Homework 2
Date due: Thursday, February 7, 2019

1. Problem 2.13 from **Ravindran** p.62: note that you are **not** asked to solve the problem, just to formulate it.

2. Problem 2.10 in **Ravindran**, p. 61.

3. Problem 2.15 in **Ravindran**, p. 63.

4. The *Notip Table Company* sells two models of its patented five-leg tables. The basic version uses a wood top, requires 0.6 hours to assemble, and sells for a profit of *$* 200. The deluxe model takes 1.5 hours to assemble because of its glass top, and sells for a profit of *$* 350. Over the next week the company has 300 legs, 50 wood tops, 35 glass tops and 63 hours of assembly available. *Notip* wishes to determine a maximum profit production plan assuming that everything produced can be sold.

(a) **Formulate** a mathematical programming model with 4 main constraints to determine an optimal production plan using decision variables $x_1 =$ number of basic models produced and $x_2 =$ number of deluxe models produced.

(b) On a two-dimensional plot, show the optimal solution if the profits are *$* 120 for the basic model and *$* 300 for the deluxe model.

5. *Wiley Wiz* is a mutual fund manager trying to decide how to divide up to *$* 12 million between domestic and foreign stocks. Domestic stocks have been returning 11 *%* per year and foreign stocks' return is 17 *%* per year. Naturally, *Wiley* would like to maximize the annual return from his investments. Still, he wants to exercise some caution. No more than *$* 10 million of the funds should go into domestic stocks and no more than *$*7 million into foreign. Also, at least half as much should be invested in foreign as domestic, and at least half as much in domestic as foreign to maintain some balance.

Formulate a mathematical programming model with 5 main constraints to determine *Wiley*'s optimal investment plan in terms of the decision variables:

$x_1 =$ million *$* invested in domestic stocks

$x_2 =$ million *$* invested in foreign stocks .

6. *Mexican Communications* is choosing cable for a new 16,000-meter telephone line. The table below shows the available diameters, along with the associated costs, resistance and attenuation of each. The company wishes to choose the least cost combination of wires that will provide a new line with at most 1600 *ohms* resistance and 8.5 decibels attenuation.

diameter (*d*) (0.1 *mm*)	costs (*$/m*)	resistance (*ohms/m*)	attenuation (*db/m*)
4	0.092	0.279	0.00175
5	0.112	0.160	0.00130
6	0.141	0.120	0.00161
9	0.420	0.065	0.00095
12	0.719	0.039	0.00048

Formulate a mathematical programming model with three main constraints to choose an optimal combination of wires using decision variables with

$x_d =$ meters of diameter d wire used $\qquad d = 4, 5, 6, 9, 12$

Assume the resistance and attenuation grow linearly with the length of the wire used.

7. A major expansion of the Brisbane airport will require moving substantial quantities of earth from 4 sites with surplus to 7 locations needing fill. The table shows the haul distances in hundreds of *m* between points, as well as the quantities of available fill in m^3 at the surplus sites. The site engineer wishes to compute a minimum total distance times volume plan for accomplishing the required earth moving.

(a) **Explain** why appropriate decision variables for a model of this problem are

$$x_{i,j} = \text{ volume in } m^3 \text{ moved from surplus site } i \text{ to need site } j \qquad \begin{matrix} i = 1, \dots, 4 \\ j = 1, \dots, 7 \end{matrix}$$

(b) **Assign** suitable symbolic names to the constants of the problem and **formulate** the objective function to minimize total distance times volume movement.

(c) **Formulate** a system of 4 main constraints, assuring that the full available amount is moved from each surplus site.

(d) **Formulate** a system of 7 main constraints, assuring that the required amount is moved to each need site.

need site	quantity (m^3)	surplus site			
		apron	terminal	cargo	Access
extension	247	22	28	20	26
dry pond	394	12	14	26	10
roads	265	10	12	20	4
parking	105	18	20	2	16
fire station	90	11	13	6	24
industrial park	85	8	10	22	14
perimeter road	145	20	22	18	21
quantity available (m^3)		660	301	271	99

8. A construction contractor has undertaken a job with 7 major tasks. Some of the tasks can begin at any time, but others have predecessors that must be completed first. The following table shows those predecessor task numbers, together with the minimum and maximum times (in days) allowed for each task, and the total costs that would be associated with accomplishing each task in its minimum and maximum times (more time usually saves expense).

j	minimum time	maximum time	cost at minimum	cost at maximum	predecessor tasks
1	6	12	1,600	1,000	None
2	8	16	2,400	1,800	None
3	16	24	2,900	2,000	2
4	14	20	1,900	1,300	1, 2
5	4	16	3,800	2,000	3
6	12	16	2,900	2,200	3
7	2	12	1,300	800	4

The contractor seeks a way to complete all the work in 40 days at the least total costs, assuming that the costs of each task are linearly interpolated for times between the minimum and maximum times. **Formulate** an *LP* model of this time/cost planning problem using the decision variables

$$\left.\begin{array}{l} s_j = \text{start time of task } j \\ t_j = \text{days to complete task } j \end{array}\right. \quad j = 1, \ldots, 7$$

Your model should have an objective function summing interpolated costs and maintain constraints to enforce precedence relationships and the time limit.

9. To assess the impact on the U.S. coal market of different pollution control strategies, the *Environmental Protection Agency* (*EPA*) wants to determine, for assumed control regimes, how much coal from supplies s_i in different mining regions $i = 1, \ldots, 24$ will be extracted, how much will then be processed into various deliverable coal types $m = 1, \ldots, 8$, and how much of each deliverable product will be sent to meet consumer demands $d_{m,j}$ in regions $j = 1, \ldots, 113$. Demands are expressed in *Btu*, with each ton of raw coal mined at i for processing into type m yielding $a_{i,m}$ *Btu*. Including the economic burden of pollution controls and transportation, the costs per ton of raw coal mined at i for processing into type m and use at j can be estimated at $c_{i,m,j}$ *$*. **Formulate**

an *LP* model to determine how coal would be mined, processed and distributed if the market seeks to minimize total costs. Use the decision variables

$x_{i,m,j}$ = tons of coal mined at region i for processing into type m and used at region j

$$i = 1, \dots, 24; \; m = 1, \dots, 8; \; j = 1, \dots, 113.$$

10. An Indian reservation irrigation project must decide how much water to release through the gate at the top of its main canal in each of the upcoming 4-hour periods $t = 1, \dots, 18$. The project relies on another gate at a lower part of the canal to provide control over the canal outflows. Ideal canal outflows are known for each time period and the total outflow over all 18 periods should equal or exceed the sum of these quantities. However, period-to-period deviations may be needed to avoid flooding. The initial canal storage is 120 units, and the net effect of releases and outflows should never leave more than U units stored after each period. Within these limits, managers would like to minimize the total absolute deviation between desired demands R_t and the actual outflows. **Formulate** an *LP* model of this irrigation control problem in terms of the following decision variables

x_t = gate release during period t
s_t = amount of water stored in the canal at the end of period t
w_t = canal outflow during period t $\qquad t = 1, \dots, 18$
d_t^+ = oversatisfaction of demand in period t
d_t^- = undersatisfaction of demand in period t.