 to city 6.

Constraints:

$x_{12} + x_{13} = 30$
$x_{46} + x_{56} = 30$
$x_{12} - x_{23} - x_{24} - x_{25} = 0$ (Node 2 conservation)
$x_{13} + x_{23} - x_{34} - x_{35} = 0$ (Node 3 conservation)
$x_{24} + x_{34} + x_{54} - x_{46} = 0$ (Node 4 conservation)
$x_{25} + x_{35} - x_{54} - x_{56} = 0$ (Node 5 conservation)
$0 \leq x_{12} \leq 17$ (Capacity of arc 12)
$0 \leq x_{13} \leq 25$ (Capacity of arc 13)
$0 \leq x_{23} \leq 5$ (Capacity of arc 23)
$0 \leq x_{24} \leq 10$ (Capacity of arc 24)
$0 \leq x_{25} \leq 5$ (Capacity of arc 25)
$0 \leq x_{34} \leq 15$ (Capacity of arc 34)
$0 \leq x_{35} \leq 20$ (Capacity of arc 35)
$0 \leq x_{46} \leq 18$ (Capacity of arc 46)
$0 \leq x_{54} \leq 5$ (Capacity of arc 54)
$0 \leq x_{56} \leq 17$ (Capacity of arc 56)

Objective:

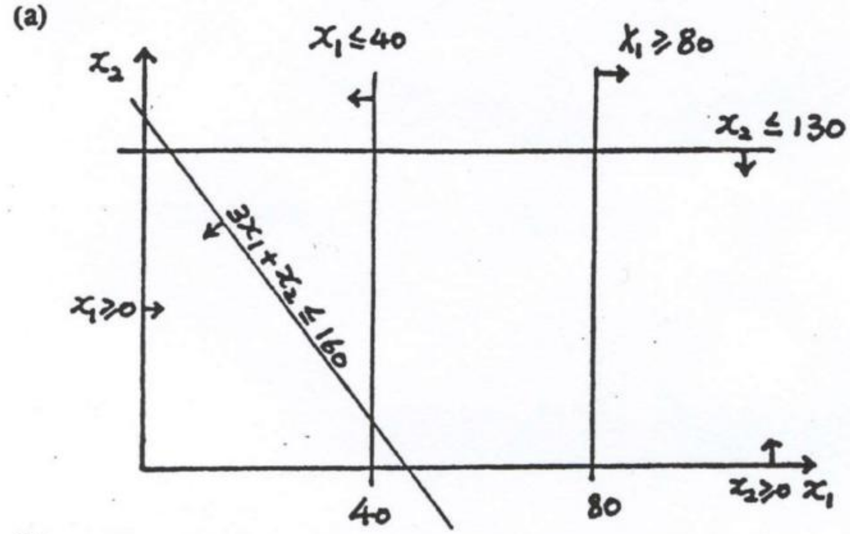
$$\text{Min } Z = 2x_{12} + x_{13} + 3x_{23} + x_{24} + 3x_{25} + 2x_{34} + 5x_{35} + 2x_{46} + 6x_{54} + 3x_{56}$$

Problem 2

The addition of a constant, K, to the objective function does not affect the optimal solution. However, the optimal value will increase (or decrease) by the magnitude of K, depending on the sign of K.

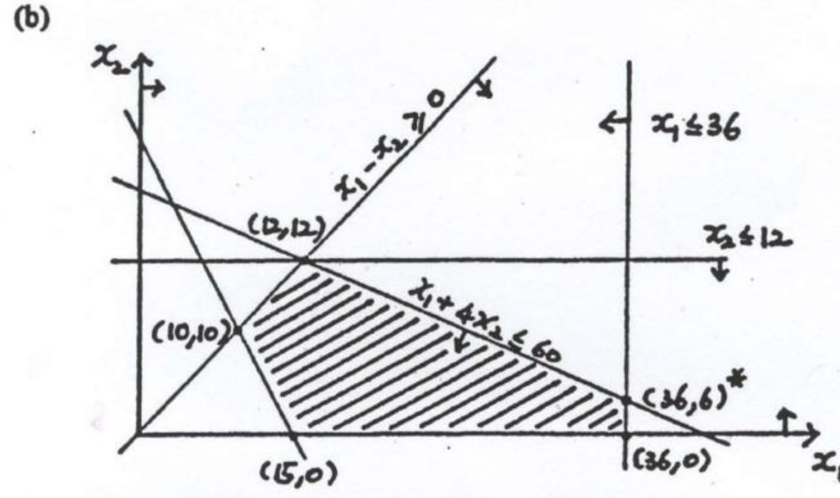
Problem 3

(a)



INFEASIBLE

(b)

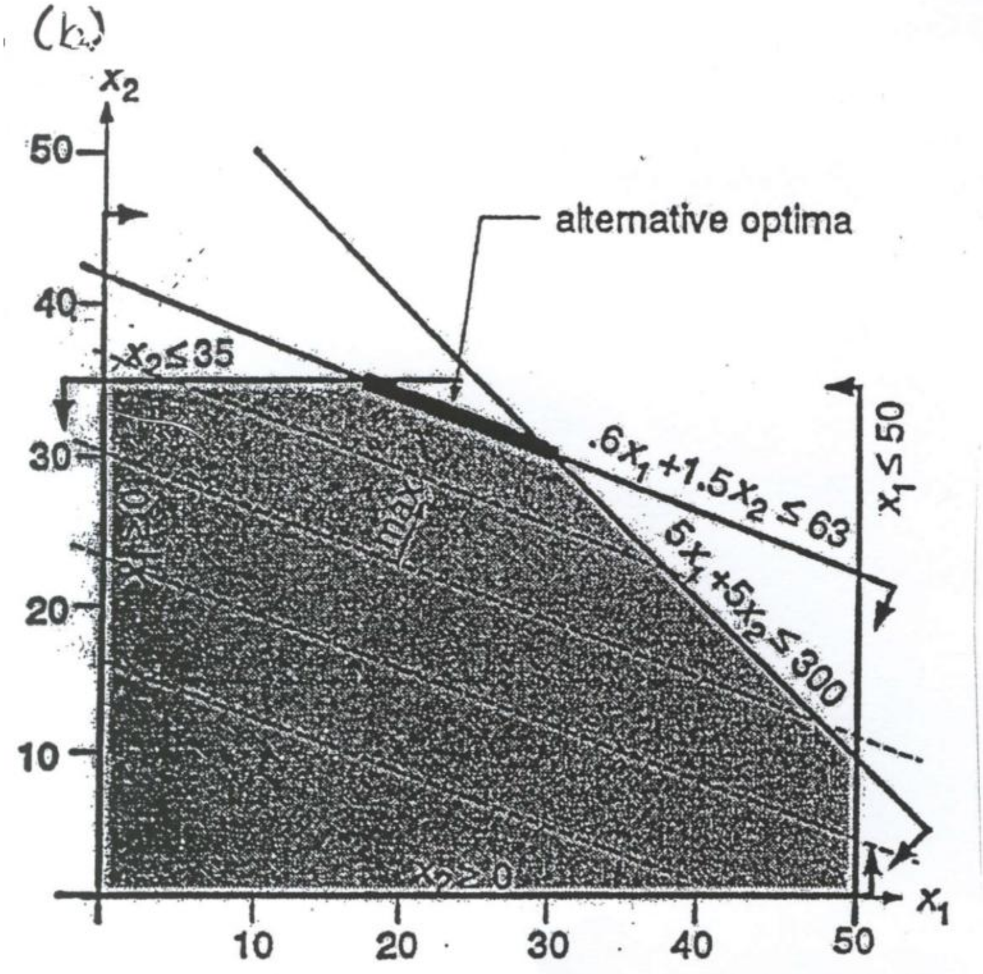


Optimal solution: $x_1=36$; $x_2=6$; Max Z=312.

Problem 4

(a) max $200x_1 + 350x_2$ (max total profit), s.t. $5x_1 + 5x_2 \leq 300$ (legs), $0.6x_1 + 1.5x_2 \leq 63$ (assembly hours), $x_1 \leq 50$ (wood tops), $x_2 \leq 35$ (glass tops), $x_1 \geq 0$, $x_2 \geq 0$

(b)



All optimal from $\mathbf{x} = (30, 30)$ to $\mathbf{x} = (17.5, 35)$.

Problem 5

max $.11x_1 + .17x_2$ (max total return),
s.t. $x_1 + x_2 \leq 12$ ($12 million investment),
$x_1 \leq 10$ (max $10 million domestic), $x_2 \leq 7$
(max $7 million foreign), $x_1 \geq .5x_2$ (domestic at
least half foreign), $x_2 \geq .5x_1$ (foreign at least half
domestic), $x_1 \geq 0$, $x_2 \geq 0$

Problem 6

min $.092x_4 + .112x_5 + .141x_6 + .420x_9 +$
$719x_{12}$ (min total cost),
s.t. $x_4 + x_5 + x_6 + x_9 + x_{12} = 16000$ (16000m
ine),
$279x_4 + .160x_5 + .120x_6 + .065x_9 + .039x_{12} \leq 1600$
(at most 1600 Ohms resistance),
$00175x_4 + .00130x_5 + .00161x_6 + .00095x_9 +$
$00048x_{12} \leq 8.5$ (at most 8.5 dBell attenuation),
$x_4, x_5, x_6, x_9, x_{12} \geq 0$

Problem 7

(a) We must decide what quantities to move from surplus sites to fulfill each need.
(b) $s_i \triangleq$ the supply available at i, $r_j \triangleq$ the quantity needed at j, $d_{i,j} \triangleq$ the distance from i to j.
(c) min $\sum_{i=1}^{4} \sum_{j=1}^{7} d_{i,j} x_{i,j}$
(c) $\sum_{j=1}^{7} x_{i,j} = s_i$, $i = 1, \ldots, 4$
(d) $\sum_{i=1}^{4} x_{i,j} = r_j$, $j = 1, \ldots, 7$

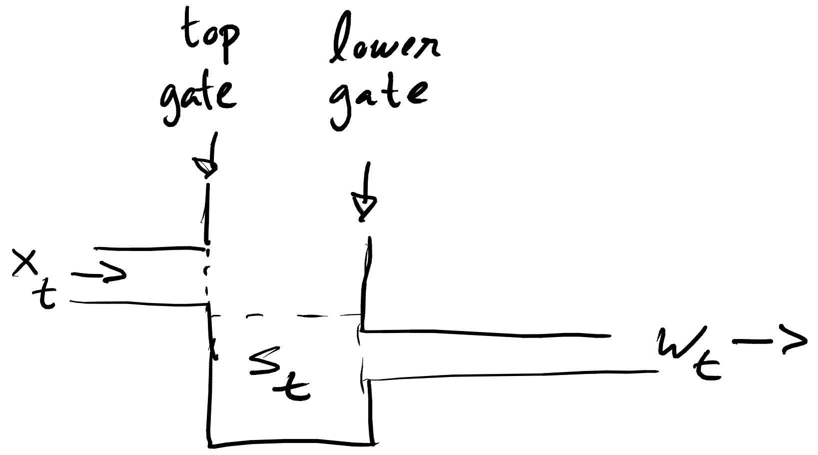
Problem 8

$\min \sum_{j=1}^{7} \left(\underline{c}_j + (t_j - \underline{t}_j)(\overline{c}_j - \underline{c}_j) / (\overline{t}_j - \underline{t}_j) \right)$ (min sum of interpolated task costs), s.t. $s_3 \geq s_2 + t_2$ (2 precedes 3); $s_4 \geq s_1 + t_1$ (1 precedes 4); $s_4 \geq s_2 + t_2$ (2 precedes 4); $s_5 \geq s_3 + t_3$ (3 precedes 5); $s_6 \geq s_3 + t_3$ (3 precedes 6); $s_7 \geq s_4 + t_4$ (4 precedes 7); $s_j + t_j \leq 40$, $j = 1, \ldots, 7$ (task j complete within 40 days); $s_j \geq 0$, $j = 1, \ldots, 7$; $t_j \geq 0$, $j = 1, \ldots, 7$; where $\underline{t}_j$ and $\overline{t}_j$ are the given min and max times for task j, and $\underline{c}_j$ and $\overline{c}_j$ are the corresponding min and max costs.

Problem 9

$\min \sum_{i=1}^{24} \sum_{m=1}^{8} \sum_{j=1}^{113} c_{i,m,j} x_{i,m,j}$ (min total cost), s.t. $\sum_{m=1}^{8} \sum_{j=1}^{113} x_{i,m,j} \leq s_i$, $i = 1, \ldots, 24$ (supply limit in mining region i); $\sum_{i=1}^{24} a_{i,m} x_{i,m,j} = d_{m,j}$, $m = 1, \ldots, 8$, $j = 1, \ldots, 113$ (demand for coal type m in region j); all variables nonnegative

Problem 10



R_t - outflow reference at time t

$$d_t^+ = \begin{cases} w_t - R_t & \text{if } w_t > R_t \\ 0 & \text{otherwise} \end{cases}$$

$$d_t^- = \begin{cases} R_t - w_t & \text{if } R_t > w_t \\ 0 & \text{otherwise} \end{cases}$$

$t = 1, \dots, 18$

$$\min_{x,w} \sum_{t=1}^{18} d_t^+ + d_t^- \leftarrow \text{total absolute deviation}$$

$$\sum_{t=1}^{18} w_t \geq \sum_{t=1}^{18} R_t \quad (3^{rd} \text{ sentence})$$

$$s_t \leq U, \quad t = 1, \dots, 18 \quad \left(\begin{matrix}\text{maximum} \\ \text{storage}\end{matrix}\right)$$

$$(s_t - s_{t-1}) = w_t - x_t, \quad t = 2, \dots, 18$$

$$s_1 - 120 = w_1 - x_1$$

(volume conservation)

↑ initial storage

Since we defined d_t^+ and d_t^- as nonlinear functions of w_t, we must replace w_t with the appropriate expression:

$$w_t = d_t^+ - d_t^- + R_t$$

The formulation then becomes:

$$\min \; d_t^+ + d_t^-$$

$$\sum_{t=1}^{18} (d_t^+ - d_t^-) \geq 0$$

$$s_t \leq U \;, \quad t = 1, \ldots, 18$$

$$(s_t - s_{t-1}) = d_t^+ - d_t^- + R_t - x_t, \quad t = 2, \ldots, 18$$

$$s_1 - 120 = d_1^+ - d_1^- + R_1 - x_1$$