Homework 10 Solutions

Problem 1

9.13. Because of independence among arrivals, the probability distribution for arrivals over the next 15 minutes is independent of how many arrived previously. Thus, for both questions,

$P_P(X = 1 \text{ in 15 minutes} \mid m = 6 \text{ per hour})$

$= P_P(X = 1 \text{ in 15 minutes} \mid m = 1.5 \text{ per 15 minutes})$

$= 0.335$ from Appendix C.

Problem 2

9.14. a. $E(T_A) = 5$ years and $E(T_B) = 10$ years.

b. $P(T_A \geq 5 \mid m = 0.2) = e^{-5(0.2)} = 0.368$

$P(T_B \geq 10 \mid m = 0.1) = e^{-10(0.1)} = 0.368.$

For exponential random variables, the probability is 0.368 that the random variable will exceed its expected value.

c.i. Average lifetime $= 0.5\,(T_A) + 0.5\,(T_B) = \overline{T}$.

$E(\overline{T}) = 0.5\,(5) + 0.5\,(10) = 7.5.$

$Var(\overline{T}) = 0.5^2\,(25) + 0.5^2\,(100) = 31.25.$

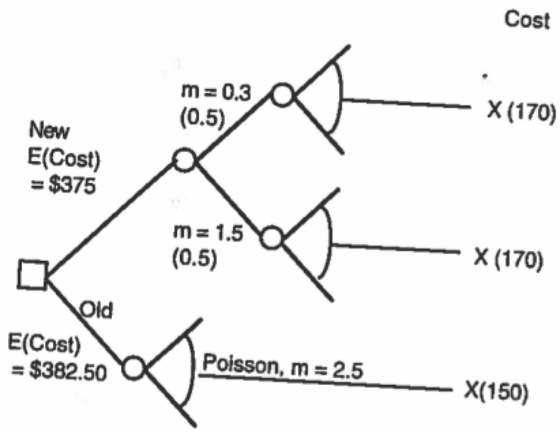
ii. Difference $\Delta T = T_B - T_A$

$E(\Delta T) = E(T_B) - E(T_A) = 10 - 5 = 5$

$Var(\Delta T) = 1^2\,Var(T_B) + (-1)^2\,Var(T_A) = 25 + 100 = 125.$

Problem 3

a.



The older machines appear to have the edge. They have both lower expected cost and less uncertainty.

b. Find P(m = 1.5 per month | X = 6 in 3 months) = P(m = 4.5 per 3 months | X = 6 in 3 months):

P(m = 4.5 per 3 months | X = 6 in 3 months)

$$= \frac{P(X = 6 \mid m = 4.5)\ P(m = 4.5 \text{ per 3 months})}{P(X = 6 \mid m = 4.5)\ P(m = 4.5\ ) + P(X = 6 \mid m = 9)\ P(m = 9)}$$

$$= \frac{P_P(X = 6 \mid m = 4.5)\ 0.5}{P_P(X = 6 \mid m = 4.5)\ 0.5 + P_P(X = 6 \mid m = 9)\ 0.5}$$

$$= \frac{0.128\ (0.50)}{0.128\ (0.50) + 0.091\ (0.50)} = 0.5845.$$

c. Now the expected cost of the new machines would be

E(Cost) = 0.5845 ($255) + 0.4155 ($510) = $360.96.

This is less than $375, so the new information suggests that the new machines would be the better choice.

Problem 4

9.24. a. The assessments give $P(Q < 0.08) = P(Q > 0.22) = 0.10$ and $P(Q < 0.14) = 0.50$. Therefore,

$$P(0.08 < Q < 0.14) = P(Q < 0.14) - P(Q < 0.08) = 0.40.$$

Likewise,

$$P(0.14 < Q < 0.22) = P(Q < 0.22) - P(Q < 0.14) = 0.40.$$

b. $n = 40, r = 6.$

$$P_\beta(Q \le 0.0829 \mid r = 6, n = 40) = 0.10$$
$$P_\beta(Q \le 0.2249 \mid r = 6, n = 40) = 0.90$$
$$P_\beta(Q \le 0.1441 \mid r = 6, n = 40) = 0.50$$

c. $E(Q) = \frac{6}{40} = 0.15$. However, there is more than a 50% chance that market share will be less than 0.15.

Problem 5

9.26. Let X denote return (in per cent), M = McDonalds, and S = US Steel. We have prior probability $P(M) = 0.80$.

a. $P(6 < X < 18 \mid M) = P_N(6 < X < 18 \mid \mu = 14, \sigma = 4)$

$$= P\left(\frac{6 - 14}{4} < Z < \frac{18 - 14}{4}\right) = P(-2 < Z < 1) = 0.8185.$$

$P(6 < X < 18 \mid S) = P_N(6 < X < 18 \mid \mu = 12, \sigma = 3)$

$$= P\left(\frac{6 - 12}{3} < Z < \frac{18 - 12}{3}\right) = P(-2 < Z < 2) = 0.9544.$$

b. $P(6 < X < 18) = P(6 < X < 18 \mid M)\,P(M) + P(6 < X < 18 \mid S)\,P(S)$

$= 0.8185\ (0.8) + 0.9544\ (0.2) = 0.84568.$

c. $P(X > 12 \mid M) = P_N(X > 12 \mid \mu = 14, \sigma = 4) = P\left(Z > \frac{12 - 14}{4}\right) = P(Z > -0.5) = 0.6915.$

$P(X > 12 \mid S) = P_N(X > 12 \mid \mu = 12, \sigma = 3) = P\left(Z > \frac{12 - 12}{3}\right) = P(Z > 0) = 0.5.$

$$P(M \mid X > 12) = \frac{P(X > 12 \mid M)\,P(M)}{P(X > 12 \mid M)\,P(M) + P(X > 12 \mid S)\,P(S)}$$

$$= \frac{0.6915\ (0.8)}{0.6915\ (0.8) + 0.5\ (0.2)} = 0.847.$$

d. $E(\text{Return}) = 0.5\,E(X \mid M) + 0.5\,E(X \mid S) = 0.5\ (14) + 0.5\ (12) = 13\%.$

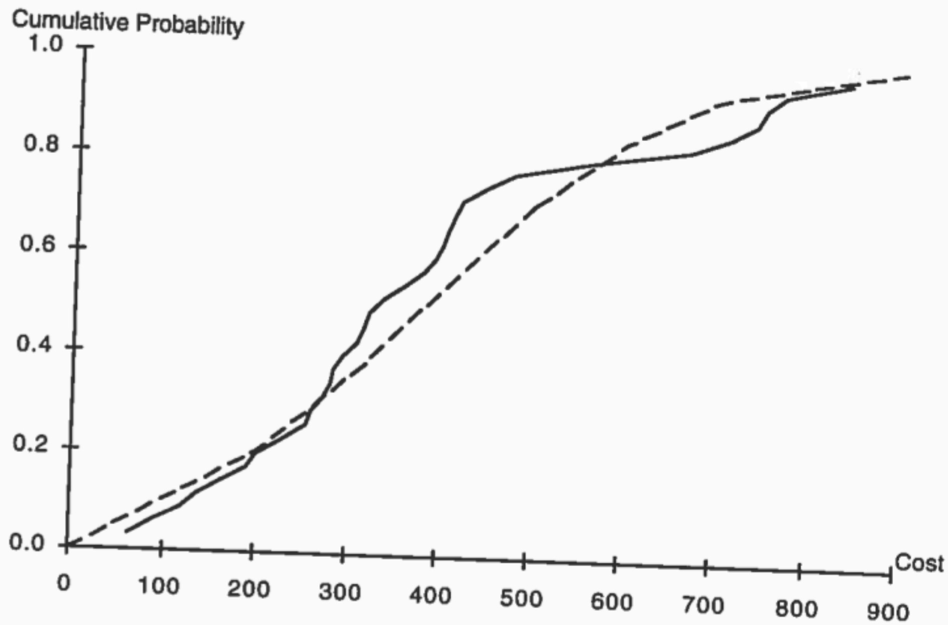
$\text{Var}(\text{Return}) = 0.5^2\,\text{Var}(X \mid M) + 0.5^2\,\text{Var}(X \mid S) = 0.25\ (4^2) + 0.25\ (3^2) = 6.25.$

Problem 6

10.8. Using the sample mean and standard deviation, we can calculate equivalent z values. For example, the z that is equivalent to 200 is $z = (200 - 380.4)/217.6 = -0.8$. Then from the standard normal table, we have $P(Z \le -0.8) = 0.203$. This is compared with the data-based cumulative probability $P(X \le 200) = 0.20$. A table might look like this:

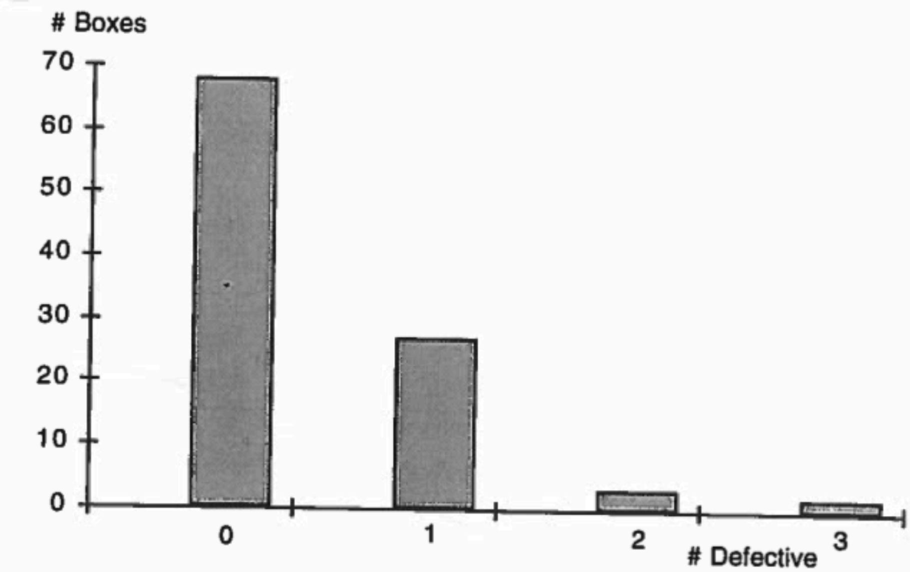
	z	P(Z ≤ z)	Data
200	-0.8	0.203	0.20
300	-0.4	0.356	0.41
400	0.1	0.536	0.63
500	0.5	0.709	0.78
600	1.0	0.844	0.81
700	1.5	0.929	0.85

The normal and data-based probabilities do not appear to be very different. The two cumulative distributions can be compared graphically as well:



Problem 7

10.10. a.



b, c. A binomial distribution with $n = 12$ and $p = 39/1200$ would be a reasonable distribution. The total number of bulbs checked is 1200, and 39 were found defective. Another possibility would be a Poisson distribution with $m = 0.4$.

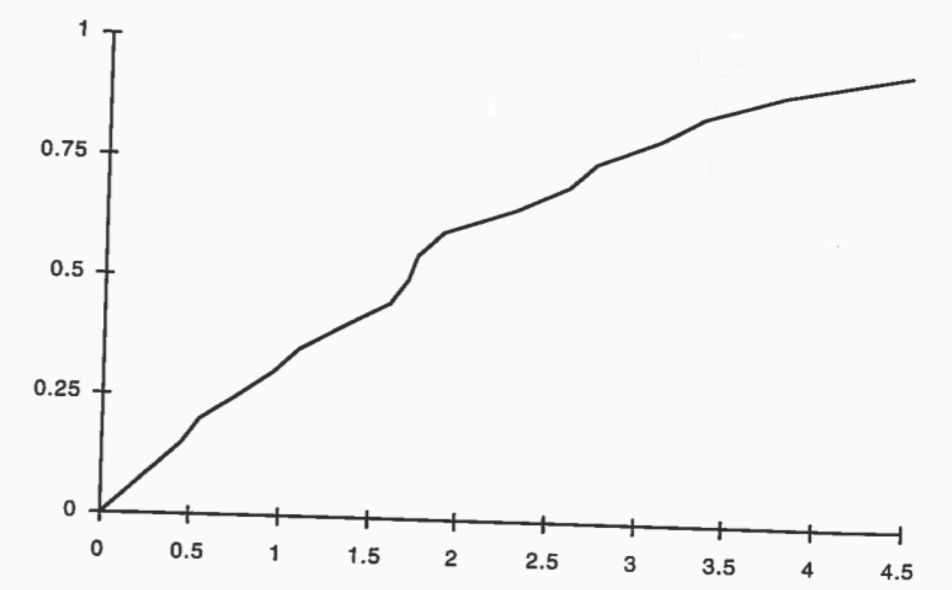
Sample mean = 0.39, sample variance = 0.4179.

	# Boxes	Relative Frequency	PROBABILITIES Binomial p=39/1200 = 0.0325	Poisson m=12*39/1200 = 0.4
0	68	0.68	0.67	0.67
1	27	0.27	0.27	0.27
2	3	0.03	0.05	0.05
3	2	0.02	0.01	0.01

Problem 8

10.11. a.

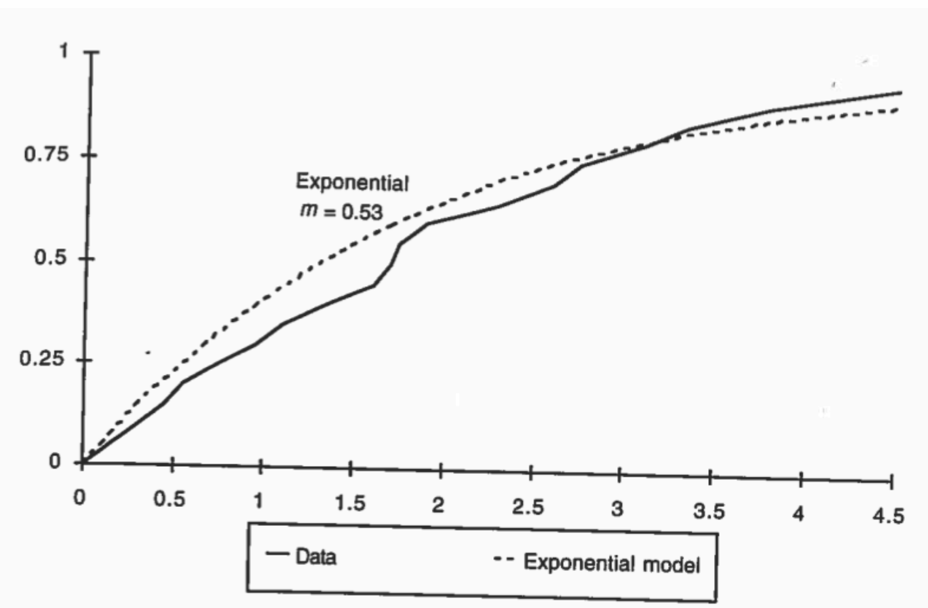
	Time	x_i	P(Time < x_i)
		0	0
1	0.1	0.15	0.05
2	0.2	0.30	0.10
3	0.4	0.45	0.15
4	0.5	0.55	0.20
5	0.6	0.75	0.25
6	0.9	0.95	0.30
7	1.0	1.10	0.35
8	1.2	1.35	0.40
9	1.5	1.60	0.45
10	1.7	1.70	0.50
11	1.7	1.75	0.55
12	1.8	1.90	0.60
13	2.0	2.30	0.65
14	2.6	2.60	0.70
15	2.6	2.75	0.75
16	2.9	3.10	0.80
17	3.3	3.35	0.85
18	3.4	3.80	0.90
19	4.2	4.50	0.95
20	4.8		



b. An exponential distribution often is used to model times between arrivals, especially when the arrivals are independent (in this case, no groups of shoppers arriving at once).

c. Sample mean = 1.87 minutes, sample standard deviation = 1.35 minutes. Thus, we will use an exponential distribution with mean $m = \frac{1}{1.87} = 0.53$. (Or $\frac{1}{1.35} = 0.74$ if you would rather use the standard deviation.)

d.



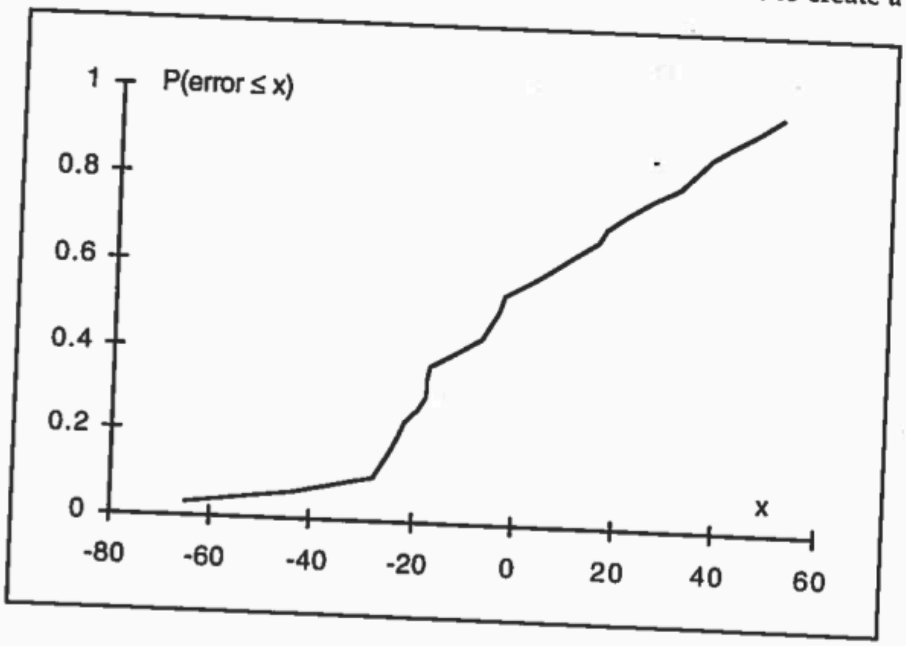
Problem 9

10.13. a. The regression equation is:

E(Sales Price | House Size, Lot Size, Attractiveness)

= 15.04 + 0.0854 (House Size) + 20.82 (Lot Size) + 2.83 (Attractiveness).

b. Creating this graph requires calculation of the residuals from the regression (which can be done automatically with Excel's regression procedure). Then use the residuals in exactly the same way that we used the halfway-house data in Table 10.2 in the text to create a CDF.



c. The expected Sales Prices for the two properties are

1. E(Sales Price | House Size = 2700, Lot Size = 1.6, Attractiveness = 75)

   = 15.04 + 0.0854 (2700) + 20.82 (1.6) + 2.83 (75)

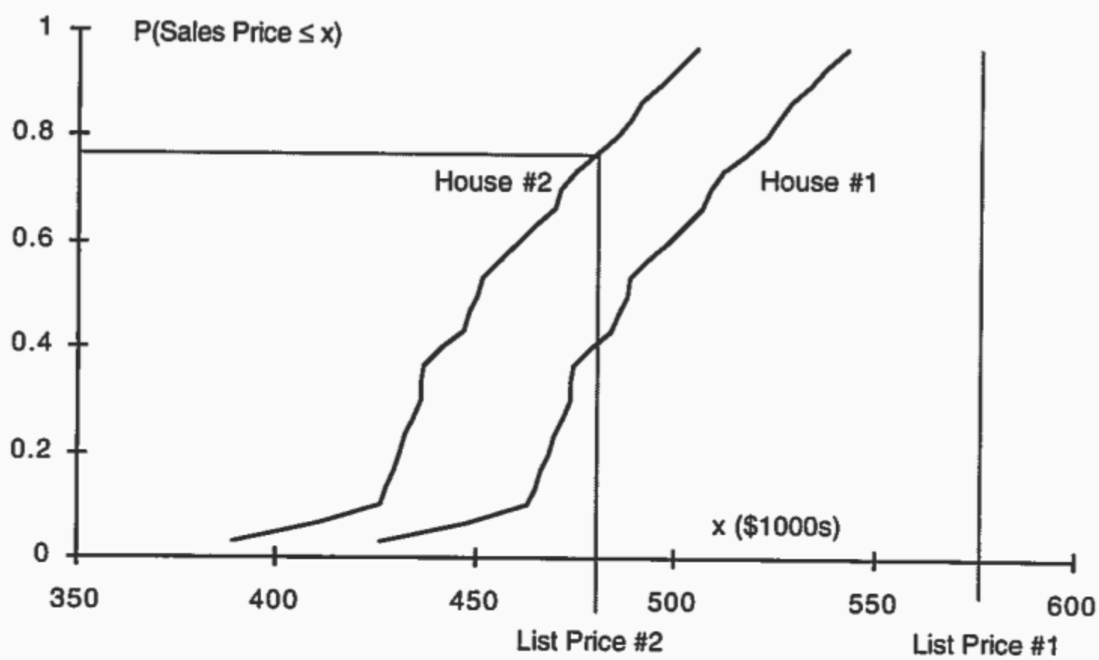
   = 491 ($1000s).

2. E(Sales Price | House Size = 2000, Lot Size = 2.0, Attractiveness = 80)

   = 15.04 + 0.0854 (2000) + 20.82 (2.0) + 2.83 (80)

   = 453.7 ($1000s).

d.



From the graph, it is easy to see that House #2 is reasonably priced, according to the model. At $480K, its list price falls just below the 0.80 fractile of the distribution. Presuming that the owners have built in some negotiating room, it looks as though Sandy may be able to make a reasonable offer and obtain the property for a price close to the price suggested by the model.

House #1 is a different story. Its list price falls well above the probability distribution given by the model. It is either way overpriced (which suggests that Sandy may have to offer a very low price), or there is something about the house that increases its value but is not reflected in the model.