ECE 307 – Techniques for Engineering Decisions

15. Value of Information

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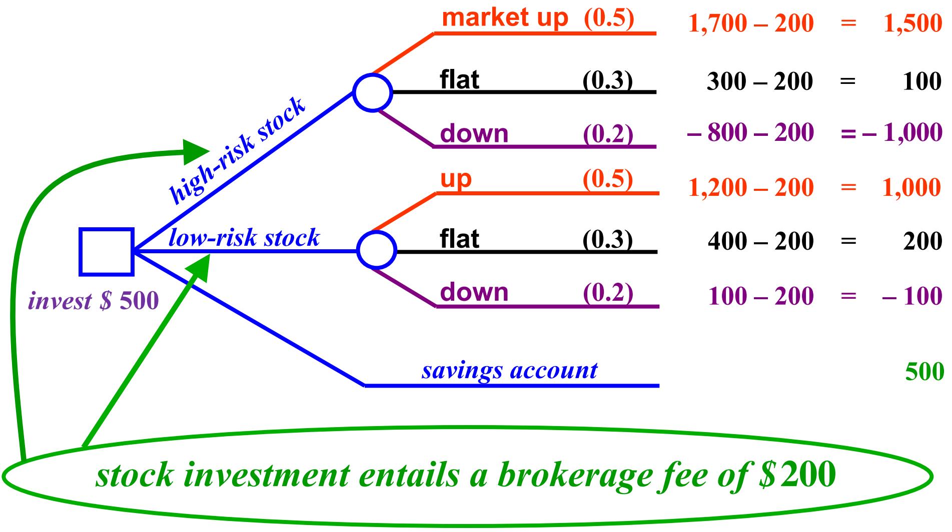
VALUE OF INFORMATION

- While we cannot do away with uncertainty, there is always a natural desire to attempt to reduce the uncertainty about future outcomes
- The quest for reduction in uncertainty about future outcomes may provide us alternatives that *strongly increase the chances* for a good outcome
- We focus this lecture on the principles behind information valuation

A SIMPLE INVESTMENT EXAMPLE



invest $ 500

high-risk stock

market up (0.5) 1,700 – 200 = 1,500

flat (0.3) 300 – 200 = 100

down (0.2) – 800 – 200 = – 1,000

low-risk stock

up (0.5) 1,200 – 200 = 1,000

flat (0.3) 400 – 200 = 200

down (0.2) 100 – 200 = – 100

savings account 500

stock investment entails a brokerage fee of $200

NOTION OF PERFECT INFORMATION

- ☐ **We say that an expert's information is perfect if it is always correct; we may view an expert as a *clairvoyant*, whose future forecasts are correct**
- ☐ **We may quantify the value of information in a decision problem with the quantification of the *expected value of information* (*EVI*)**

NOTION OF PERFECT INFORMATION

- We consider the role of *perfect information* in the simple investment example
- In this decision problem, the optimal policy is to invest in high–risk stock since it has the highest returns on an expected basis
- Suppose an expert predicts that the market goes up: this implies the investor still chooses the high–risk stock investment and consequently the *perfect information* of the expert appears to be of no added value

NOTION OF PERFECT INFORMATION

- On the other hand, suppose the expert predicts a market decrease or a flat market: under this information, the investor's choice is the savings account and the *perfect information* brings value as it leads to a *changed* outcome with improved results over those in the case without the expert
- Under worst case conditions, regardless of the information, we make the identicale decision as

NOTION OF PERFECT INFORMATION

without the information and consequently $EVI = 0$; the interpretation, then, is that we are equally well off without the expert

- ☐ Cases in which we have information and we change to a different optimal decision lead to $EVI > 0$, since we make a decision that improves the outcome using the available information

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EVI ASSESSMENT

- ☐ It follows that the value of information is always nonnegative, $EVI \geq 0$
- ☐ Indeed, *perfect information* removes all uncertainty, and the *expected value of perfect information EVPI* provides an upper bound for *EVI*

$$EVI \leq EVPI$$

A SIMPLE INVESTMENT EXAMPLE: COMPUTATION OF *EVPI*

- Absent any expert information, a value–maximizing investor selects the high–risk stock option
- The introduction of an expert or clairvoyant brings in *perfect information* since there is perfect *a priori* knowledge of how the market will fare before the investor makes his decision and the investor's decision is based on this information

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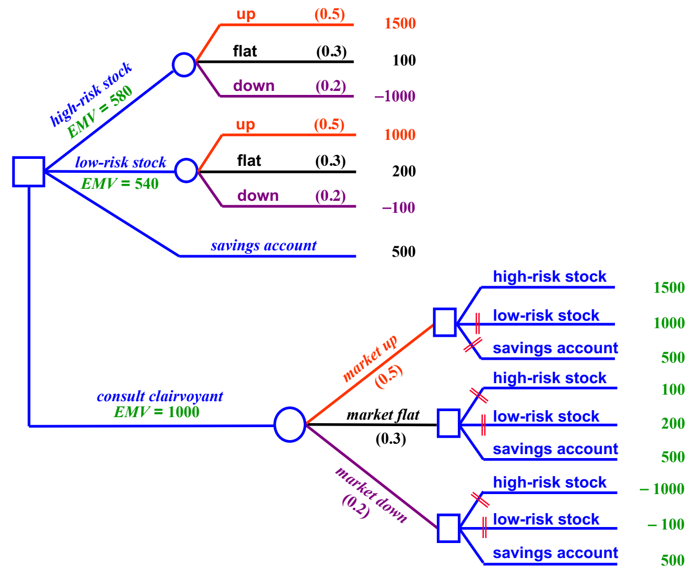
COMPUTATION OF *EVPI*

- We use a decision tree approach to compute *EVPI* and reverse the decision and uncertainty order: we view the value of information in an *a priori* sense and define

$$EVPI = E\ \{decision\ with\ perfect\ information\} -$$

$$E\ \{decision\ absent\ additional\ information\}$$

COMPUTATION OF *EVPI*



COMPUTATION OF *EVPI*

- ❑ For the investment problem,

$$EVPI = 1{,}000 - 580 = 420$$

- ❑ We may view *EVPI* to represent the bound on the amount that the investor is *willing to pay* an expert for the *perfect information* resulting in the improved outcome

EXPECTED VALUE OF IMPERFECT INFORMATION

- ☐ In practice, however, we cannot obtain *perfect information*; rather, the information is *imperfect* since there exist no *clairvoyants*
- ☐ We evaluate the expected value of *imperfect information, EVII*
- ☐ For example, we engage an expert economist to forecast the future stock market trends; the economist's forecasts constitute *imperfect information*: the track record based on past performance is tabulated below

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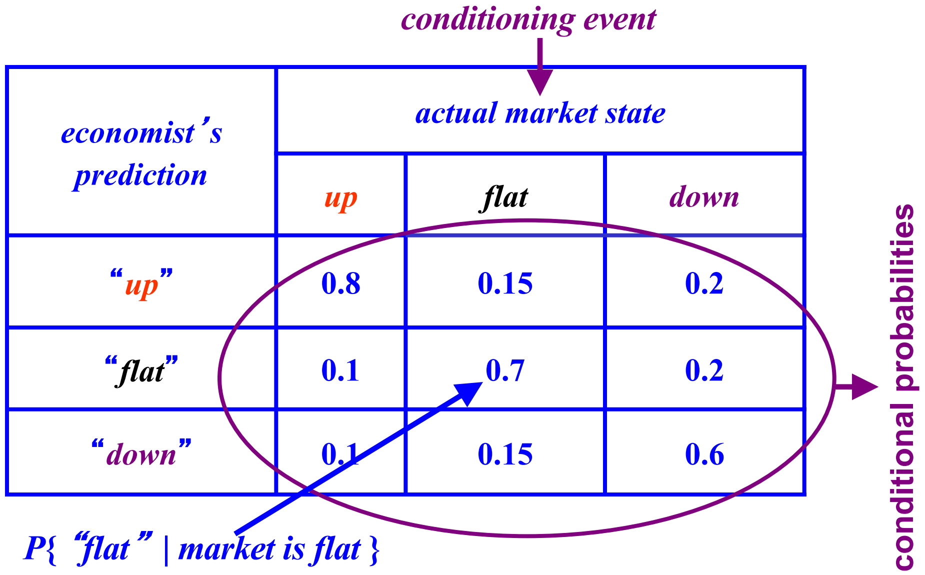
EXPECTED VALUE OF IMPERFECT INFORMATION



conditioning event

economist's prediction	*actual market state*		
	up	*flat*	*down*
"up"	0.8	0.15	0.2
"flat"	0.1	0.7	0.2
"down"	0.1	0.15	0.6

P{ "flat" | market is flat }

conditional probabilities

EVII ASSESSMENT

- We use the decision tree approach to compute the *EVII* and so need to evaluate the *reverse conditional probabilities* via the application of Bayes' theorem
- Under imperfect information, we define the *r.v.*

$$\underset{\sim}{M} = \begin{array}{c}\text{market}\\ \text{performance}\end{array} = \begin{cases} up & with \ \ probability \ \ 0.5 \\ flat & with \ \ probability \ \ 0.3 \\ down & with \ \ probability \ \ 0.2 \end{cases}$$

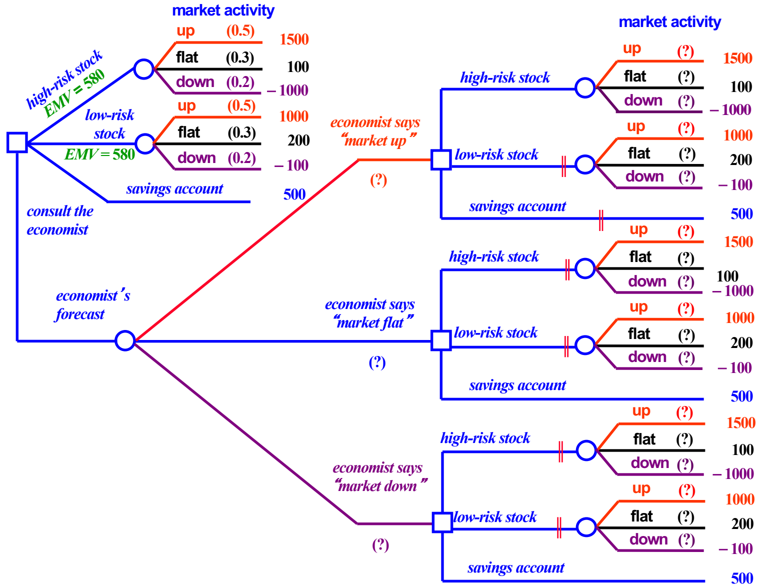
EVII ASSESSMENT

and the forecast *r.v.*

$$\underset{\sim}{F} = \begin{cases} "up" \\ "flat" \\ "down" \end{cases}$$

- We have no knowledge of the probabilities of the forecast *r.v.*; all we know is the prior probabilities of $\underset{\sim}{F}$ given $\underset{\sim}{M}$

EVII COMPUTATION: INCOMPLETE DECISION TREE



COMPUTATION OF REVERSE CONDITIONAL PROBABILITIES

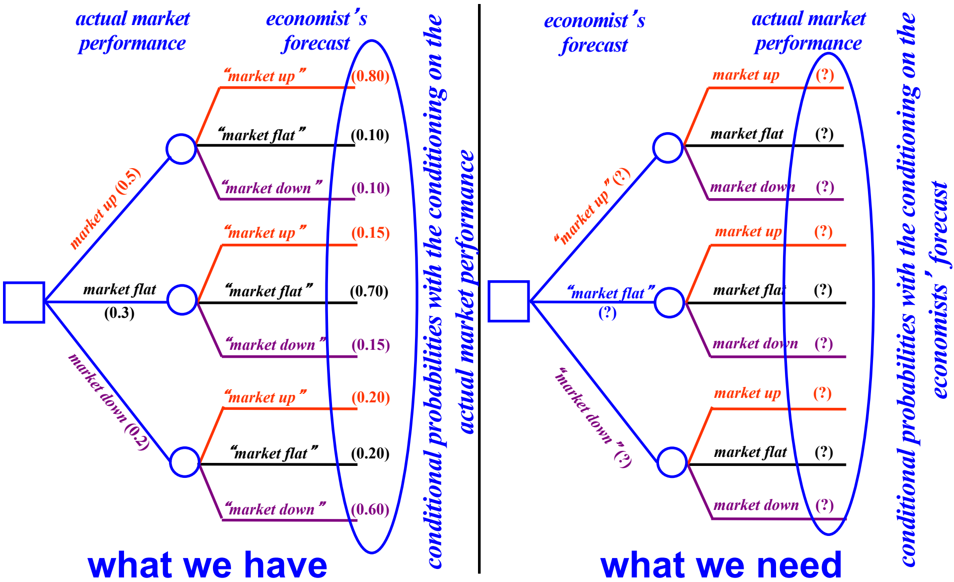
$$P\left\{\underset{\sim}{M} = down \middle| \underset{\sim}{F} = "up"\right\} = \frac{P\left\{\underset{\sim}{F} = "up" \middle| \underset{\sim}{M} = down\right\} P\left\{\underset{\sim}{M} = down\right\}}{\left[P\left\{\underset{\sim}{F} = "up" \middle| \underset{\sim}{M} = down\right\} P\left\{\underset{\sim}{M} = down\right\} + P\left\{\underset{\sim}{F} = "up" \middle| \underset{\sim}{M} = up\right\} P\left\{\underset{\sim}{M} = up\right\} + P\left\{\underset{\sim}{F} = "up" \middle| \underset{\sim}{M} = flat\right\} P\left\{\underset{\sim}{M} = flat\right\}\right]}$$

$$P\left\{\underset{\sim}{F} = "up"\right\} \leftarrow$$

$$= \frac{0.2\left(0.2\right)}{0.2\left(0.2\right) + 0.15\left(0.3\right) + 0.8\left(0.5\right)}$$

In this way, we *flip* the conditional probabilities

EVII COMPUTATION: FLIPPING THE PROBABILITY TREE



what we have **what we need**

POSTERIOR PROBABILITIES

economist's prediction	*posterior probability for:*		
	market up	*market flat*	*market down*
"up"	0.8247	0.0928	0.0825
"flat"	0.1667	0.7000	0.1333
"down"	0.2325	0.2093	0.5581

conditional probabilities on economists forecast

EVII COMPUTATION

- ☐ We use conditional probabilities in the table to build the posterior probabilities
- ☐ For example

$$P\left\{ market\ up \,\middle|\, economist\ \ predicts\ "up" \right\} = 0.8247$$

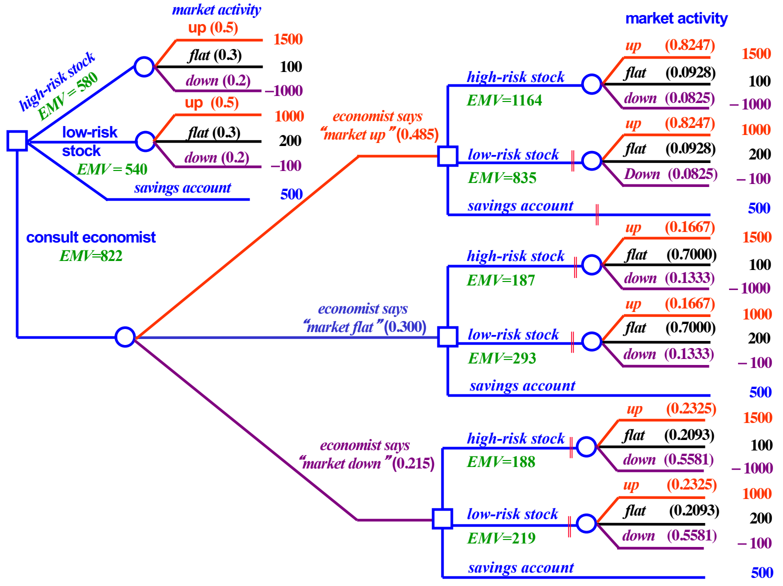
- ☐ We then compute

$$P\left\{ \underset{\sim}{F} = "up" \right\} = 0.485$$

$$P\left\{ \underset{\sim}{F} = "flat" \right\} = 0.300$$

$$P\left\{ \underset{\sim}{F} = "down" \right\} = 0.215$$

EXPECTED VALUE OF IMPERFECT INFORMATION



EVII COMPUTATION

- The expected mean value for the decision made with the economist information is

$$EMV\,|_{economist} \;=\; 1{,}164(0.485) + 500(0.515) = 822$$

- The expected mean value without information is 580
- Consequently,

$$EVII \;=\; 822 \;-\; 580 \;=\; 242$$

- This value represents the bound on the worth of the economist's forecast

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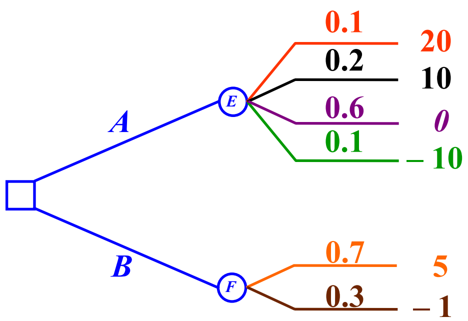
EXAMPLE OF VALUE OF INFORMATION

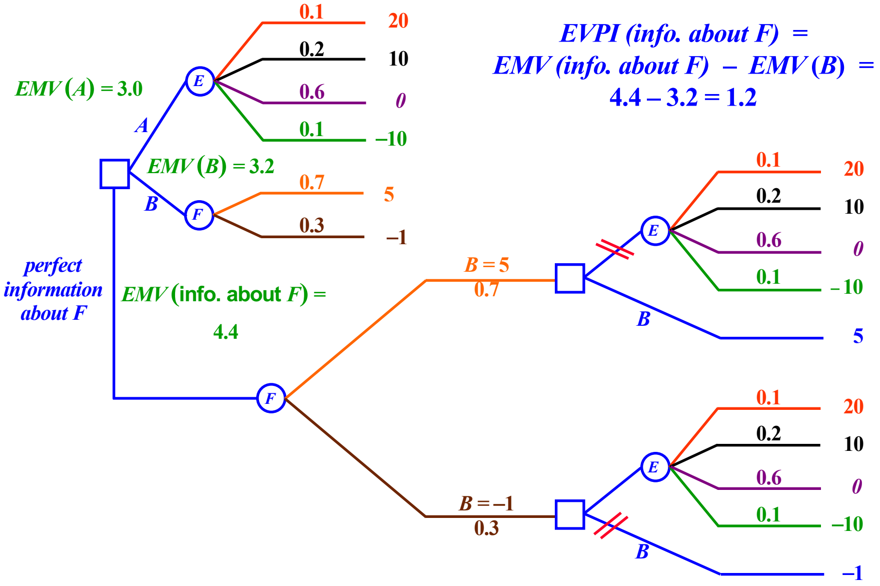
- We consider the following decision tree



with the events at *E* and *F* as independent

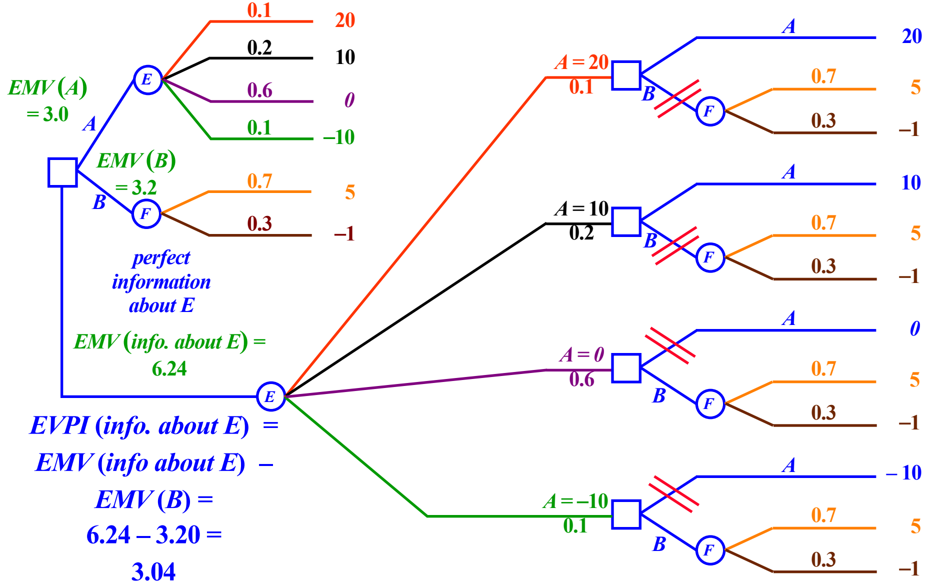
- We perform a number of valuations of *EVPI* for this simple decision problem

EVPI FOR *F* ONLY



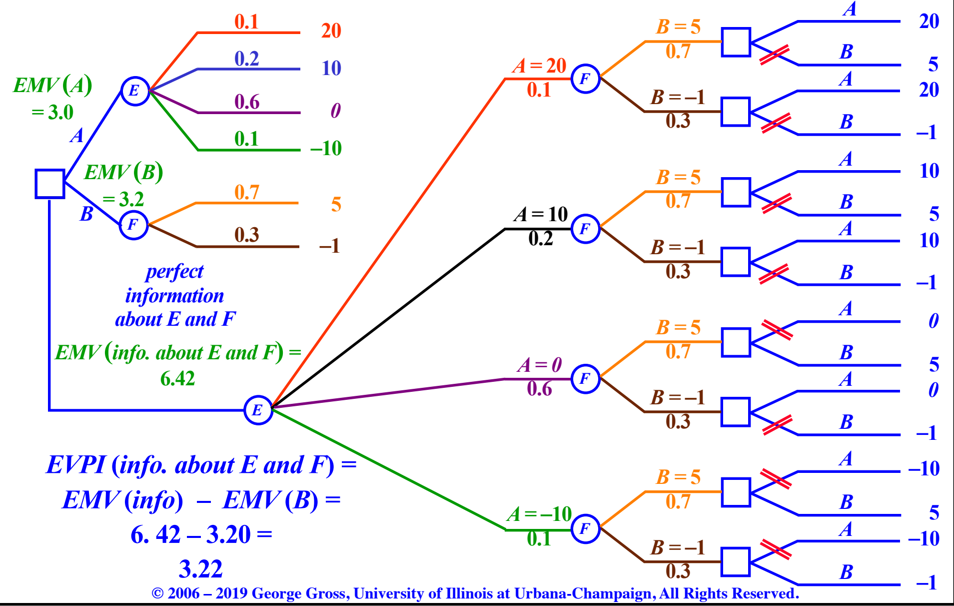
EVPI (info. about F) =
EMV (info. about F) – *EMV* (*B*) =
4.4 – 3.2 = 1.2

EVPI FOR *E* ONLY



EVPI (*info. about E*) =
EMV (*info about E*) –
EMV (*B*) =
6.24 – 3.20 =
3.04

EVPI FOR BOTH *E* AND *F*



EVPI (*info. about E and F*) =
EMV (*info*) − *EMV* (*B*) =
6. 42 − 3.20 =
3.22