HOMEWORK 11 SOLUTIONS

Problem 1

11.6. Use bracket medians for the brackets defined by the break points.

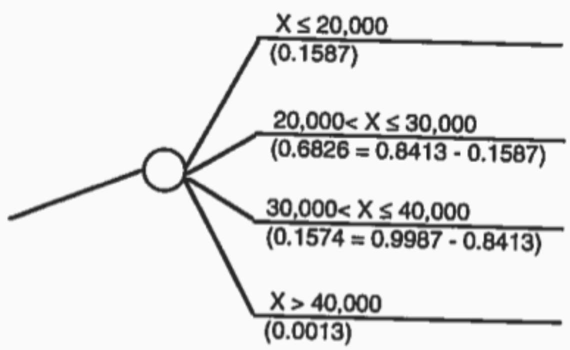
$P(X \leq 20{,}000) \mid \mu = 25{,}000, \sigma = 5000) = P(Z \leq -1) = 0.1587$

$P(X \leq 30{,}000) \mid \mu = 25{,}000, \sigma = 5000) = P(Z \leq 1) = 0.8413$

$P(X \leq 40{,}000) \mid \mu = 25{,}000, \sigma = 5000) = P(Z \leq 3) = 0.9987$

$P(X > 40{,}000) \mid \mu = 25{,}000, \sigma = 5000) = P(Z \leq -1) = 0.0013$

Thus, we have the following discrete chance node:



To find the representative *X*s for these branches, take the bracket medians. For the first bracket,

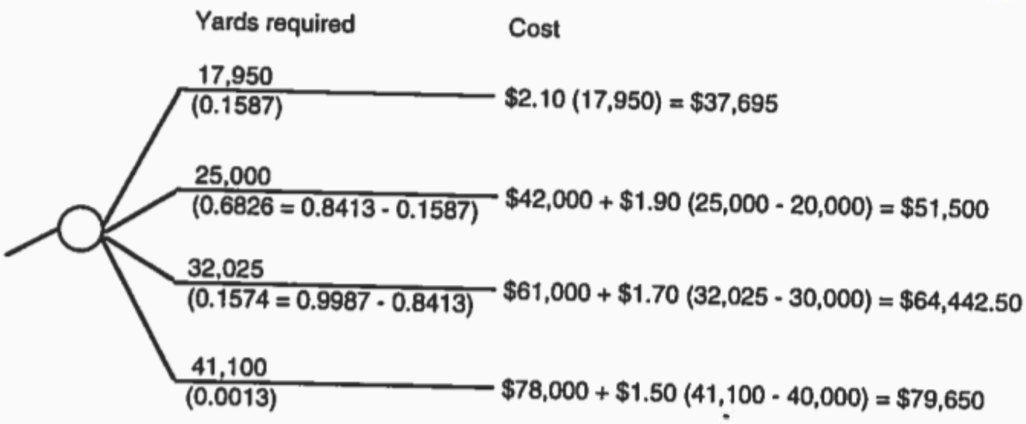
$$X_{(0 + 0.1587)/2} = X_{0.0794}.$$

We can find in Appendix E that $P(Z \leq -1.41) \approx 0.0794$. Therefore,

$$X_{0.0794} = 25{,}000 - 1.41(5000) = 17{,}950.$$

Likewise, we can calculate $X_{0.5000} = 25{,}000$; $X_{0.9200} = 32{,}025$; and $X_{0.99955} = 41{,}000$.

Using these representative points to calculate costs:



Rolling back this chance node, we obtain E(cost) = $51,383.

Problem 2

The process for simulating the call option is as follows:

1. Generate a normal random number (Stock price) with mean $45.50 and standard deviation $5.00.

2. Calculate the option value with the following IF statement:

   IF(Stock price < $45, 0, 100*(Stock price - $45.00))

   This statement checks the price of the stock. If the price is less than $45, then the option is worthless (0). If the price is greater than $45, then the value is 100 times the difference between the stock price and the exercise price.

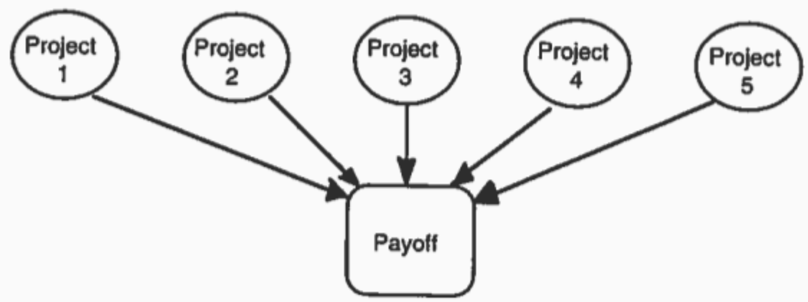
3. Repeat steps 1 and 2 many times.

4. With 1000 replications, I obtained E(Option value) = $221.89, standard deviation = $329.65.

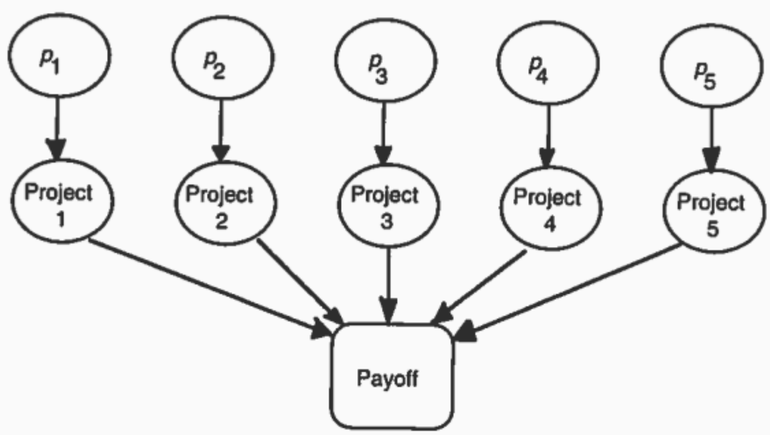
Problem 3

11.9. a. Each probability can be a random variable itself. For example, for Project 1, first generate p_1, its probability of success, from a uniform distribution that ranges from 0.45 to 0.55. Then, once p_1 is determined, determine whether Project 1 succeeds using p_1 as the probability of success. In influence diagram form:

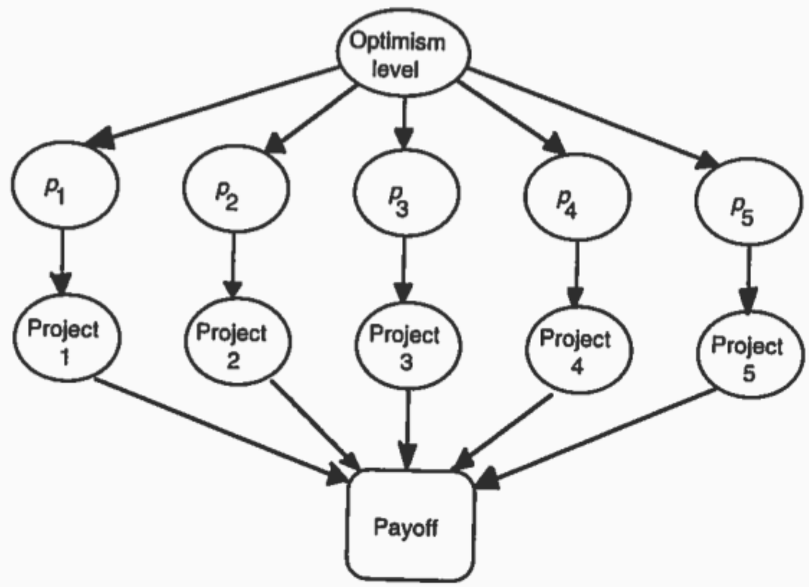
No uncertainty or vagueness about probabilities:



With uncertainty or vagueness about probabilities:



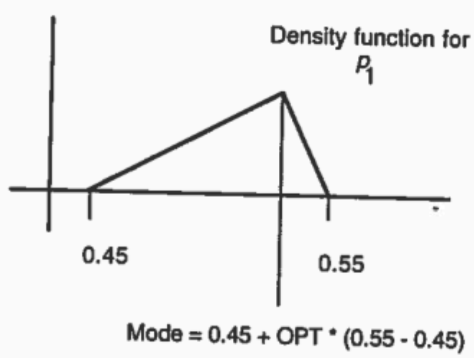
b. In part b, the probabilities are dependent. We can link them through an "optimism" node to represent the decision maker's unknown level of optimism/pessimism regarding the chance of success of the projects:



How could such a model be implemented? One possibility would be the following:

1. Generate an "optimism parameter" (OPT) according to a distribution. For example, OPT could come from a uniform distribution between 0 and 1. Of course, this distribution should be assessed by the decision maker.

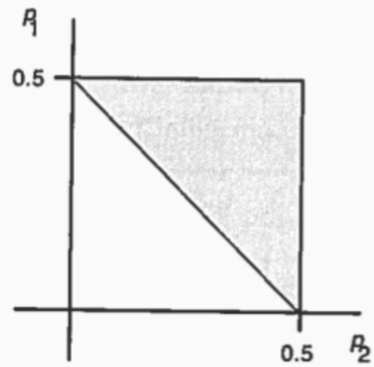
2. Now generate $p_1, \ldots, p_5$ from triangular distributions that have modes that are defined by OPT:



In this way, all of the distributions for the p_is will "lean" the same way, depending on the value of OPT. Thus, $p_1, \ldots, p_5$ all will be more likely to come from the upper (or lower) ends of their respective ranges.

Problem 4

11.10. a. A straightforward sensitivity analysis would calculate expected values with the probabilities set at various values. The results would be a triangular grid that could be plotted in terms of p_1 = P($10,000) and p_2 = P($5000):



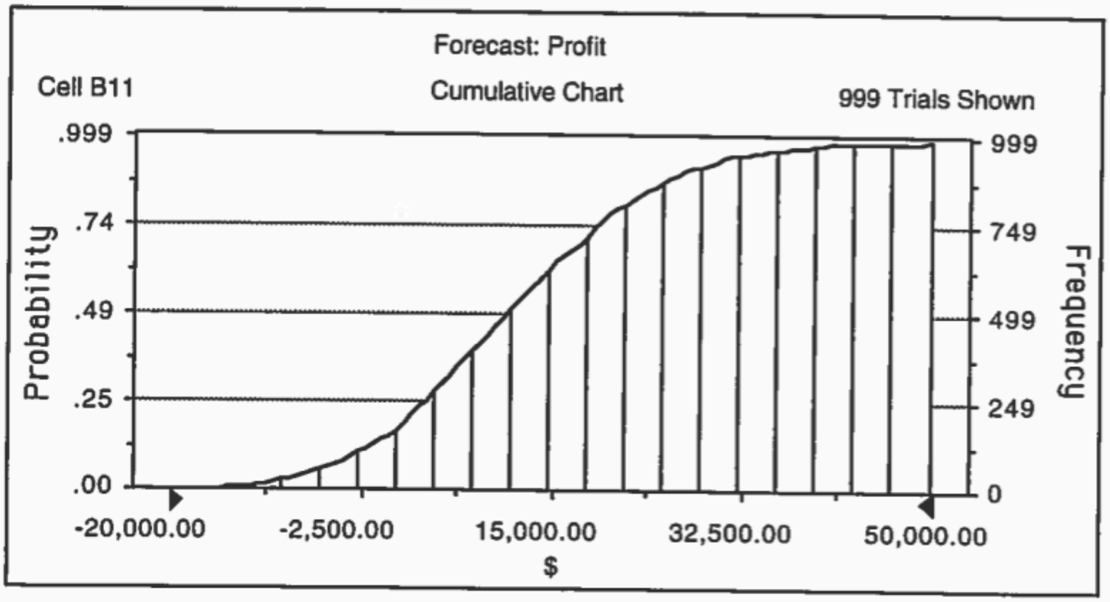
The expected values could be generated using simulation, but that is not necessary in this case.

To use simulation to incorporate the uncertainty about the probabilities, it would be possible to generate the probabilities randomly. For example, first, generate p_1 from a uniform distribution between 0 and 0.5. Second, generate p_2 from a uniform distribution between $(0.5 - p_1)$ and 0.5, and finally calculate $p_3 = 1 - p_1 - p_2$.

b. No, it would not be possible to have each of the three probabilities chosen from a uniform distribution between zero and one, because the three probabilities would never sum to one. Some kind of building-up process, like that described in part a, must be followed.

Problem 5

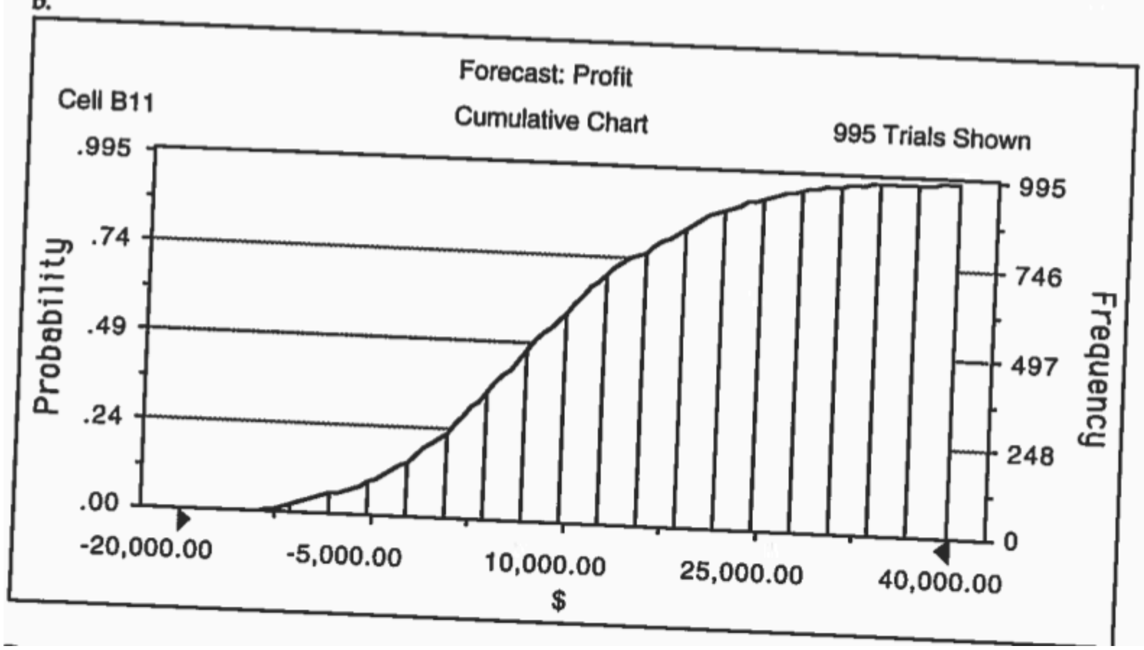
11.11. a. From Crystal Ball:



This risk profile has P(Profit ≤ 0) = 0.153. The fractiles are:

0.05	-6887
0.25	3593
0.50	10,951
0.75	19,142
0.95	31,147

b.

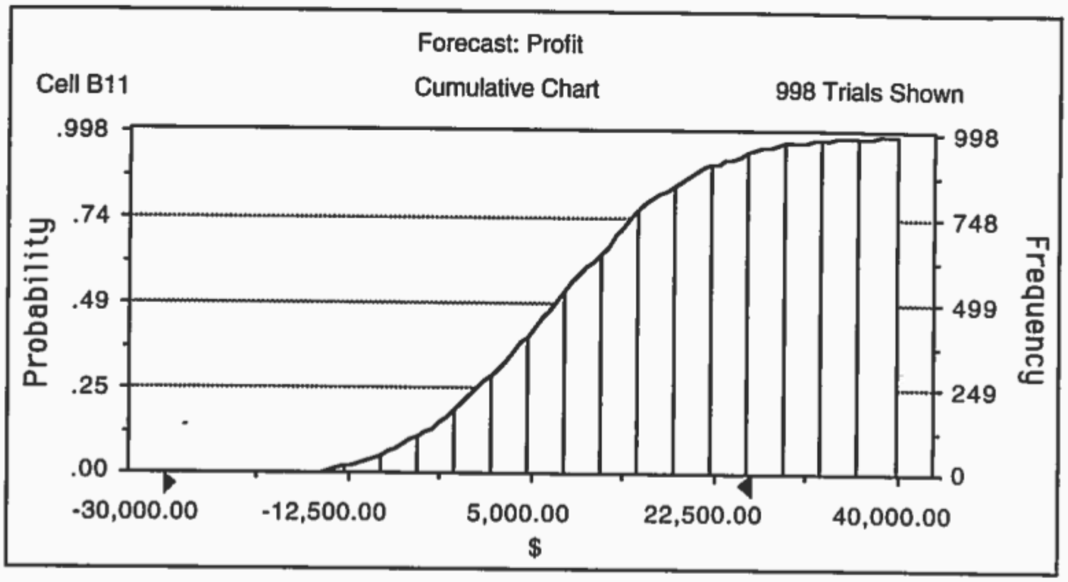


For this simulation, the risk profile has P(Profit $\leq$ 0) = 0.215. The fractiles are:

0.05	-8733
0.25	1182
0.50	7380
0.75	14,543
0.95	26,200

The distribution is similar, and most of the variability remains, reflecting the importance of the four variables included in the simulation. Thus, the results here confirm the sensitivity-analysis approach to variable selection discussed in Chapter 5. The distribution in part b is slightly lower than the one in part a, however, because we have eliminated some of the variables that have particularly skewed distributions.

c.



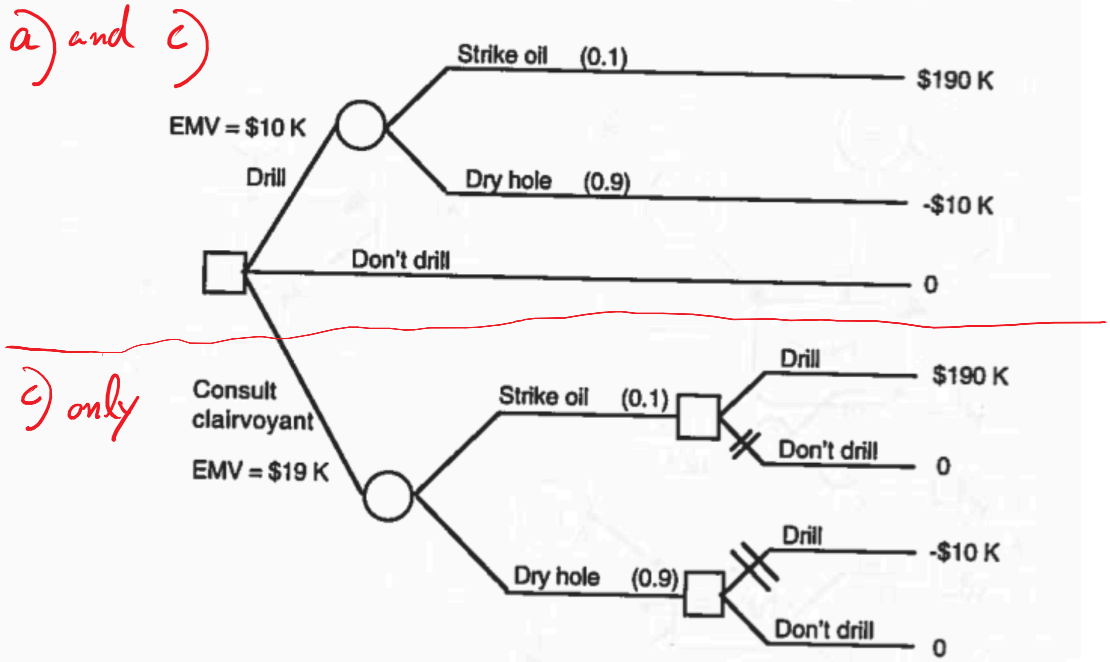
With Hours Flown and Capacity correlated (and still running the reduced simulation as in part b), the risk profile has P(Profit $\leq$ 0) = 0.256. The fractiles are:

0.05	-9467
0.25	-56
0.50	7495
0.75	14,625
0.95	26,000

This distribution is very similar to the one from part b. Adding the correlations has not changed the results dramatically. Part of the reason for this is that the two variables offset each other. For example, as Capacity increases (adding revenue), Hours Flown increases (adding to cost). Likewise, as capacity (and hence revenue) decreases, the amount of Hours Flown also decreases, saving on cost.

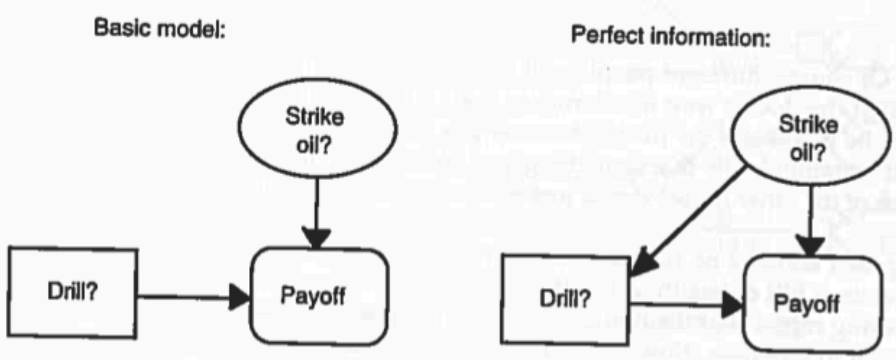
Problem 6

12.7. The decision tree for parts a and c:



a. The expected value of drilling is $10 K, versus $0 for not drilling.

b.



c. See the decision tree above.

EVPI = EMV(Clairvoyant) - EMV(Drill) = $19 K - $10 K = $9 K.

d. We have

$$P(\text{"good"} \mid \text{oil}) = 0.95 \qquad P(\text{oil}) = 0.1$$
$$P(\text{"poor"} \mid \text{dry}) = 0.85 \qquad P(\text{dry}) = 0.9$$

We can find P("good") and P("poor") with the law of total probability:

$$P(\text{"good"}) = P(\text{"good"} \mid \text{oil})\,P(\text{oil}) + P(\text{"good"} \mid \text{dry})\,P(\text{dry})$$

$$= 0.95\,(0.1) + 0.15\,(0.9)$$

$$= 0.23$$

$$P(\text{"poor"}) = 1 - P(\text{"good"})$$

$$= 1 - 0.23$$

$$= 0.77.$$

Now we can find

$$P(\text{oil} \mid \text{"good"}) = \frac{P(\text{"good"} \mid \text{oil})\,P(\text{oil})}{P(\text{"good"} \mid \text{oil})\,P(\text{oil}) + P(\text{"good"} \mid \text{dry})\,P(\text{dry})}$$

$$= \frac{0.95\,(0.1)}{0.95\,(0.1) + 0.15\,(0.9)}$$

$$= 0.41$$

$$P(\text{dry} \mid \text{"good"}) = 1 - P(\text{oil} \mid \text{"good"})$$

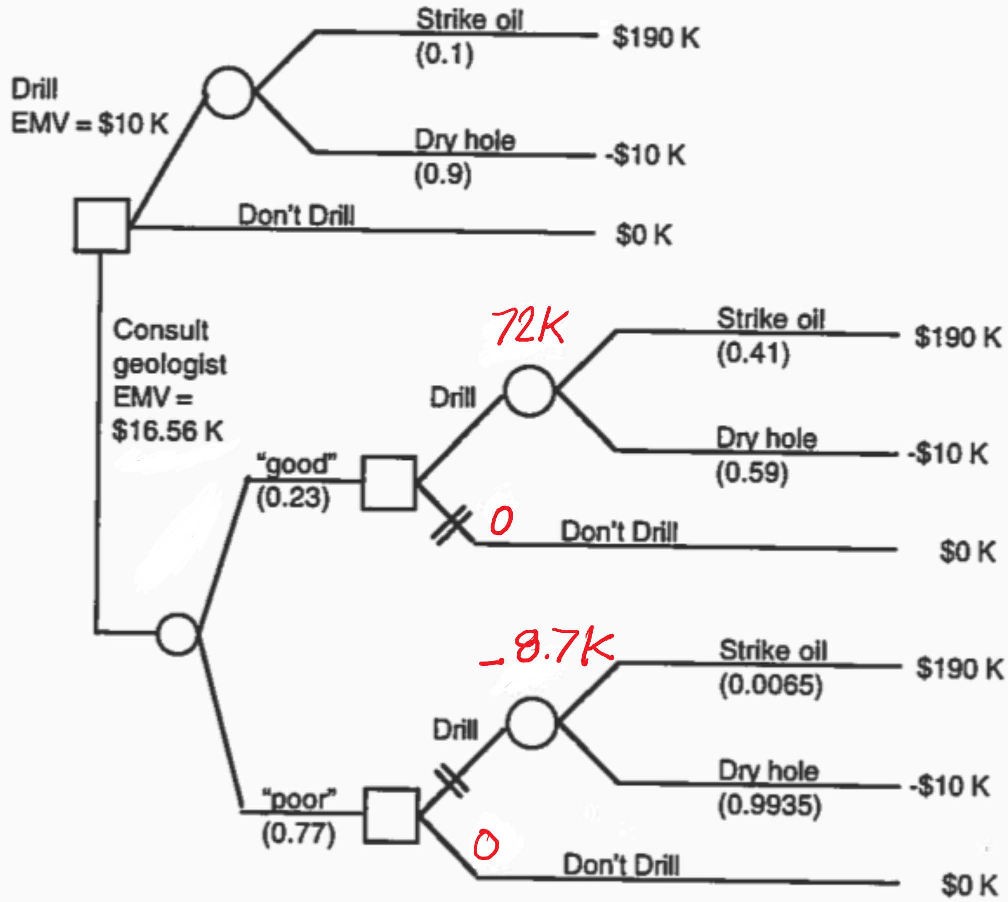
$$= 0.59$$

$$P(\text{oil} \mid \text{"poor"}) = \frac{P(\text{"poor"} \mid \text{oil})\,P(\text{oil})}{P(\text{"poor"} \mid \text{oil})\,P(\text{oil}) + P(\text{"poor"} \mid \text{dry})\,P(\text{dry})}$$

$$= \frac{0.05\,(0.1)}{0.05\,(0.1) + 0.85\,(0.9)}$$

$$= 0.0065$$

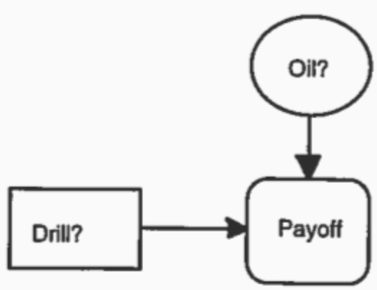
P(dry | "poor") = 0.9935.Now the decision tree



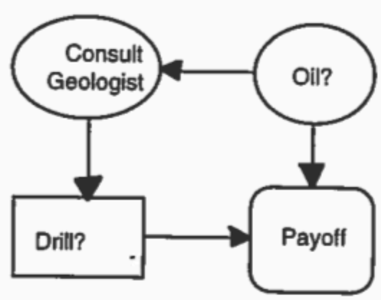
is:

The influence diagram solution:

Without Geologist:



With Geologist:

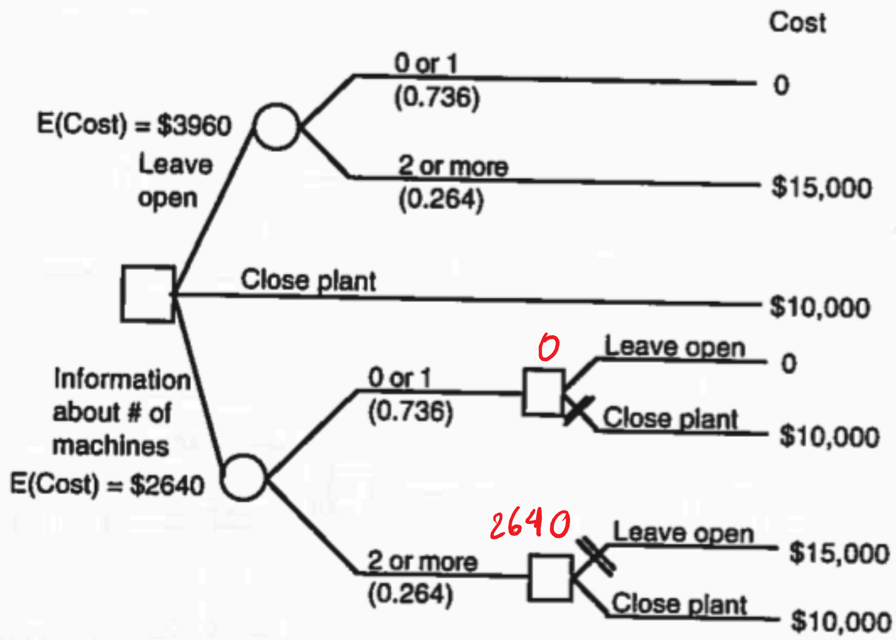


EVII = EMV(Consult Geologist) - EMV(Drill) = $16.56 K - $10 K = $6.56 K.

Because EVII is less than $7000, which the geologist would charge, this is a case where the expected value of the geologist's information is less than what it would cost. Don't consult her.

Problem 7

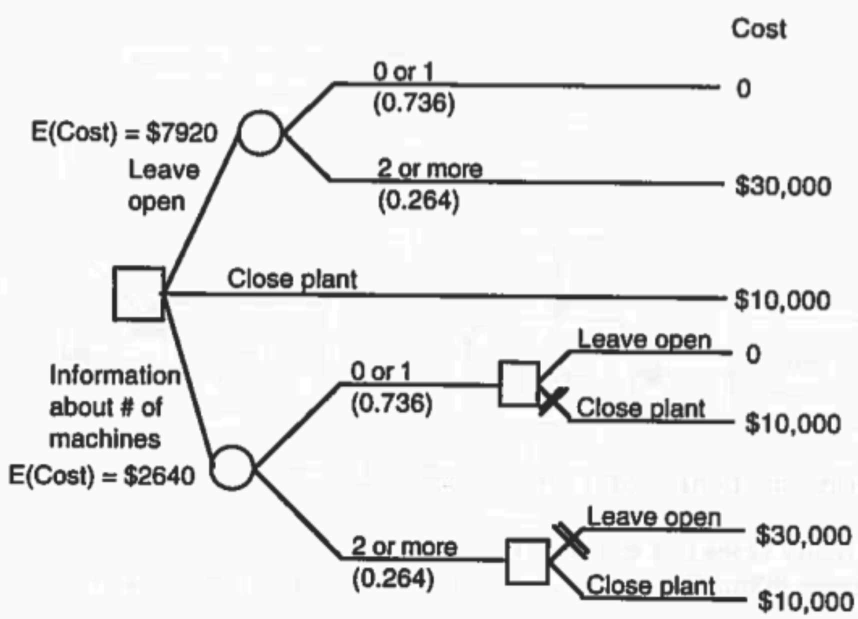
a.



Note that because we are minimizing cost in this problem, we need to find the expected cost savings due to the information. For that reason, the formula for EVPI appears reversed:

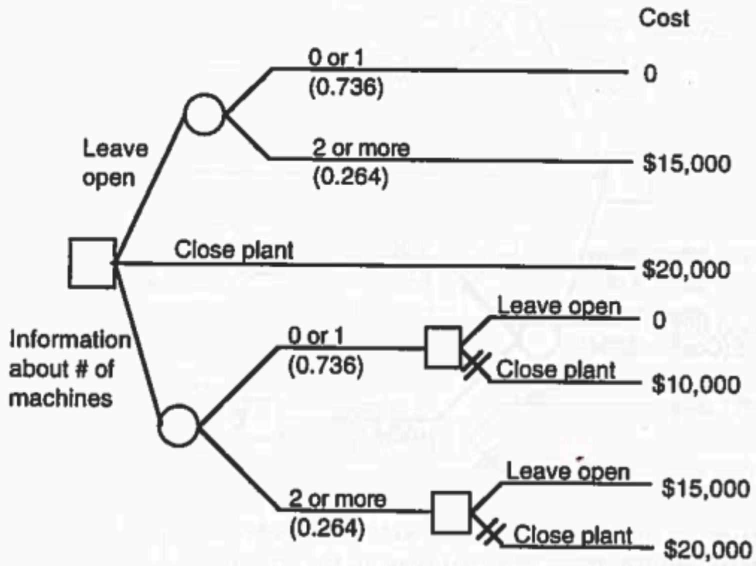
EVPI = E(Cost for leaving plant open) - E(Cost for information) = $3960 - $2640 = $1320.

b.



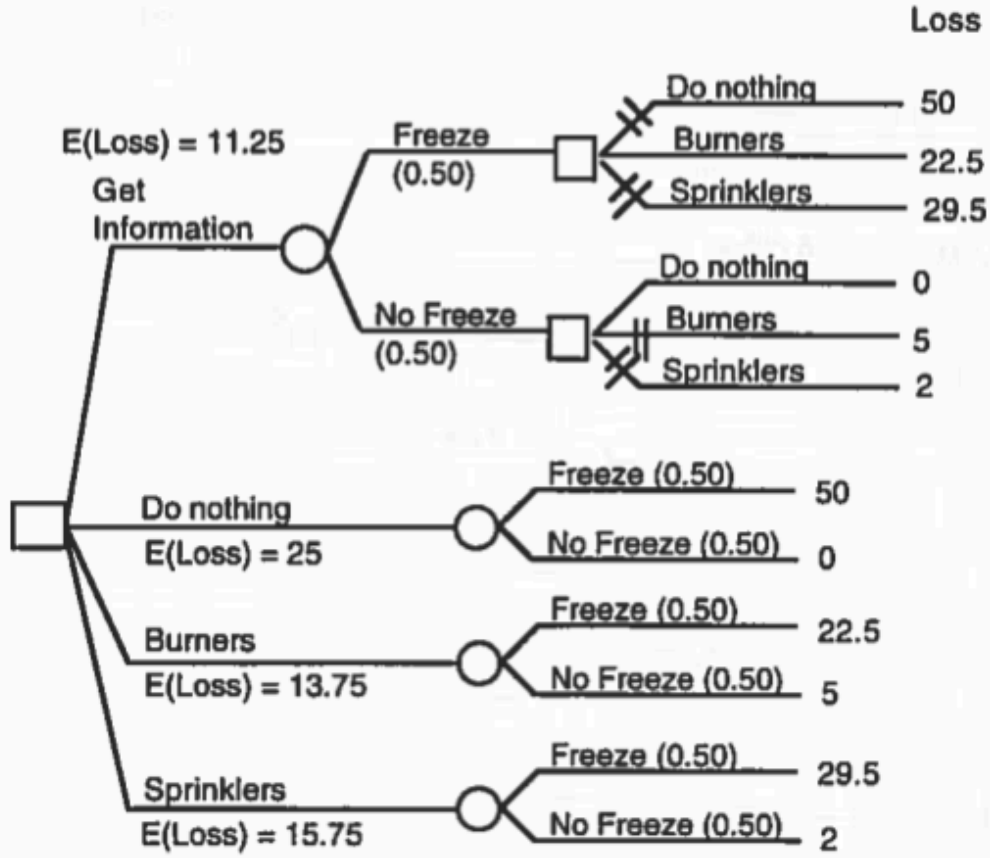
EVPI = E(Cost for leaving plant open) - E(Cost for information) = \$7920 - \$2640 = \$5280.

c.

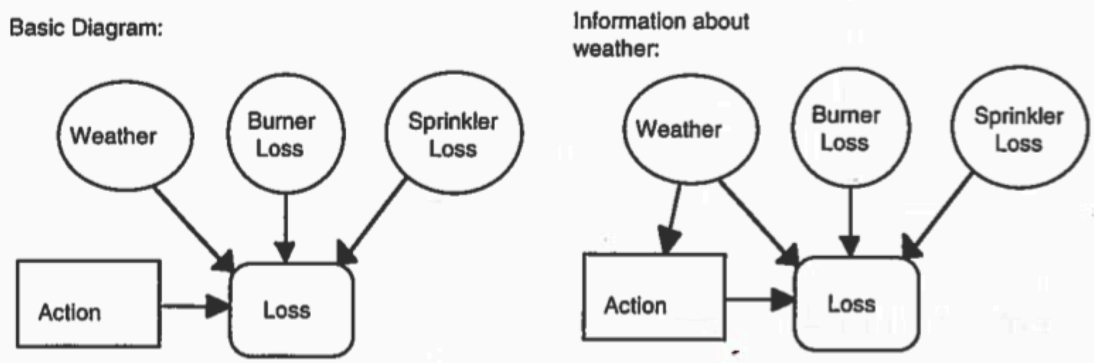


Now EVPI = 0 because leaving the plant open deterministically dominates closing the plant. The manager would leave the plant open regardless of the number of broken machines.

Problem 8



Influence diagram:



EVPI = E(Loss | Burner) - E(Loss | Information) = 13.75 - 11.25 = $2.5 K = $2500.