ECE 307 – Techniques for Engineering Decisions

16. Forward Contracts

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RISK

- There are many definitions of risk; we use the conceptual definition from Webster's dictionary that ***risk is the possibility of suffering loss***
- People measure risk with a wide variety of **specific metrics**
- A rational market player aims to minimize the risks faced

RISK

- **Market players use specific *financial tools* to keep their risks below a specified threshold**
- **Such actions constitute *risk management* and are carried out with financial instruments called *risk management tools***
- ***Financial derivatives* are some of the most widely-used risk management tools in financial markets**

FINANCIAL DERIVATIVES

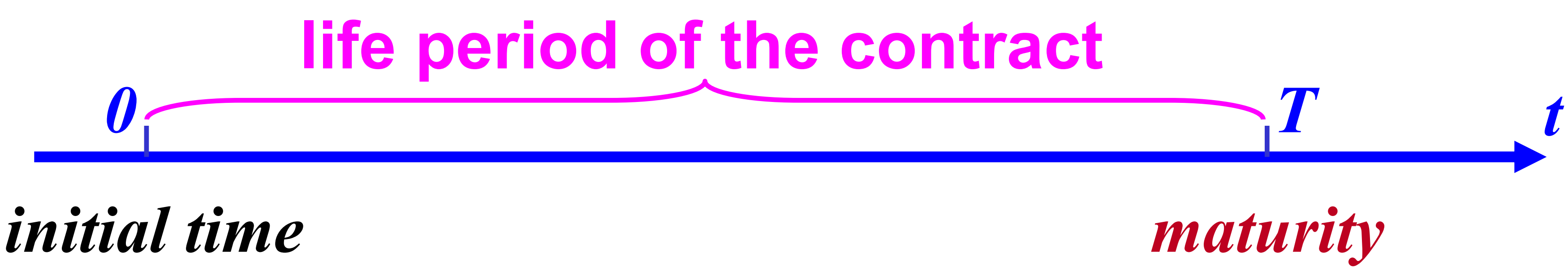
- Basic definition: a derivative is a financial tool whose value depends on the value of other, more basic underlying variables
- The basic derivatives we examine are
  - forward contracts
  - future contracts
  - options
    - puts
    - calls

EXAMPLE: A FLOUR CONTRACT

- A farmer in Illinois and a restaurant in Wisconsin enter into a contract on January 1, 2015, under which the farmer agrees to sell 1 *ton* of flour for *$* 400 to the restaurant on September 1, 2015
- The contract involves two parties
  - the farmer is the *issuer* of the contract and holds a *short position*
  - the restaurant is the *holder* of the contract and has a *long position*

EXAMPLE: A FLOUR CONTRACT

- The contract is signed on January 1 for the actual sale that occurrs on September 1 and
  - we call January 1 the ***initial time*** of the contract and denote it by $t = 0$, the origin of the time line;
  - we call September 1 the ***maturity*** of the contract and denote it by $t = T$



EXAMPLE: A FLOUR CONTRACT

- This contract is on the trading of a single specified commodity – the flour; we call the 1 *ton* of flour the *underlying asset*
- The contract provides the holder with
  - the delivery of the underlying asset at T
  - the fixed price K for the asset – the so–called *delivery price*

EXAMPLE: A FLOUR CONTRACT

- **In the absence of a forward contract, the restaurant needs to buy the flour from the spot market at an *uncertain price* to meet its needs; this price can be high so that the restaurant bears *price risks***
- **With the forward contract, the price is fixed and known and, therefore, the holder is protected from price risks**
- **This forward contract is a *physical contract* since the actual delivery of the asset is involved**

EXAMPLE: A FLOUR CONTRACT

- Next, we assume the existence of a *spot market price* s_T for flour at time T so that one can buy or sell the flour at that *spot price* on the September 1 date
- The *flour forward contract* may also be signed as a *purely financial contract* with the flour commodity as the underlying asset, September 1, 2015 as the maturity time T and the specification of the set of following payments:

EXAMPLE: A FLOUR CONTRACT

- if $s_T > K$, the issuer reimburses the holder the difference $s_T - K$ to purchase the flour at s_T
- if $s_T < K$, the holder must pay to the issuer in the amount of $K - s_T$ and buys flour at s_T

- These specified payments constitute the *payoffs* of the financial contract
- Thus, the *net price* to the holder is the *delivery price* K and is independent of the market spot price s_T

EXAMPLE: A FLOUR CONTRACT

- Since the issuer can sell the flour in the spot market, its *net price* also equals *K*
- Therefore, this purely financial contract provides precisely the same function as the *physical contract* to both the issuer and the holder but involves no actual delivery
- Typically, many forwards are purely financial contracts that do not involve *physical deliverability*

FORWARD CONTRACTS

- A forward contract is a binding agreement to buy or sell an *asset* at the designated future time at the specified price
- An *asset* is a general term for any good, service or commodity
- The buyer – holder – is said to hold a *long position* and the seller – issuer – holds a *short position*
- The specified price is called the *delivery price*

FORWARD CONTRACTS

- A forward contract is *settled at maturity* – the designated future time at which the purchase/sale is consummated
- The *short position delivers* the *asset* to the *long position* in return for the cash payment in the amount of the *delivery price* times the *contract amount*
- The value of the forward contract is a function of the *market price* of the asset and its *maturity*

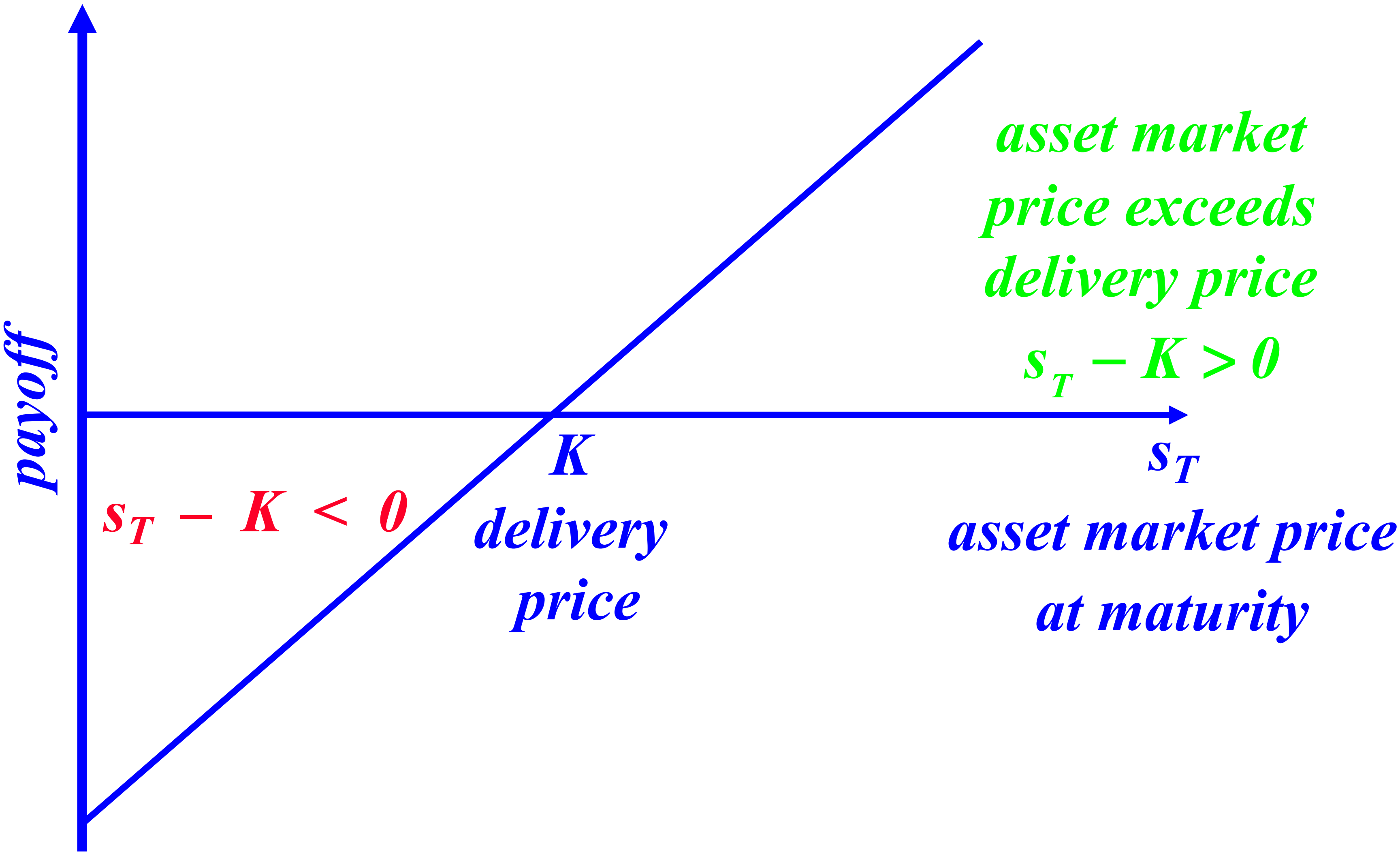
FORWARD VALUE AND PRICE

- The value of a forward contract is *0* for both the *short* and the *long* positions at the time the contract is signed; thereafter, its value may be positive, *0* or negative, depending on market conditions

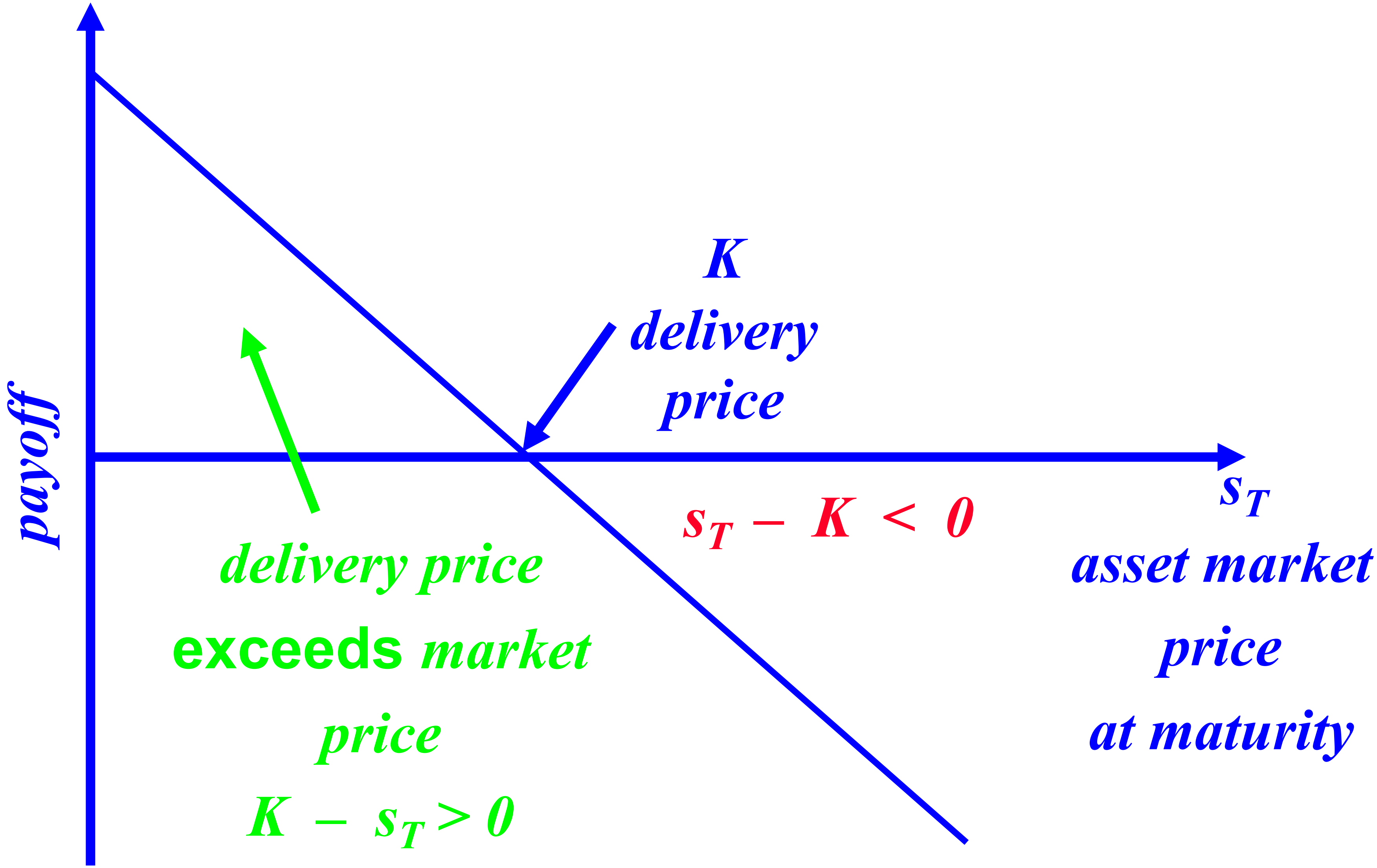
FORWARD VALUE AND PRICE

- The *forward price* of a forward contract is the *delivery price* that makes the forward contract have *0* value at the time the contract is signed
- By definition, the *forward price* equals the *delivery price* at the time the contract is signed; thereafter, the *delivery price K* remains fixed while the *forward price* may change as a function of the *market price* and the *maturity* of the contract

THE *LONG POSITION* FORWARD CONTRACT *PAYOFF*



THE *SHORT POSITION* FORWARD CONTRACT *PAYOFF*



EXAMPLE: FOREIGN EXCHANGE

May 8, 1995 spot and forward foreign exchange for *British £* and *US $*

foreign exchange	*price in US $*
spot	1.6080
30 – *day forward*	1.6076
90 – *day forward*	1.6056
180 – *day forward*	1.6018

EXAMPLE: FOREIGN EXCHANGE

- Investor signs a 90-day contract for *£* 1,000,000 on May 8, 1995
- Investor pays *$* 1,605,600 in 90 days and receives *£* 1,000,000
- Consider two hypothetical cases:

case	s_{90}	*investor payoff* $(s_{90} - K)$ *in $*
1	1.6500	1,650,000 – 1,605,600 = 44,400
2	1.5500	1,550,000 – 1,605,600 = – 55,600

- The investor *payoff* represents the investor's total gains ($s_T - K > 0$) or total losses ($s_T - K < 0$)

FUTURES CONTRACTS

- **A *futures contract* is a *standardized forward contract* that is, typically, traded on an exchange; the exchange provides a mechanism that guarantees the contract is honored by the two parties**
- **A key aspect in which a futures contract differs from a forward contract is that a precise delivery date is not specified; typically, the futures contract specifies the delivery month**

EXAMPLE: WHEAT FUTURES CONTRACT

- **Traded on the *Chicago Board of Trade* (*CBT*)**
- **Size: 5,000 *bushels***
- **Delivery months: March, May, July, September, and December**
- **Maturity: up to 18 months in the future**
- **Quality: grades of wheat specified by *CBT***
- **Delivery locations: specified by *CBT***

FORWARD vs. FUTURES CONTRACTS

forward contract	*futures contract*
customized	*standardized*
private bilateral agreements	*publicly traded on an exchange*
the specified delivery date	*range of delivery dates*
settled at maturity (contract end)	*settled daily*
long position takes delivery; short position gets cash settlement	*typically contracts are closed out prior to maturity and do not involve delivery*

FINANCIAL DERIVATIVES : FORMAL DEFINITION

- A *financial derivative* $\mathcal{D}$ is a financial instrument that derives its values from a *related* or *underlying asset*

- Financial derivative attributes are
  - the *underlying asset* $\mathcal{S}$
  - the *maturity time* T
  - the *payoff function* $f^{\mathcal{D}}(\bullet)$

POSITIONS AND MATURITY TIME

- Two parties are involved in a financial derivative
  - the issuer: *short position*
  - the holder: *long position*
- The *maturity* is the derivative expiration time T
- The derivative may be exercised at
  - anytime $t \in \left[0,T \right]$ for *American* derivatives
  - only at $t = T$ for *European*–type derivatives
- We focus on the use of *European* derivatives: for example, in electricity T is chosen to be the time the energy is needed

THE UNDERLYING ASSETS AND ASSET MARKETS

- The derivative is written on the price movement of a traded *underlying asset* S
- The *underlying asset* may be any good, service or variable whose value is well defined, such as a stock, a bond, a commodity, currency, or a financial contract

THE UNDERLYING ASSETS AND ASSET MARKETS

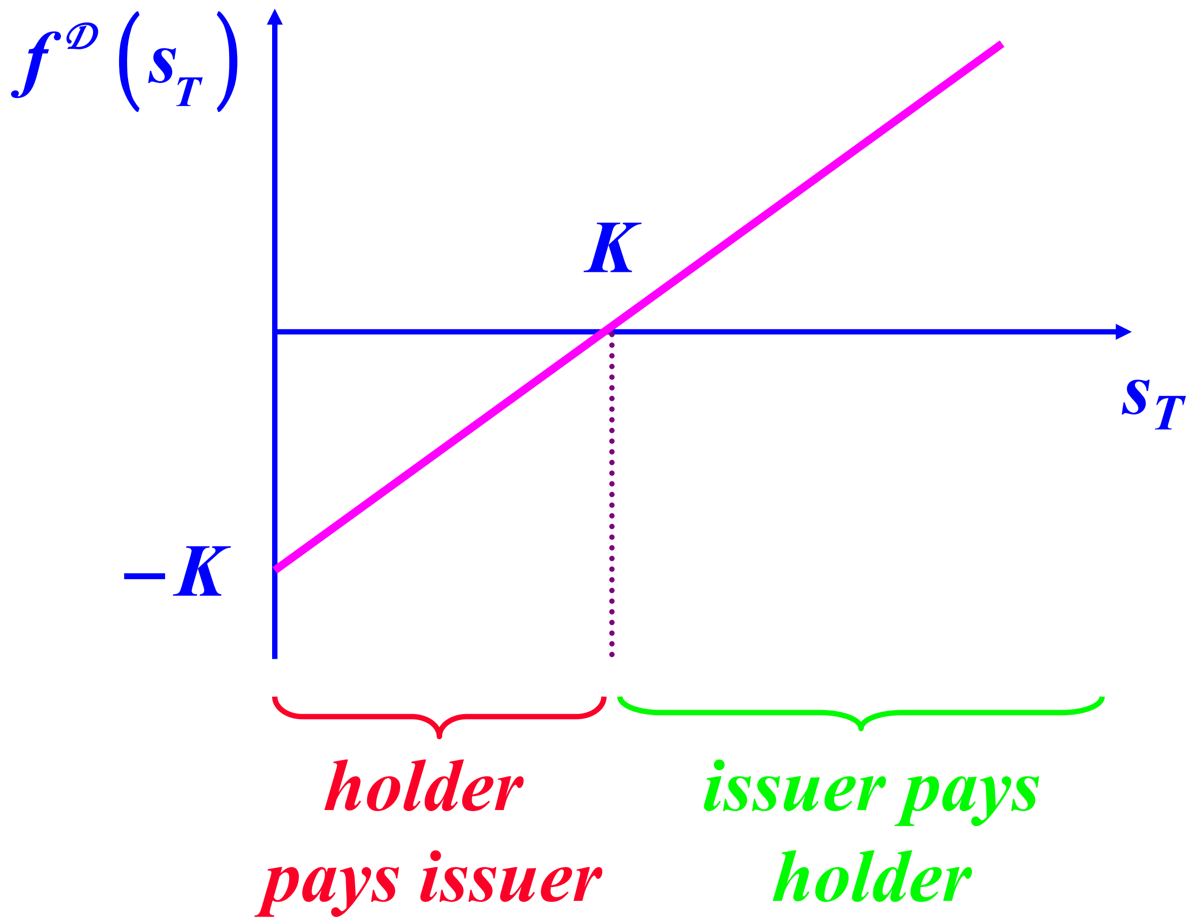
- **We assume the existence of spot markets for the underlying asset at all times during the contract life; at any time *t*, a single spot price s_t exists for the particular asset $\mathcal{S}$**
- **Short selling is allowed in the asset markets, i.e., the investor may borrow an asset from a bank and sell it, with the explicit *obligation* to purchase the asset at a later time to return it to the bank**

PAYOFF FUNCTION OF THE DERIVATIVES

- **Each derivative specifies a payment of the *payoff* from the issuer to the holder; the value of the *payoff* is expressed by the function $f^{\mathcal{D}}(\cdot)$**
- **The *payoff* is a function of the *underlying asset* spot price; for *European* derivatives, it is simply a function of s_T**

PAYOFF EXAMPLE: THE FLOUR CONTRACT

$$f^{\mathcal{D}}\left(s_T\right) = s_T - K$$



RIGHTS AND *OBLIGATIONS*

- In the forward flour contract example, the contract must be exercised at time T: the holder of the contract must buy the flour from the issuer who must deliver it at the time T
- The *payoff* of the forward is either nonnegative or negative, so that *two-sided* payments may exist
- Forward contracts impose *obligations* on both the issuer and the holder

RIGHTS AND *OBLIGATIONS*

- There exist other types of derivatives, for which, the holder has the option to choose whether or not to exercise the contract
  - the holder has the *right* but not the *obligation* to exercise the contract
  - the issuer has the *obligation* to perform as the contract dictates
- Such derivatives are called *options*

OPTION CONTRACTS

- An *option* is a financial derivative that provides the holder the *right* but not the *obligation* to buy or sell the underlying asset at maturity at the specified strike price
- Types of options
  - *call* option $\mathcal{C}$: rights to buy
  - *put* option $\mathcal{P}$: rights to sell
  - combinations of *call* and *put* options at various strike prices

EXAMPLE : ELECTRICITY *CALL*

- A generation company G issues to a broker B a *call* option $\mathcal{C}$; the option provides B the right to buy 1 MWh electricity at time T at 20 $\$/MWh$
- Attributes of the electricity *call* option
  - the *underlying asset* $\mathcal{S}$ is the 1 MWh electricity
  - the *maturity* is the time T
  - the *payoff* function is $f^{C}\left(s_T\right)$ with $K = 20\ \$/MWh$

EXAMPLE : ELECTRICITY *CALL*

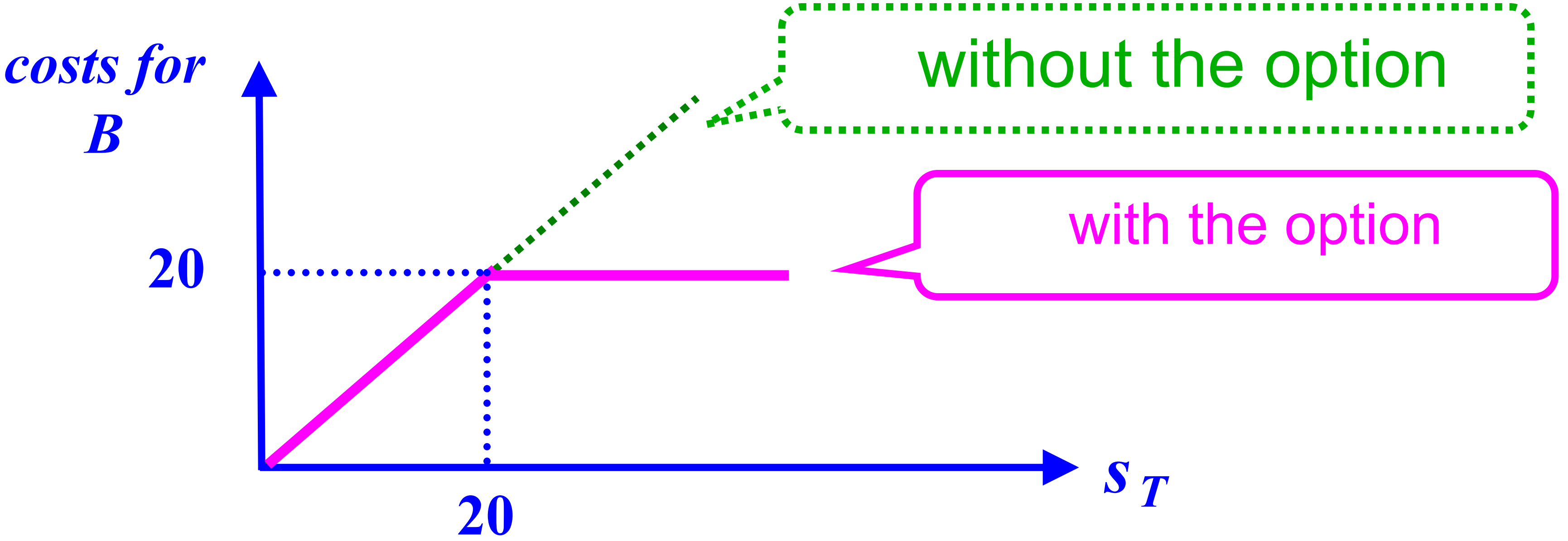
- The contractual aspects of this option are
  - at the time T, broker B has the *right* but not the *obligation* to buy the 1 MWh from the generator G
  - G has the *obligation* to provide the energy if requested by B at the *delivery price* of $K = 20\ \$/MWh$
  - the negotiated *strike price* 20 $\$/MWh$ is totally independent of the *spot price* s_T at time T

EXAMPLE : ELECTRICITY *CALL*

- We assume there is $1 \; MWh$ need at time T so that B needs to buy $1 \; MWh$ to meet its demand
- We assume B is rational so that it minimizes the costs to meet its demand
  - $s_T > 20$: B exercises the option and buys the $1 \; MWh$ from G at the fixed price $\$ \, 20$
  - $s_T \leq 20$: B discards the option and buys the $1 \; MWh$ from the spot market for the spot energy price s_T

EXAMPLE : ELECTRICITY *CALL*

- The costs for B to purchase the 1 MWh is a function of s_T



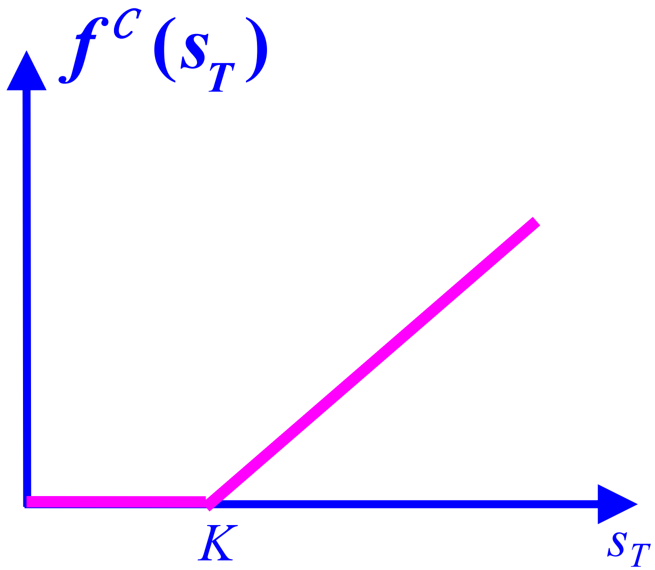
- The *call* option protects the holder B from exposure to any spot price above its *strike price*

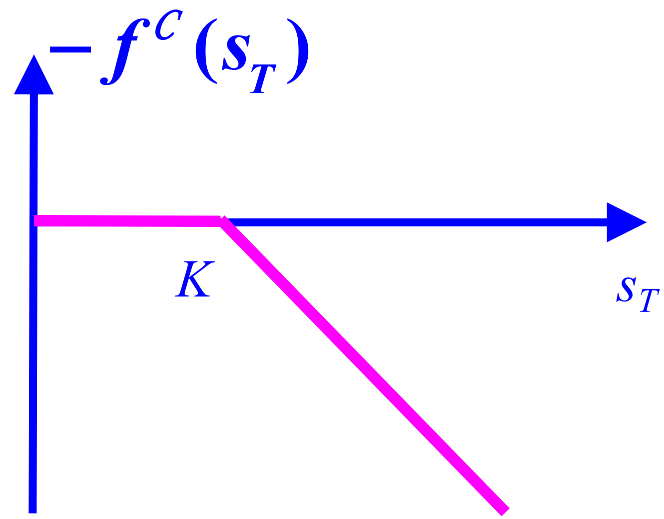
EXAMPLE : ELECTRICITY *CALL*

- In financial markets, typically, the option does not involve the physical delivery of the underlying asset; instead, the following payoff is specified
  - if $s_T > 20$: G pays B the price difference $s_T - 20$
  - if $s_T \leq 20$: no payment takes place
- This *payoff function* provides precisely the same outcomes for G and B financially as if the electricity were physically delivered

CALL OPTION *PAYOFF* DIAGRAM

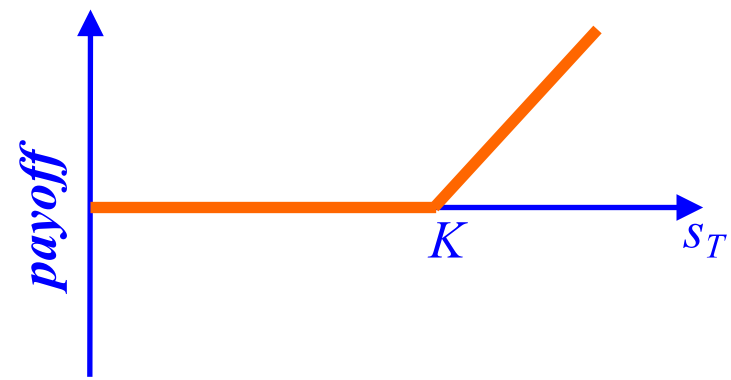
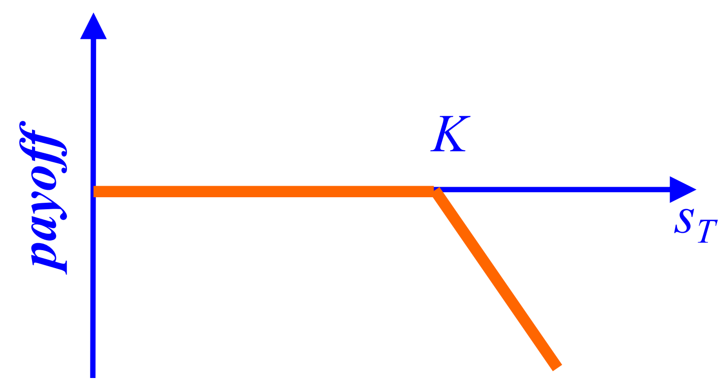
$$f^{C}(s_T) = max\{0, s_T - K\}$$





long position ***short position***

EUROPEAN CALL OPTION PAYOFF

position	*payoff at maturity*	
	functional form	*plot*
long	$\max\{(s_T - K), 0\}$	
short	$\min\{(K - s_T), 0\}$	

PUT OPTIONS

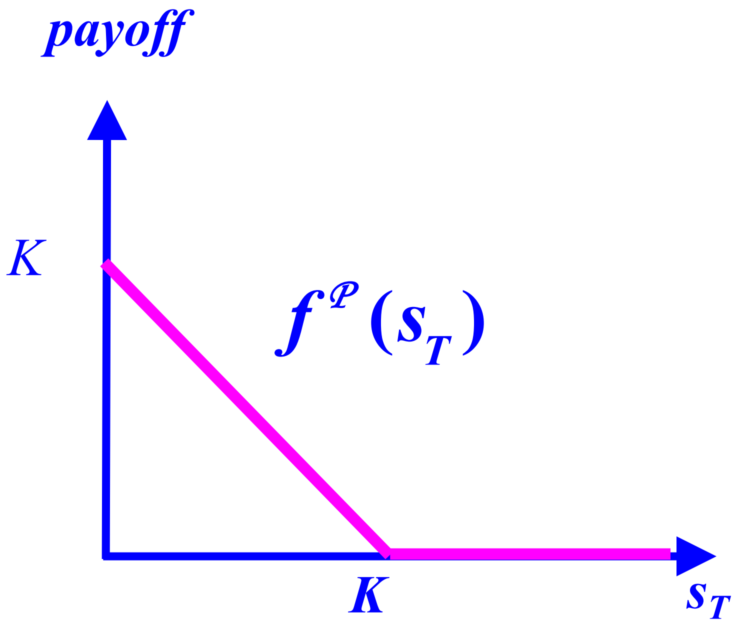
- A *put* option gives the holder the *right* to sell the underlying asset at the specified *strike price*

- For a *put* option $\mathcal{P}$ with the underlying asset $\mathcal{S}$, *strike price* K and *maturity* T, the *payoff* to the holder is given by

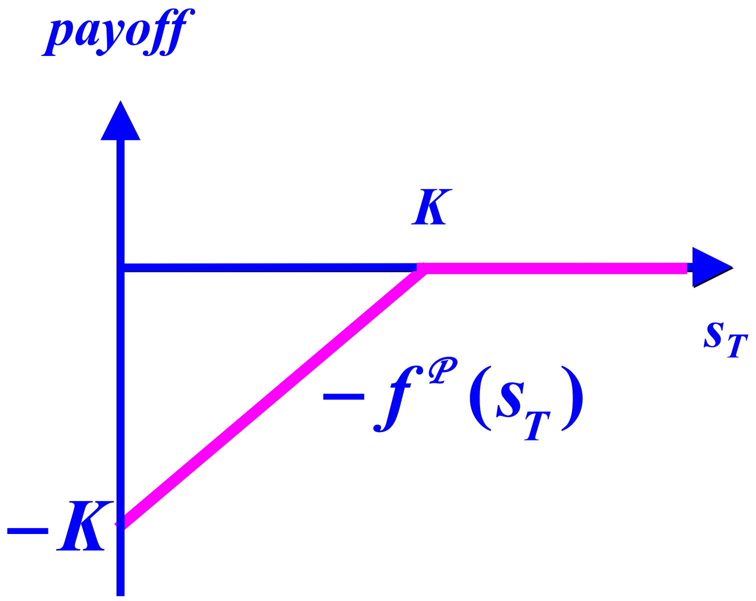
$$f^{\mathcal{P}}(s_T) = max\left\{0,\ K - s_T\right\}$$

PUT OPTION PAYOFF FUNCTION

$$f^{\mathcal{P}}(s_T) = max\{0, K - s_T\}$$

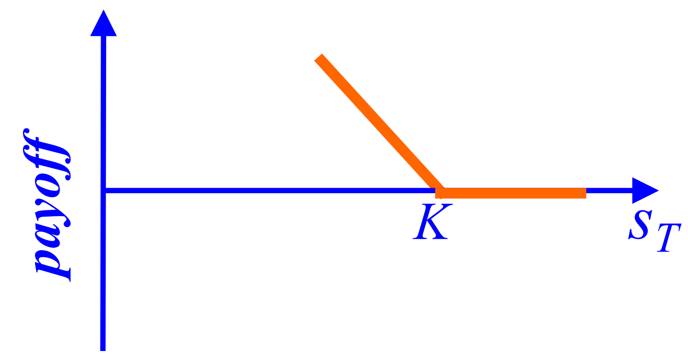
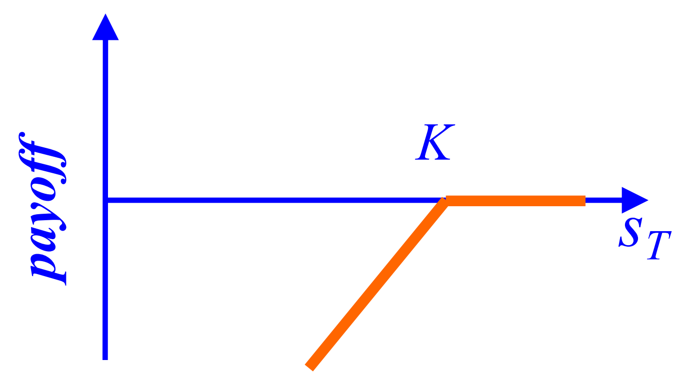


long position

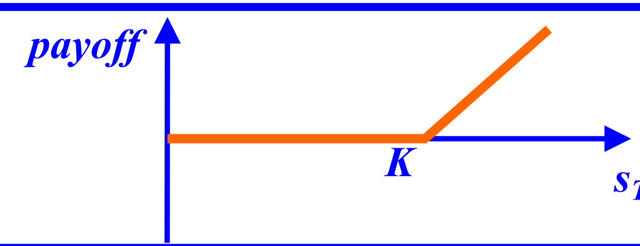
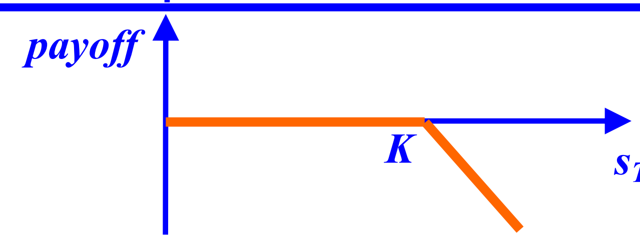
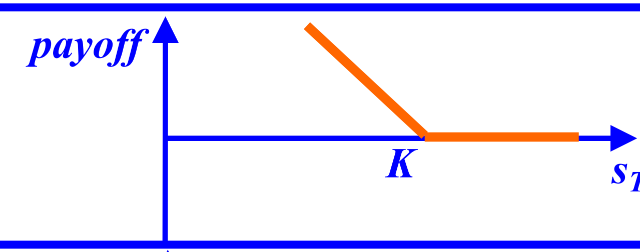
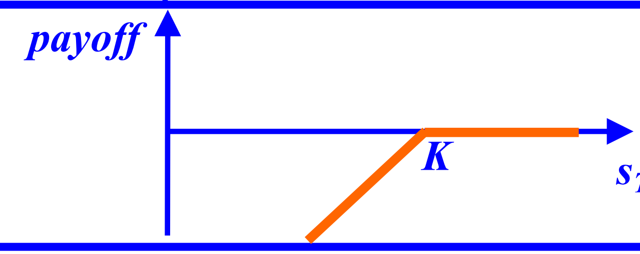


short position

EUROPEAN PUT OPTION PAYOFF

position	*payoff at maturity*	
	functional form	*plot*
long	$\max\{(K - s_T), 0\}$	
short	$\min\{(s_T - K), 0\}$	

EUROPEAN OPTION *PAYOFF*

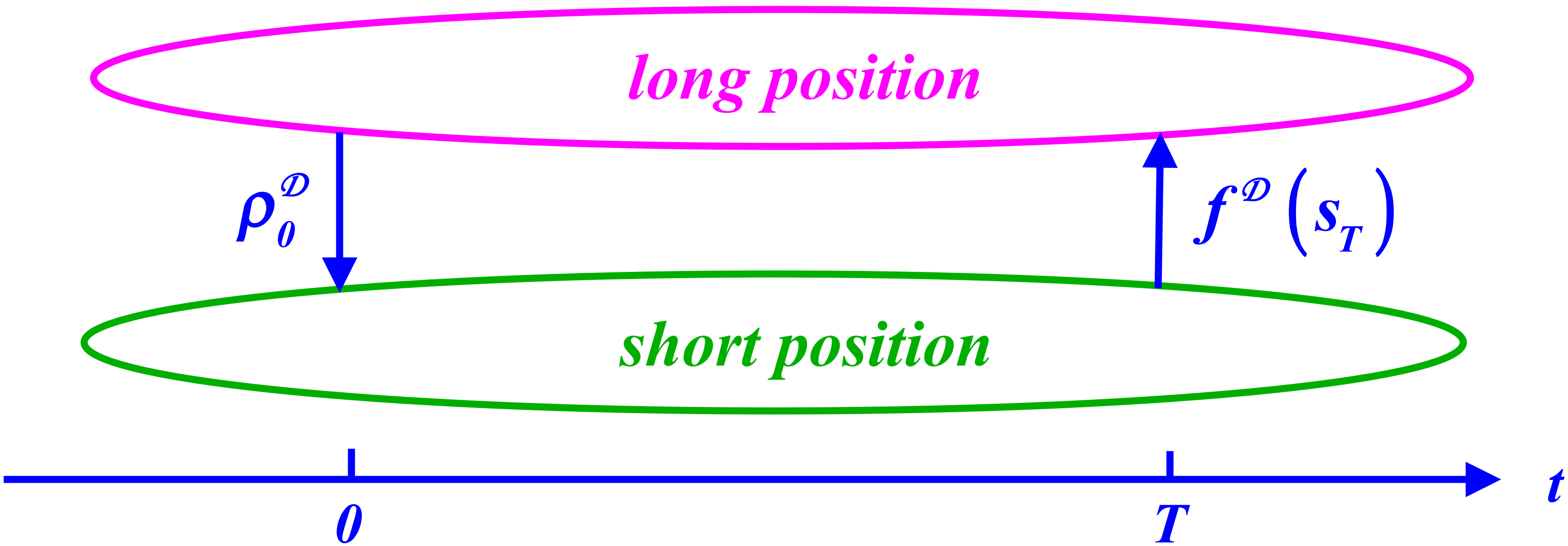
option type	position	*payoff* at maturity	
		functional form	plot
call	*long*	$max\{(s_T - K), 0\}$	
	short	$min\{(K - s_T), 0\}$	
put	*long*	$max\{(K - s_T), 0\}$	
	short	$min\{(s_T - K), 0\}$	

THE OPTION *PREMIUM*

- By definition, the *payoff* to the holder of an option contract is nonnegative, thereby providing the protection or hedge against the uncertain *spot market prices*
- In return for such protection, the holder pays a *premium* to the issuer
- The *premium* is the *price* $\rho_0^{\mathcal{D}}$ of the option contract derivative $\mathcal{D}$ at $t = 0$
- $\rho_0^{\mathcal{D}} > 0$ for every option $\mathcal{D}$

DERIVATIVE *PROFITS AND LOSSES*

- The *profits and losses* (*P&L*) for a derivative $\mathcal{D}$, denoted by $\pi^{\mathcal{D}}$, are defined as the net cash flow into a position – *long* or *short* – during the life $[0, T]$ of the derivative



DERIVATIVE *PROFITS AND LOSSES*

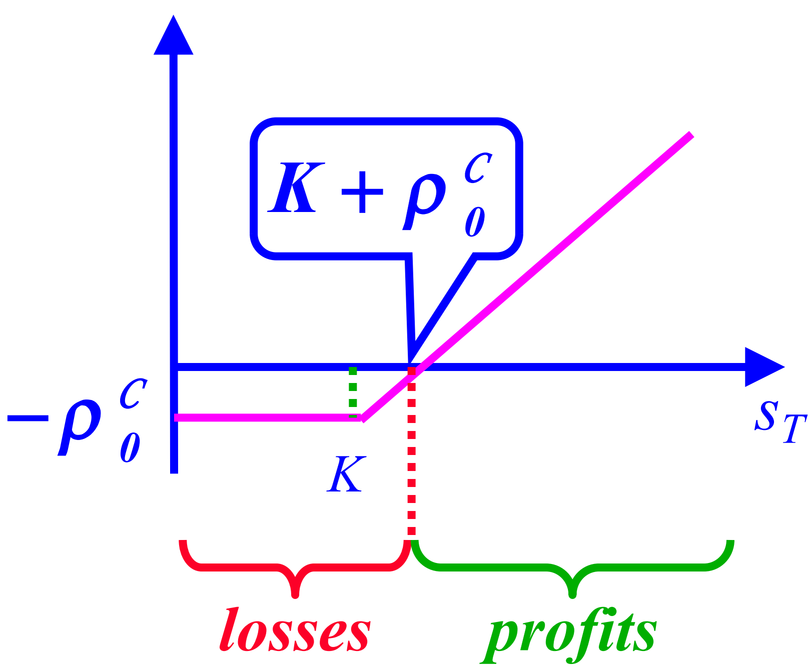
- For derivative $\mathcal{D}$ with *payoff* $f^{\mathcal{D}}\left(s_T\right)$ and *premium* $\rho_0^{\mathcal{D}}$, the *P&L* are defined as
  - *long position*: $\pi_+^{\mathcal{D}} = f^{\mathcal{D}}\left(s_T\right) - \rho_0^{\mathcal{D}}$
  - *short position*: $\pi_-^{\mathcal{D}} = -f^{\mathcal{D}}\left(s_T\right) + \rho_0^{\mathcal{D}}$
- Note that by definition

$$\pi_+^{\mathcal{D}} = -\pi_-^{\mathcal{D}} \qquad \forall\ \mathcal{D}$$

CALL OPTION *PROFITS AND LOSSES*

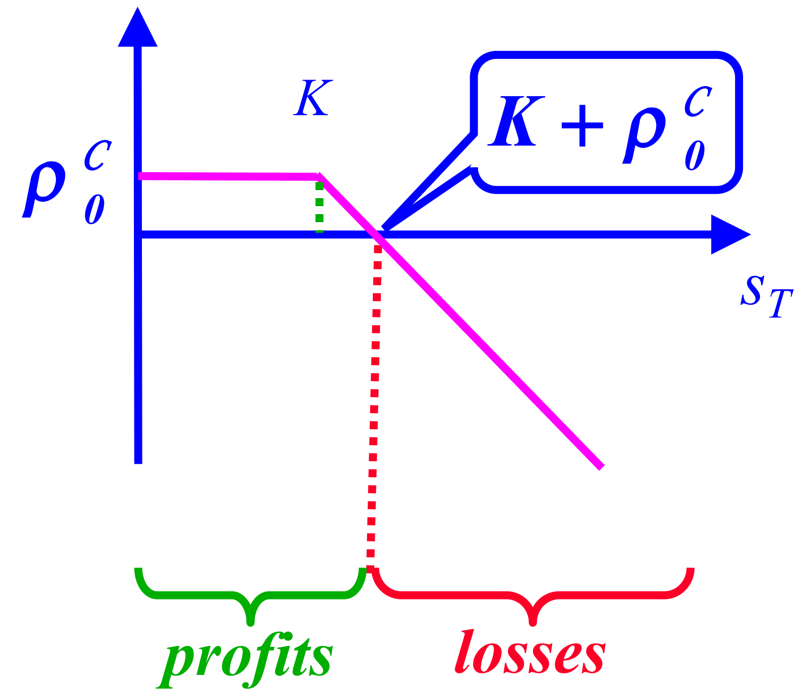
long position

$$\pi_{+}^{C} = f^{C}(s_T) - \rho_0^{C}$$



short position

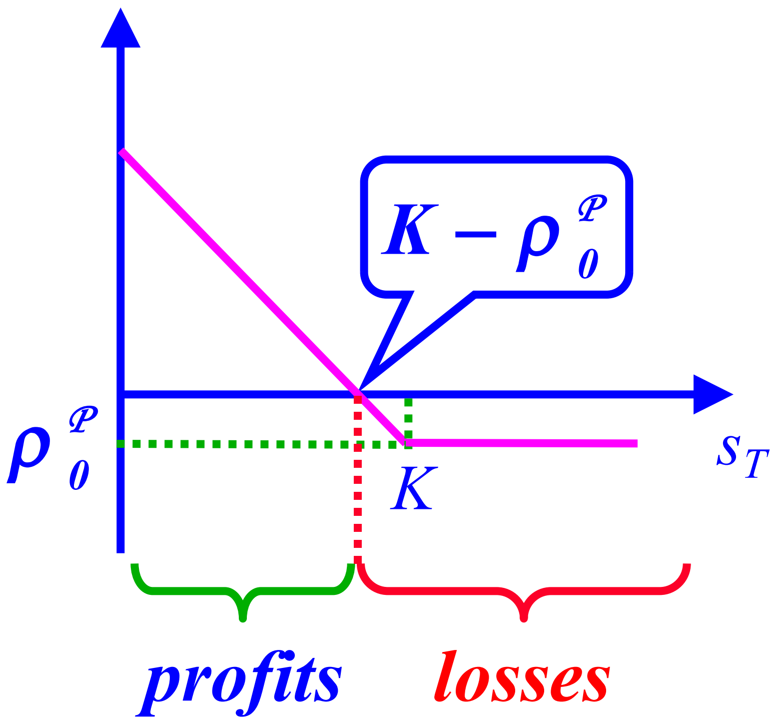
$$\pi_{-}^{C} = -f^{C}(s_T) + \rho_0^{C}$$



PUT OPTION *PROFITS AND LOSSES*

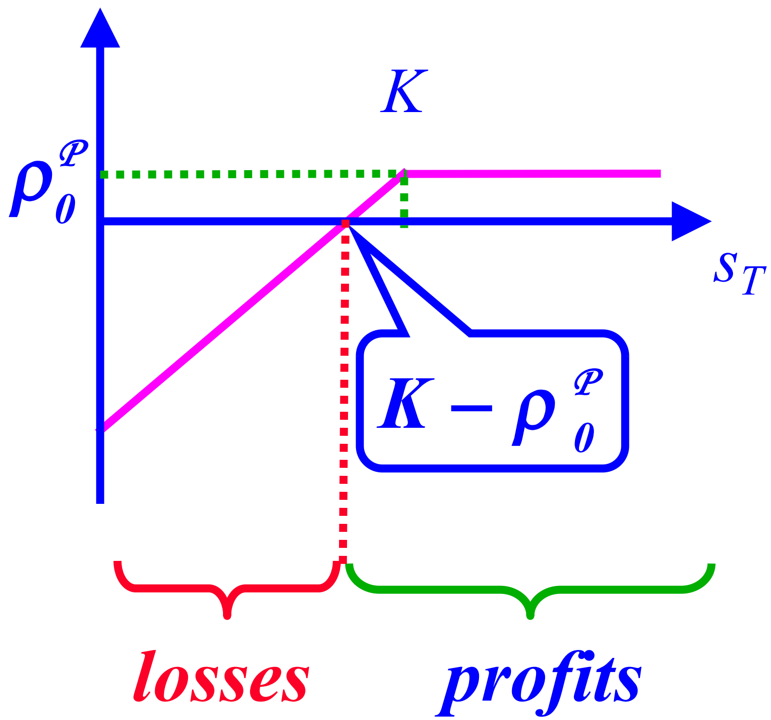
long position

$$\pi_{+}^{\mathcal{P}} = f^{\mathcal{P}}(s_T) - \rho_0^{\mathcal{P}}$$



short position

$$\pi_{-}^{\mathcal{P}} = -f^{\mathcal{P}}(s_T) + \rho_0^{\mathcal{P}}$$



HEDGING

- A *hedger* is a trader interested to reduce the risk he faces; a *hedger* uses financial derivatives to reduce faced exposure to movements in price
- We revisit the currency exchange example: an investor needs to make £ 1,000,000 payment in 180 days and so is faced with significant foreign exchange risks in the volatile currency markets since the investor pays in *US $*

EXAMPLE : FOREIGN EXCHANGE

May 8, 1995 spot and forward foreign exchange for British £ and *US $*

spot	**1.6080**
30 – *day forward*	**1.6076**
90 – *day forward*	**1.6018**
180 – *day forward*	**1.6056**

HEDGING STRATEGY

- Investor signs a forward contract to buy in 180 days £ 1,000,000 for $ 1,605,600; this hedge
  - requires no initial payments
  - provides certainty for the exchange rate
  - need not ensure better outcomes

case	*rate ($/£)*	*investor's gains/losses ($)*
1	1.7000	94,400
2	1.5000	– 105,600

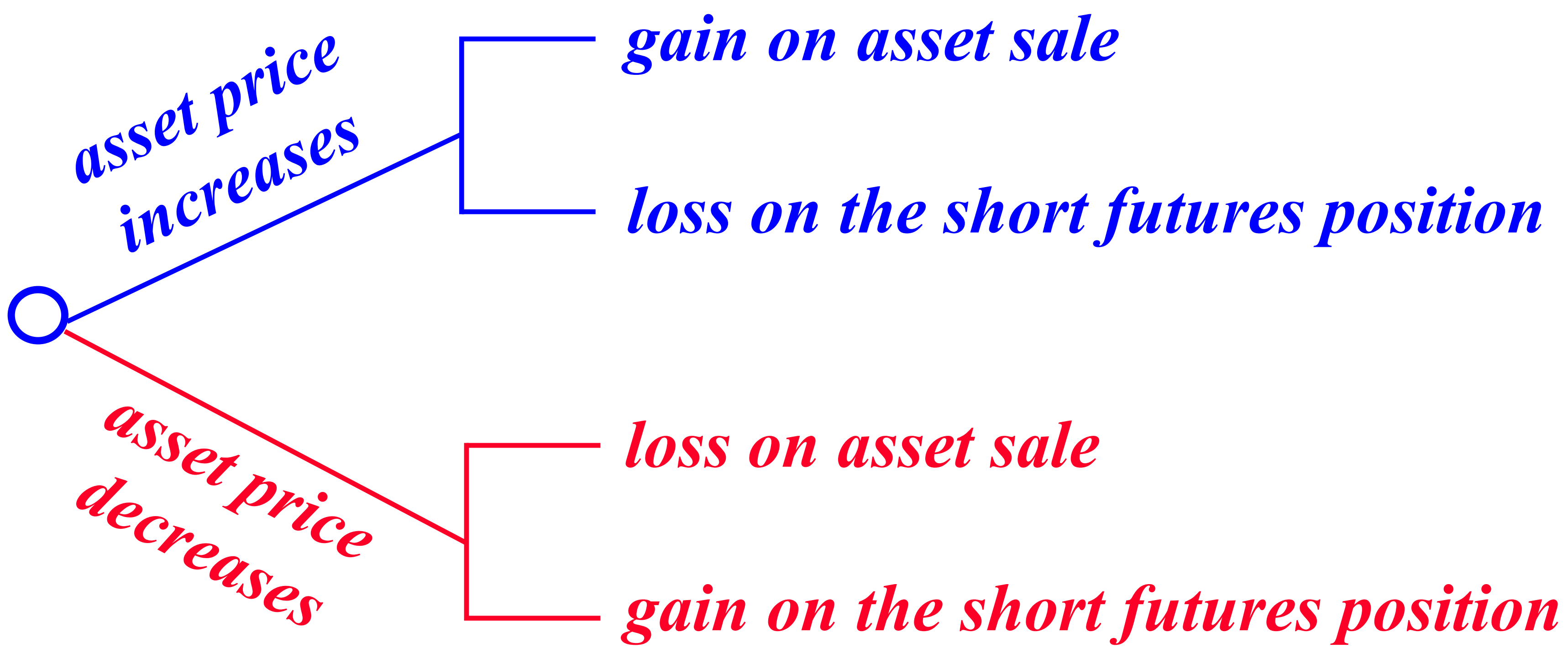
HEDGING STRATEGY

- Investor buys a *call* option to acquire £ 1,000,000 at the exchange rate of 1.6000; this hedge
  - requires an initial outlay of cash for the *call* option premiums
  - provides protection to the investor against adverse exchange rate movements and benefits from favorable movements

case	*exchange rate*	*investor's action*
1	≥ 1.6	exercise option
2	< 1.6	buy £ in market

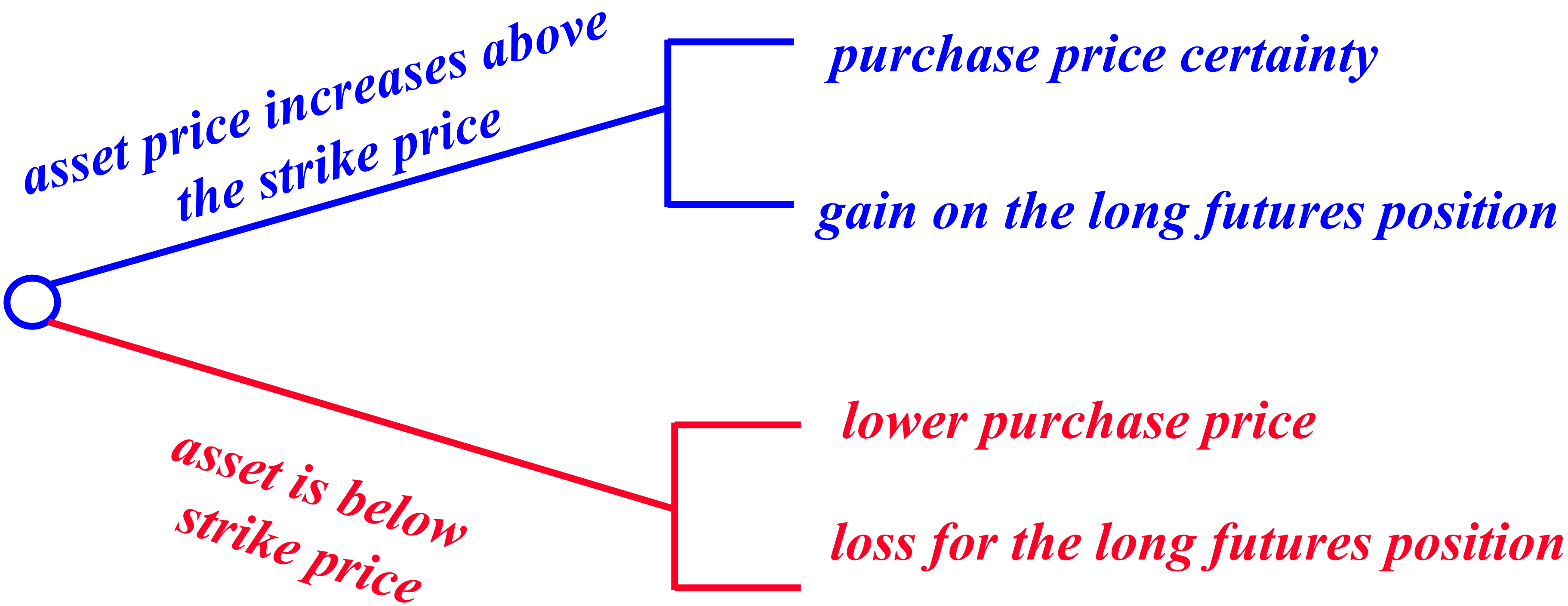
USING *FUTURES* FOR *HEDGING*

- An entity that sells an asset at some given future time can hedge by taking a *short futures position*; this is a *short hedge*



USING *FUTURES* FOR *HEDGING*

- An entity that wishes to buy an asset at some future time can hedge by taking a *long futures position*; this is a *long hedge*



USING *FUTURES* FOR *HEDGING*

- Futures hedging does not necessarily improve the overall financial outcome; in fact, on the average the outcome is worse 50 % of the time
- Futures hedging, however, reduces the risk since it provides price certainty

EXAMPLE : *SHORT HEDGE*

- We consider a generator whose plan is the sale of its $1\ MWh$ energy production at an hour T at a future time
- We assume the production costs for the $1\ MWh$ energy are at $20\ \$ / MWh$
- We assume the energy spot price s_T at hour T has the following discrete distribution

$$s_T = \begin{cases} 18\ \ \$ / MWh & \textit{with probability } 0.5 \\ 26\ \ \$ / MWh & \textit{with probability } 0.5 \end{cases}$$

EXAMPLE : *SHORT HEDGE*

- If the generator sells its energy directly in the spot market, its profits are

$$\pi_T = \begin{cases} -2\ \$ & \textit{with probability } 0.5 \\ 6\ \$ & \textit{with probability } 0.5 \end{cases}$$

- The generator suffers a loss in the case the spot price is 18 *\$/MWh* since his marginal costs are 20 *\$/MWh*
- The generator may protect himself from such a loss by taking a *short futures position*

EXAMPLE : *SHORT HEDGE*

- If the generator sells a *futures* contract $\mathcal{A}$ on 1 *MWh* electricity with the *maturity* T and the *delivery price* $K = 22$ *\$/MWh*, the generator receives the net *payoff* at time T of

$$f^{\mathcal{A}}(s_T) = -(s_T - K) = \begin{cases} 4\ \$ & \text{if} \quad s_T = 18\ \$/MWh \\ -4\ \$ & \text{if} \quad s_T = 26\ \$/MWh \end{cases}$$

- The generator's net profits then become

$$\pi_T^{net} = \pi_T + f^{\mathcal{A}}(s_T) = \begin{cases} 2\ \$ & \textit{with probability } 0.5 \\ 2\ \$ & \textit{with probability } 0.5 \end{cases}$$

EXAMPLE : *SHORT HEDGE*

- So, as the holder of the *short futures position*, the generator's *net profits* become independent of the *spot price*; the generator gains in either case
- In this way, we say the generator's position is *fully hedged*

EXAMPLE : *LONG HEDGE*

- We consider a 1 MW load planning its energy purchases for the hour T at some future time
- We assume the energy spot price s_T at hour T has the following distribution

$$s_T = \begin{cases} 18 \ \ \$ / MWh & with\ probability\ 0.5 \\ \\ 26 \ \ \$ / MWh & with\ probability\ 0.5 \end{cases}$$

EXAMPLE : *LONG HEDGE*

- If the load purchases its energy directly in the spot market, its costs are

$$c_T = \begin{cases} 18\ \$ & \textit{with probability } 0.5 \\ \\ 26\ \$ & \textit{with probability } 0.5 \end{cases}$$

- The load faces uncertainty in the supply costs
- The load may get price certainty by taking a *long futures position*

EXAMPLE : *LONG HEDGE*

- If the load holds a *long futures position* $\mathcal{A}$ on 1 *MWh* energy with the *maturity* T and the *delivery price* $K = 22\ \$ / MWh$ at T, the load receives the net *payoff*

$$f^{\mathcal{A}}(s_T) = s_T - K = \begin{cases} -4\ \$ & \text{if} \quad s_T = 18\ \$ / MWh \\ 4\ \$ & \text{if} \quad s_T = 26\ \$ / MWh \end{cases}$$

- The load's net costs then become

$$c_T^{net} = c_T - f^{\mathcal{A}}(s_T) = \begin{cases} 22\ \$ & \textit{with probability } 0.5 \\ 22\ \$ & \textit{with probability } 0.5 \end{cases}$$

EXAMPLE : *LONG HEDGE*

- **In other words, the load's *long futures position* makes the net costs to be independent of the spot prices; the load gets its price certainty**

- **In this way, we say the load's position is *fully hedged***

SPECULATION

- A speculator takes a position in a market by betting that either that a price increases or a price decreases
- A speculator may purchase the asset on the spot market and rely on
  - future spot markets
  - use *forward contracts* with a higher level of leverage
  - use *options* for additional leverage

SPECULATION

- In the *sterling exchange* example, the investor can speculate and take a long position in a 180-day forward contract on sterling: suppose the speculator buys a 180-day forward contract at conversion rate of 1.6056 and the exchange rate rises to 1.7000, then he makes a profit of

  $ 94,440

EXAMPLE : SPECULATION USING OPTIONS

- A stock price is *$* 43 and an investor is betting that the price will rise and buys *call* options with a *strike price* of *$* 48 at *$* 1 per option
- If the price fails to go above *$* 48 during the life of the option, the speculator loses *$* 1 per option
- If the price rises to *$* 55, the speculator realizes net *profits* of *$* 6 per option – a 600% gain on the original premium investment

ARBITRAGE

- Arbitrageurs lock in *riskless profits* by undertaking transactions in two or more markets
- As an example we consider a situation where a stock is traded on stock exchanges both in *NY* and *London*: suppose stock price is *\$* 172 in *NY* and *£* 100 in *London* when the exchange rate is *\$* 1.750 per *£*:
  - arbitrageur buys 1,000 shares in *NY* and immediately sells them in *London* to realize *risk-free profits* of *\$* 3,000

ARBITRAGE

- such arbitrage opportunities do not last very long because as arbitrageurs buy more stocks in *NY*, the law of supply and demand causes the price to rise and, similarly, as they sell more stock in *London*, the *£* price will decrease so that in a short time the prices become the same at the current exchange rate