16. Forward Contracts

George Gross
Department of Electrical and Computer Engineering
University of Illinois at Urbana–Champaign
RISK

- There are many definitions of risk; we use the conceptual definition from Webster's dictionary that *risk is the possibility of suffering loss*

- People measure risk with a wide variety of specific metrics

- A rational market player aims to minimize the risks faced
Market players use specific financial tools to keep their risks below a specified threshold. Such actions constitute risk management and are carried out with financial instruments called risk management tools. Financial derivatives are some of the most widely-used risk management tools in financial markets.
FINANCIAL DERIVATIVES

- Basic definition: a derivative is a financial tool whose value depends on the value of other, more basic underlying variables

- The basic derivatives we examine are
  - forward contracts
  - future contracts
  - options
    - puts
    - calls
EXAMPLE: A FLOUR CONTRACT

A farmer in Illinois and a restaurant in Wisconsin enter into a contract on January 1, 2015, under which the farmer agrees to sell 1 ton of flour for $400 to the restaurant on September 1, 2015.

The contract involves two parties:

- the farmer is the *issuer* of the contract and holds a *short position*
- the restaurant is the *holder* of the contract and has a *long position*
EXAMPLE: A FLOUR CONTRACT

- The contract is signed on January 1 for the actual sale that occurs on September 1 and
  - we call January 1 the *initial time* of the contract and denote it by $t = 0$, the origin of the time line;
  - we call September 1 the *maturity* of the contract and denote it by $t = T$

Initial time | Maturity  
---|---
$t = 0$ | $t = T$

Life period of the contract
EXAMPLE: A FLOUR CONTRACT

- This contract is on the trading of a single specified commodity – the flour; we call the 1 ton of flour the underlying asset.

- The contract provides the holder with:
  - the delivery of the underlying asset at \( T \)
  - the fixed price \( K \) for the asset – the so-called delivery price.
EXAMPLE: A FLOUR CONTRACT

- In the absence of a forward contract, the restaurant needs to buy the flour from the spot market at an *uncertain price* to meet its needs; this price can be high so that the restaurant bears *price risks*.

- With the forward contract, the price is fixed and known and, therefore, the holder is protected from price risks.

- This forward contract is a *physical contract* since the actual delivery of the asset is involved.
EXAMPLE: A FLOUR CONTRACT

Next, we assume the existence of a \textit{spot market price} \( s_T \) for flour at time \( T \) so that one can buy or sell the flour at that \textit{spot price} on the September 1 date.

The \textit{flour forward contract} may also be signed as a \textit{purely financial contract} with the flour commodity as the \textit{underlying asset}, September 1, 2015 as the maturity time \( T \) and the specification of the set of following payments:
EXAMPLE: A FLOUR CONTRACT

○ if \( s_T > K \), the issuer reimburses the holder the difference \( s_T - K \) to purchase the flour at \( s_T \)

○ if \( s_T < K \), the holder must pay to the issuer in the amount of \( K - s_T \) and buys flour at \( s_T \)

These specified payments constitute the *payoffs* of the financial contract

Thus, the *net price* to the holder is the *delivery price* \( K \) and is independent of the market spot price \( s_T \)
EXAMPLE: A FLOUR CONTRACT

- Since the issuer can sell the flour in the spot market, its net price also equals $K$

- Therefore, this purely financial contract provides precisely the same function as the physical contract to both the issuer and the holder but involves no actual delivery.

- Typically, many forwards are purely financial contracts that do not involve physical deliverability.
FORWARD CONTRACTS

- A forward contract is a **binding** agreement to buy or sell an **asset** at the designated **future time** at the specified price.

- An **asset** is a general term for any good, service or commodity.

- The buyer – holder – is said to hold a **long position** and the seller – issuer – holds a **short position**.

- The specified price is called the **delivery price**.
FORWARD CONTRACTS

- A forward contract is *settled at maturity* – the designated future time at which the purchase/sale is consummated.

- The *short position delivers the asset to the long position* in return for the cash payment in the amount of the *delivery price times the contract amount*.

- The value of the forward contract is a function of the *market price of the asset and its maturity*.
FORWARD VALUE AND PRICE

- The value of a forward contract is $0$ for both the short and the long positions at the time the contract is signed; thereafter, its value may be positive, $\theta$, or negative, depending on market conditions.
The forward price of a forward contract is the delivery price that makes the forward contract have 0 value at the time the contract is signed.

By definition, the forward price equals the delivery price at the time the contract is signed; thereafter, the delivery price $K$ remains fixed while the forward price may change as a function of the market price and the maturity of the contract.
THE *LONG POSITION* FORWARD CONTRACT *PAYOFF*

- **payoff**
  - \( s_T - K < 0 \)
  - \( s_T - K > 0 \)

- **asset market price**
  - delivery price
  - \( K \)
  - \( s_T \)

- **asset market price at maturity**
  - \( s_T - K > 0 \)
THE SHORT POSITION FORWARD CONTRACT PAYOFF

$K - s_T > 0$

delivery price

$K < 0$

asset market price at maturity

$s_T - K < 0$

delivery price

exceeds market price
# EXAMPLE: FOREIGN EXCHANGE

May 8, 1995 spot and forward foreign exchange for *British £* and *US $*

<table>
<thead>
<tr>
<th>foreign exchange</th>
<th>price in US $</th>
</tr>
</thead>
<tbody>
<tr>
<td>spot</td>
<td>1.6080</td>
</tr>
<tr>
<td>30 – day forward</td>
<td>1.6076</td>
</tr>
<tr>
<td>90 – day forward</td>
<td>1.6056</td>
</tr>
<tr>
<td>180 – day forward</td>
<td>1.6018</td>
</tr>
</tbody>
</table>
EXAMPLE: FOREIGN EXCHANGE

- Investor signs a 90-day contract for £1,000,000 on May 8, 1995
- Investor pays $1,605,600 in 90 days and receives £1,000,000
- Consider two hypothetical cases:

<table>
<thead>
<tr>
<th>case</th>
<th>$s_{90}$</th>
<th>investor payoff ( $s_{90} - K$) in $</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.6500</td>
<td>1,650,000 – 1,605,600 = 44,400</td>
</tr>
<tr>
<td>2</td>
<td>1.5500</td>
<td>1,550,000 – 1,605,600 = –55,600</td>
</tr>
</tbody>
</table>

- The investor payoff represents the investor’s total gains ( $s_T - K > 0$) or total losses ( $s_T - K < 0$)
FUTURES CONTRACTS

- A futures contract is a standardized forward contract that is, typically, traded on an exchange; the exchange provides a mechanism that guarantees the contract is honored by the two parties.
- A key aspect in which a futures contract differs from a forward contract is that a precise delivery date is not specified; typically, the futures contract specifies the delivery month.
EXAMPLE: WHEAT FUTURES CONTRACT

- Traded on the *Chicago Board of Trade (CBT)*

- Size: 5,000 *bushels*

- Delivery months: March, May, July, September, and December

- Maturity: up to 18 months in the future

- Quality: grades of wheat specified by *CBT*

- Delivery locations: specified by *CBT*
# FORWARD vs. FUTURES CONTRACTS

<table>
<thead>
<tr>
<th>forward contract</th>
<th>futures contract</th>
</tr>
</thead>
<tbody>
<tr>
<td>customized</td>
<td>standardized</td>
</tr>
<tr>
<td>private bilateral agreements</td>
<td>publicly traded on an exchange</td>
</tr>
<tr>
<td>the specified delivery date</td>
<td>range of delivery dates</td>
</tr>
<tr>
<td>settled at maturity (contract end)</td>
<td>settled daily</td>
</tr>
<tr>
<td>long position takes delivery; short position gets cash settlement</td>
<td>typically contracts are closed out prior to maturity and do not involve delivery</td>
</tr>
</tbody>
</table>
A financial derivative $D$ is a financial instrument that derives its values from a related or underlying asset.

Financial derivative attributes are:

- the underlying asset $S$
- the maturity time $T$
- the payoff function $f^D(\cdot)$
Two parties are involved in a financial derivative:
- the issuer: short position
- the holder: long position

The maturity is the derivative expiration time $T$.

The derivative may be exercised at:
- anytime $t \in [0, T]$ for American derivatives
- only at $t = T$ for European-type derivatives

We focus on the use of European derivatives: for example, in electricity $T$ is chosen to be the time the energy is needed.
The derivative is written on the price movement of a traded underlying asset $S$.

The underlying asset may be any good, service or variable whose value is well defined, such as a stock, a bond, a commodity, currency, or a financial contract.
THE UNDERLYING ASSETS AND ASSET MARKETS

- We assume the existence of spot markets for the underlying asset at all times during the contract life; at any time $t$, a single spot price $S_t$ exists for the particular asset $S$.

- Short selling is allowed in the asset markets, i.e., the investor may borrow an asset from a bank and sell it, with the explicit obligation to purchase the asset at a later time to return it to the bank.
PAYOFF FUNCTION OF THE DERIVATIVES

- Each derivative specifies a payment of the payoff from the issuer to the holder; the value of the payoff is expressed by the function $f^\varphi(\cdot)$.

- The payoff is a function of the underlying asset spot price; for European derivatives, it is simply a function of $S_T$. 
PAYOFF EXAMPLE: THE FLOUR CONTRACT

\[ f^\phi (s_T) = s_T - K \]
In the forward flour contract example, the contract must be exercised at time $T$: the holder of the contract must buy the flour from the issuer who must deliver it at the time $T$.

The payoff of the forward is either nonnegative or negative, so that two-sided payments may exist.

Forward contracts impose obligations on both the issuer and the holder.
There exist other types of derivatives, for which, the holder has the option to choose whether or not to exercise the contract:

- the holder has the *right* but not the *obligation* to exercise the contract

- the issuer has the *obligation* to perform as the contract dictates

Such derivatives are called *options*
An option is a financial derivative that provides the holder the right but not the obligation to buy or sell the underlying asset at maturity at the specified strike price.

Types of options:
- *call* option $C$: rights to buy
- *put* option $P$: rights to sell
- combinations of call and put options at various strike prices
EXAMPLE: ELECTRICITY CALL

- A generation company $G$ issues to a broker $B$ a call option $C$; the option provides $B$ the right to buy 1 MWh electricity at time $T$ at 20 $/MWh

- Attributes of the electricity call option
  - the underlying asset $S$ is the 1 MWh electricity
  - the maturity is the time $T$
  - the payoff function is $f^C(S_T)$ with $K = 20$ $/MWh$
EXAMPLE : ELECTRICITY CALL

☐ The contractual aspects of this option are

☐ at the time \( T \), broker \( B \) has the right but not the obligation to buy the 1 MWh from the generator \( G \)

☐ \( G \) has the obligation to provide the energy if requested by \( B \) at the delivery price of \( K = 20 \) \$/MWh

☐ the negotiated strike price 20 \$/MWh is totally independent of the spot price \( s_T \) at time \( T \)
EXAMPLE : ELECTRICITY  CALL

We assume there is 1 $\text{MWh}$ need at time $T$ so that $B$ needs to buy 1 $\text{MWh}$ to meet its demand.

- We assume $B$ is rational so that it minimizes the costs to meet its demand.
  - $s_T > 20$ : $B$ exercises the option and buys the 1 $\text{MWh}$ from $G$ at the fixed price $\$20$
  - $s_T \leq 20$ : $B$ discards the option and buys the 1 $\text{MWh}$ from the spot market for the spot energy price $s_T$
The costs for $B$ to purchase the 1 $\text{MWh}$ is a function of $s_T$.

The *call* option protects the holder $B$ from exposure to any spot price above its *strike price*.
In financial markets, typically, the option does not involve the physical delivery of the underlying asset; instead, the following payoff is specified:

- If \( s_T > 20 \): \( G \) pays \( B \) the price difference \( s_T - 20 \)
- If \( s_T \leq 20 \): no payment takes place

This payoff function provides precisely the same outcomes for \( G \) and \( B \) financially as if the electricity were physically delivered.
CALL OPTION PAYOFF DIAGRAM

\[ f^c(s_T) = \max\{0, s_T - K\} \]

**long position**

**short position**
### EUROPEAN CALL OPTION PAYOFF

<table>
<thead>
<tr>
<th>Position</th>
<th>Payoff at Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>long</strong></td>
<td>( \max{ (s_T - K), 0 } )</td>
</tr>
<tr>
<td><strong>short</strong></td>
<td>( \min{ (K - s_T), 0 } )</td>
</tr>
</tbody>
</table>

#### Functional Form

- \( s_T \): Stock Price at Maturity
- \( K \): Strike Price

#### Plot

- **Payoff**: Graph showing the payoff for both long and short positions as a function of \( s_T \) and \( K \).
PUT OPTIONS

- A put option gives the holder the right to sell the underlying asset at the specified strike price.

- For a put option $P$ with the underlying asset $S$, strike price $K$ and maturity $T$, the payoff to the holder is given by

$$f^P(s_T) = \max \left\{ 0, K - s_T \right\}$$
**PUT OPTION PAYOFF FUNCTION**

\[ f^\varphi(s_T) = \max \left\{ 0, K - s_T \right\} \]

- **payoff**
  - **long position**
    - \( f^\varphi(s_T) \)
    - \( K \)
  - **short position**
    - \( -f^\varphi(s_T) \)
    - \( -K \)

Payoff diagrams for long and short positions.
### EUROPEAN PUT OPTION PAYOFF

<table>
<thead>
<tr>
<th>position</th>
<th>payoff at maturity</th>
<th>functional form</th>
<th>plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>long</td>
<td>max{(K - s_T), 0}</td>
<td>min{0}</td>
<td>![Plot of long position]</td>
</tr>
<tr>
<td>short</td>
<td>min{(s_T - K), 0}</td>
<td>max{0}</td>
<td>![Plot of short position]</td>
</tr>
</tbody>
</table>
### EUROPEAN OPTION PAYOFF

<table>
<thead>
<tr>
<th>Option Type</th>
<th>Position</th>
<th>Payoff at Maturity</th>
<th>Functional Form</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Call</strong></td>
<td>long</td>
<td>max({s_T - K, 0})</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>short</td>
<td>min({K - s_T, 0})</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Put</strong></td>
<td>long</td>
<td>max({K - s_T, 0})</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>short</td>
<td>min({s_T - K, 0})</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
THE OPTION PREMIUM

- By definition, the payoff to the holder of an option contract is nonnegative, thereby providing the protection or hedge against the uncertain spot market prices.

- In return for such protection, the holder pays a premium to the issuer.

- The premium is the price \( \rho_0^D \) of the option contract derivative \( D \) at \( t = 0 \).

- \( \rho_0^D > 0 \) for every option \( D \).
The profits and losses (P&L) for a derivative $\mathcal{D}$, denoted by $\pi^\mathcal{D}$, are defined as the net cash flow into a position – long or short – during the life $[0, T]$ of the derivative.
DERIVATIVE PROFITS AND LOSSES

For derivative $D$ with payoff $f^D(s_T)$ and premium $\rho_0^D$, the P&L are defined as

- long position: $\pi_+^D = f^D(s_T) - \rho_0^D$
- short position: $\pi_-^D = -f^D(s_T) + \rho_0^D$

Note that by definition

$$\pi_+^D = -\pi_-^D \quad \forall \ D$$
CALL OPTION PROFITS AND LOSSES

\[ \pi_+^c = f^c(s_T) - \rho_0^c \]

long position

\[ \pi_-^c = -f^c(s_T) + \rho_0^c \]

short position

\[ -\rho_0^c \]

losses

profits

\[ K + \rho_0^c \]

\[ K \]

\[ K + \rho_0^c \]
PUT OPTION PROFITS AND LOSSES

**long position**

\[ \pi_+ = f^\varphi (s_T) - \varrho_0^\varphi \]

**short position**

\[ \pi_- = - f^\varphi (s_T) + \varrho_0^\varphi \]
HEDGING

- A *hedger* is a trader interested to reduce the risk he faces; a *hedger* uses financial derivatives to reduce faced exposure to movements in price

- We revisit the currency exchange example: an investor needs to make £1,000,000 payment in 180 days and so is faced with significant foreign exchange risks in the volatile currency markets since the investor pays in **US $**
May 8, 1995 spot and forward foreign exchange for British £ and US $

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>spot</strong></td>
<td>1.6080</td>
</tr>
<tr>
<td><strong>30 – day forward</strong></td>
<td>1.6076</td>
</tr>
<tr>
<td><strong>90 – day forward</strong></td>
<td>1.6018</td>
</tr>
<tr>
<td><strong>180 – day forward</strong></td>
<td>1.6056</td>
</tr>
</tbody>
</table>
Investor signs a forward contract to buy in 180 days £1,000,000 for $1,605,600; this hedge

- requires no initial payments
- provides certainty for the exchange rate
- need not ensure better outcomes

<table>
<thead>
<tr>
<th>case</th>
<th>rate ($/£)</th>
<th>investor’s gains/losses ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.7000</td>
<td>94,400</td>
</tr>
<tr>
<td>2</td>
<td>1.5000</td>
<td>– 105,600</td>
</tr>
</tbody>
</table>
**HEDGING STRATEGY**

- Investor buys a *call* option to acquire £1,000,000 at the exchange rate of 1.6000; this hedge
  - requires an initial outlay of cash for the *call* option premiums
  - provides protection to the investor against adverse exchange rate movements and benefits from favorable movements

<table>
<thead>
<tr>
<th>case</th>
<th>exchange rate</th>
<th>investor’s action</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>≥ 1.6</td>
<td>exercise option</td>
</tr>
<tr>
<td>2</td>
<td>&lt; 1.6</td>
<td>buy £ in market</td>
</tr>
</tbody>
</table>
An entity that sells an asset at some given future time can hedge by taking a short futures position; this is a short hedge.

- If the asset price increases, there is a gain on the asset sale and a loss on the short futures position.
- If the asset price decreases, there is a loss on the asset sale and a gain on the short futures position.
An entity that wishes to buy an asset at some future time can hedge by taking a long futures position; this is a long hedge.

- If the asset price increases above the strike price, there is purchase price certainty and gain on the long futures position.
- If the asset is below the strike price, there is a lower purchase price and loss for the long futures position.
Futures hedging does not necessarily improve the overall financial outcome; in fact, on the average the outcome is worse 50% of the time.

Futures hedging, however, reduces the risk since it provides price certainty.
EXAMPLE: *SHORT HEDGE*

- We consider a generator whose plan is the sale of its $1\ MWh$ energy production at an hour $T$ at a future time.
- We assume the production costs for the $1\ MWh$ energy are at $20\ $/MWh.
- We assume the energy spot price $s_T$ at hour $T$ has the following discrete distribution:

$$s_T = \begin{cases} 18\ $/MWh & \text{with probability } 0.5 \\ 26\ $/MWh & \text{with probability } 0.5 \end{cases}$$
EXAMPLE: SHORT HEDGE

- If the generator sells its energy directly in the spot market, its profits are

\[ \pi_T = \begin{cases} 
-2 \$/MWh & \text{with probability 0.5} \\
6 \$/MWh & \text{with probability 0.5} 
\end{cases} \]

- The generator suffers a loss in the case the spot price is 18 \$/MWh since his marginal costs are 20 \$/MWh.

- The generator may protect himself from such a loss by taking a short futures position.
EXAMPLE: *SHORT HEDGE*

- If the generator sells a futures contract $A$ on 1 MWh electricity with the maturity $T$ and the delivery price $K = 22 \$/MWh, the generator receives the net payoff at time $T$ of

$$f^A(s_T) = -(s_T - K) = \begin{cases} 
4 \$ & \text{if } s_T = 18 \$/MWh \\
-4 \$ & \text{if } s_T = 26 \$/MWh
\end{cases}$$

- The generator’s net profits then become

$$\pi_{net}^T = \pi_T + f^A(s_T) = \begin{cases} 
2 \$ & \text{with probability } 0.5 \\
2 \$ & \text{with probability } 0.5
\end{cases}$$
EXAMPLE : *SHORT HEDGE*

- So, as the holder of the *short futures position*, the generator’s *net profits* become *independent* of the *spot price*; the generator gains in either case.

- In this way, we say the generator’s position is *fully hedged*.
EXAMPLE: *LONG HEDGE*

- We consider a 1 MW load planning its energy purchases for the hour $T$ at some future time.

- We assume the energy spot price $s_T$ at hour $T$ has the following distribution:

$$s_T = \begin{cases} 
18 \, \text{\$/MWh} & \text{with probability 0.5} \\
26 \, \text{\$/MWh} & \text{with probability 0.5} 
\end{cases}$$
EXAMPLE: *LONG HEDGE*

- If the load purchases its energy directly in the spot market, its costs are:

  \[ c_T = \begin{cases} 
  18 \text{ $} & \text{with probability 0.5} \\
  26 \text{ $} & \text{with probability 0.5} 
\end{cases} \]

- The load faces uncertainty in the supply costs.

- The load may get price certainty by taking a *long* futures position.
EXAMPLE: LONG HEDGE

- If the load holds a long futures position $A$ on 1 MWh energy with the maturity $T$ and the delivery price $K = 22 \$/MWh at $T$, the load receives the net payoff $f^A(s_T) = s_T - K = \begin{cases} -4 \$ & \text{if } s_T = 18 \$/MWh \\ 4 \$ & \text{if } s_T = 26 \$/MWh \end{cases}$

- The load’s net costs then become $c_T^{net} = c_T - f^A(s_T) = \begin{cases} 22 \$ & \text{with probability 0.5} \\ 22 \$ & \text{with probability 0.5} \end{cases}$
EXAMPLE : *LONG HEDGE*

- In other words, the load’s *long futures position*

  makes the net costs to be *independent of the*

  *spot prices; the load gets its price certainty*

- In this way, we say the load’s position is *fully*

  *hedged*
SPECULATION

- A speculator takes a position in a market by betting that either that a price increases or a price decreases.
- A speculator may purchase the asset on the spot market and rely on:
  - future spot markets
  - use *forward contracts* with a higher level of leverage
  - use *options* for additional leverage
In the *sterling exchange* example, the investor can speculate and take a long position in a 180-day forward contract on sterling: suppose the speculator buys a 180-day forward contract at conversion rate of 1.6056 and the exchange rate rises to 1.7000, then he makes a profit of $\,94,440.$
EXAMPLE: SPECULATION USING OPTIONS

- A stock price is $43 and an investor is betting that the price will rise and buys call options with a strike price of $48 at $1 per option.

- If the price fails to go above $48 during the life of the option, the speculator loses $1 per option.

- If the price rises to $55, the speculator realizes net profits of $6 per option – a 600% gain on the original premium investment.
Arbitrageurs lock in *riskless profits* by undertaking transactions in two or more markets.

As an example, we consider a situation where a stock is traded on stock exchanges both in *NY* and *London*: suppose stock price is $172 in *NY* and £100 in *London* when the exchange rate is $1.750 per £:

- Arbitrageur buys 1,000 shares in *NY* and immediately sells them in *London* to realize *risk-free profits* of $3,000.
ARBITRAGE

- such arbitrage opportunities do not last very long because as arbitrageurs buy more stocks in $NY$, the law of supply and demand causes the price to rise and, similarly, as they sell more stock in $London$, the £ price will decrease so that in a short time the prices become the same at the current exchange rate.