ECE 307 – Techniques for Engineering Decisions

14. Simulation

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SIMULATION

- Simulation provides a *systematic* approach to deal with uncertainty by "*flipping a coin*" or "*rolling a die*" to represent the outcome or realization of each uncertain event
- In many real world situations, simulation may be the *only viable means* to quantitatively deal with a problem under uncertainty
- Effective simulation requires implementation of appropriate approximations at many and, some-times, at possibly every stage of the problem

SIMULATION EXAMPLE

- The problem is concerned with the purchase of fabric by a fashion designer
- The two choices offered by textile suppliers are:

  supplier 1: *fixed price – constant* 2 *\$/yd*

  supplier 2: *variable price dependent on quantity at*

  2.10 *\$/yd for the first* 20,000 *yd;*

  1.90 *\$/yd for the next* 10,000 *yd;*

  1.70 *\$/yd for the next* 10,000 *yd;*

  1.50 *\$/yd thereafter*

SIMULATION EXAMPLE

- The purchaser is uncertain about the demand $\underset{\sim}{D}$ but determines an appropriate model is:

$$\underset{\sim}{D} \sim \mathcal{N}(25{,}000\ yd, 5{,}000\ yd)$$

- The decision may be represented in form of the following decision branches:

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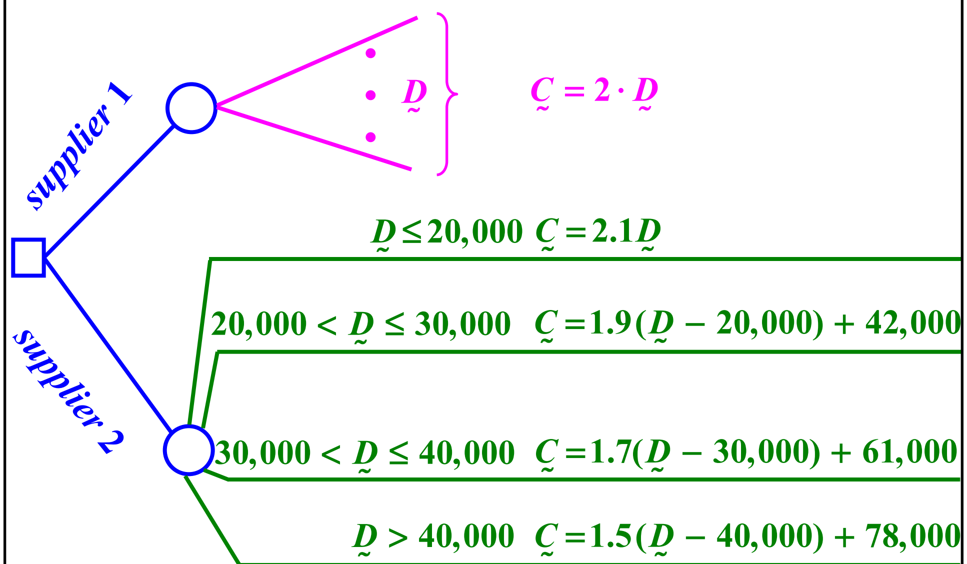
SIMULATION EXAMPLE



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SIMULATION EXAMPLE

- Supplier 1 has a simple linear cost function $\underset{\sim}{C}$
- Supplier 2 has a more complicated scheme to evaluate costs: in effect, the range of the demand and the corresponding probability for $\underset{\sim}{D}$ to be in a particular segment of the range must be known, as well as the expected value of $\underset{\sim}{D}$ for each range

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SIMULATION EXAMPLE

- We simulate the situation in the decision tree by "*drawing multiple samples from the appropriate population*"
- We systematically tabulate the results and evaluate the required statistics
- The algorithm for the simulation consists of a few simple steps which are repeated until an *appropriatly sized* sample is constructed

BASIC ALGORITHM

Step *0* : store the distribution $\mathcal{N}\left(25{,}000,\ 5{,}000\right)$; determine $\bar{k}$, the maximum number of draws; set $k = 0$

Step 1 : if $k > \bar{k}$, stop; else set $k = k + 1$

Step 2 : draw a random sample from the normal distribution $\mathcal{N}\left(25{,}000,\ 5{,}000\right)$

Step 3 : evaluate the outcomes on both branches; enter each outcome into the data base and return to Step 1

SIMULATION EXAMPLE

- Application of the algorithm allows the determination of the histogram of the cost figures and then the evaluation of the expected costs
- For the assumed demand, for supplier 1, we have the straightforward case of

$$\mu_{\underset{\sim}{C}} = E\{\underset{\sim}{C}\} = 2 \cdot E\{\underset{\sim}{D}\} = 50,000$$

and

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SIMULATION EXAMPLE

$$\sigma_{\underset{\sim}{C}} = 10,000$$

and the use of the algorithm may be bypassed

- For the supplier 2, the algorithm is applied for the $\bar{k}$ random draws
- The actual simulation is an exercise left to the reader

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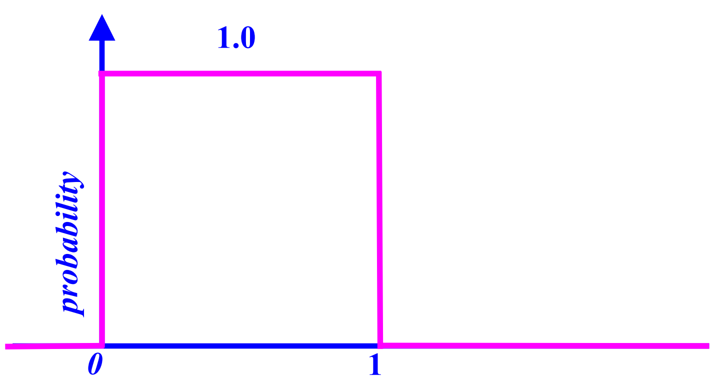
GENERATION OF RANDOM DRAWS

- A key issue is the generation of random draws for which we need a random number generator
- There are various random number generator algorithms
- One intuitive scheme is based on the use of a uniformly distributed *r.v.* between *0* and 1

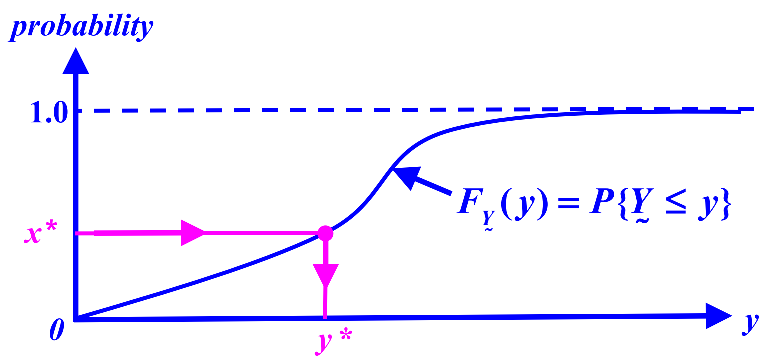
GENERATION OF RANDOM DRAWS



$$\underset{\sim}{X} = \begin{cases} x \in [0,1] \text{ with probability } 1 \\ x \notin [0,1] \text{ with probability } 0 \end{cases}$$

GENERATION OF RANDOM DRAWS

- We draw a random value of x, say x^* and work through the *c.d.f.* $F_{\underset{\sim}{Y}}(y)$ to get the value y^* of the *r.v.* $\underset{\sim}{Y}$ *with* $F_{\underset{\sim}{Y}}(y^*) = x^*$



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SOFT PRETZEL EXAMPLE

- The market size is unknown but we assume that the market size is a normally distributed *r.v.* with

$$\underset{\sim}{S} \sim \mathcal{N}\left(\underbrace{100{,}000}_{\mu_{\underset{\sim}{S}}}, \underbrace{10{,}000}_{\sigma_{\underset{\sim}{S}}}\right)$$

- We are interested to determine the fraction $\underset{\sim}{F}$ of the market the new company is able to capture
- We model the distribution of $\underset{\sim}{F}$ using the discrete distribution tabulated below:

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SOFT PRETZEL EXAMPLE

$\underset{\sim}{F} = x\ \%$	$P\{\underset{\sim}{F} = x\}$
16	0.15
19	0.35
25	0.35
28	0.15

SOFT PRETZEL EXAMPLE

- Sales price of a pretzel is $\$\ 0.50$
- Variable costs $\underset{\sim}{V}$ are represented by a uniformly distributed *r.v.* in the range $[0.08\ ,\ 0.12]$ *\$/pretzel*
- Fixed costs $\underset{\sim}{C}$ are also random
- The contributions to profits are given by

$$\underset{\sim}{\Pi} = (\underset{\sim}{S} \cdot \underset{\sim}{F}) \cdot (0.5 - \underset{\sim}{V}) - \underset{\sim}{C}$$

  and may be evaluated via simulation
- We can use simulation to approximate $F_{\underset{\sim}{\Pi}}(\bullet)$

MANUFACTURING CASE STUDY

- We focus on the selection of one of two manufacturing *processes* based on net present value (*NPV*) using a 3–year horizon – the current year *0* plus the years 1 and 2 – under a 10 % discount rate
- The *process* is used to manufacture a product whose sales price is 8 *$/unit*

PROCESS 1 DESCRIPTION

- This *process* uses the current machinery for manufacturing
- The annual fixed costs are *$* 12,000
- The yearly variable costs are represented by the *r.v.*

$$\underset{\sim}{V}_i \sim \mathcal{N}\left(4, 0.4\right) \qquad i = 0, 1, 2$$

PROCESS 1 DESCRIPTION

- Machine in the *process* can fail randomly and the number failures $\underset{\sim}{Z}_i$ in year $i = 0,1,2$ is a *r.v.* with

$$\underset{\sim}{Z}_i \sim Poisson(m = 4) \qquad i = 0,1,2$$

- Each failure incurs constant costs of *\$* 8,000 over the 3-year period
- Total costs are the sum of $\underset{\sim}{V}_i$ and $8,000\ \underset{\sim}{Z}_i$

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PROCESS 1: SALES FORECAST UNCERTAINTY DATA

current year $i = 0$		*next year* $i = 1$		*year after next* $i = 2$	
d_0	$P\{\underset{\sim}{D}_0 = d_0\}$	d_1	$P\{\underset{\sim}{D}_1 = d_1\}$	d_2	$P\{\underset{\sim}{D}_2 = d_2\}$
11,000	0.2	8,000	0.2	4,000	0.1
16,000	0.6	19,000	0.4	21,000	0.5
21,000	0.2	27,000	0.4	37,000	0.4

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PROCESS 2: DESCRIPTION

- *Process* 2 involves an investment of $\$60{,}000$ paid in cash to buy the new equipment and doing away with the worthless current machinery; the fixed costs of $\$12{,}000$ per year remain unchanged
- The yearly variable costs $\underset{\sim}{V}_i$ are normal *r.v.*s

$$\underset{\sim}{V}_i \sim \mathcal{N}\left(\$3.50,\ \$1.0\right) \qquad i = 0, 1, 2$$

- The number of machine failures $\underset{\sim}{Z}_i$ for year

$$\underset{\sim}{Z}_i \sim Poisson\left(m = 3\right) \qquad i = 0, 1, 2$$

and the costs per failure are $\$\ 6{,}000$

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PROCESS 1: SALES FORECAST UNCERTAINTY DATA

current year $i = 0$		*next year* $i = 1$		*year after next* $i = 2$	
d_0	$P\{\underset{\sim}{D}_0 = d_0\}$	d_1	$P\{\underset{\sim}{D}_1 = d_1\}$	d_2	$P\{\underset{\sim}{D}_2 = d_2\}$
14,000	0.3	12,000	0.36	9,000	0.4
19,000	0.4	23,000	0.36	26,000	0.1
24,000	0.3	31,000	0.28	42,000	0.5

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NET PROFITS

- The net profits $\underset{\sim}{\pi}_i$ each year are a function

$$\underset{\sim}{\pi}_i = f\left(\underset{\sim}{D}_i, \underset{\sim}{V}_i, \underset{\sim}{Z}_i\right) \qquad i = 0, 1, 2$$

- While for each *process,* the $F_{\underset{\sim}{\pi}_i}(\cdot)$ approximation requires the evaluation of all the possible outcomes, both $E\left\{\underset{\sim}{\pi}_i\right\}$ and $var\left\{\underset{\sim}{\pi}_i\right\}$ may be estimated by simulation by drawing an appropriate number of samples from the underlying distribution

NPV

- The *NPV* of these profits needs to be assessed and expressed in terms of the current *year 0 dollars*
- The profits are collected at the end of each year or equivalently, at the beginning of the following year
- We use the $d = 10\ \%$ discount factor to express the $var\left\{\underset{\sim}{\pi}_i\right\}$ in year *0* (*current*) *dollars* squared

NPV

- We can evaluate for *processes* 1 and 2 the profits for each year; we use superscript to denote each specific *process*

$$process\ 1:\ \underset{\sim}{\pi}_i^1 = 8\underset{\sim}{D}_i - \underset{\sim}{D}_i\underset{\sim}{V}_i - 8{,}000\underset{\sim}{Z}_i - 12{,}000$$

$$i = 0, 1, 2$$

$$process\ 2:\ \underset{\sim}{\pi}_i^2 = 8\underset{\sim}{D}_i - \underset{\sim}{D}_i\underset{\sim}{V}_i - 6{,}000\underset{\sim}{Z}_i - 12{,}000$$

and we also need to account for the *\$* 60,000 *investment* in year *0* for *process* 2

NPV

- The *NPV* evaluation then is stated as the *r.v.*

$$\underset{\sim}{\Pi}^1 = \sum_{i=0}^{2} \underset{\sim}{\pi}_i^1 (1.1)^{-(i+1)}$$

and

evaluated in year 0 dollars

$$\underset{\sim}{\Pi}^2 = -60{,}000 + \sum_{i=0}^{2} \underset{\sim}{\pi}_i^2 (1.1)^{-(i+1)}$$

- Simulation is used to evaluate

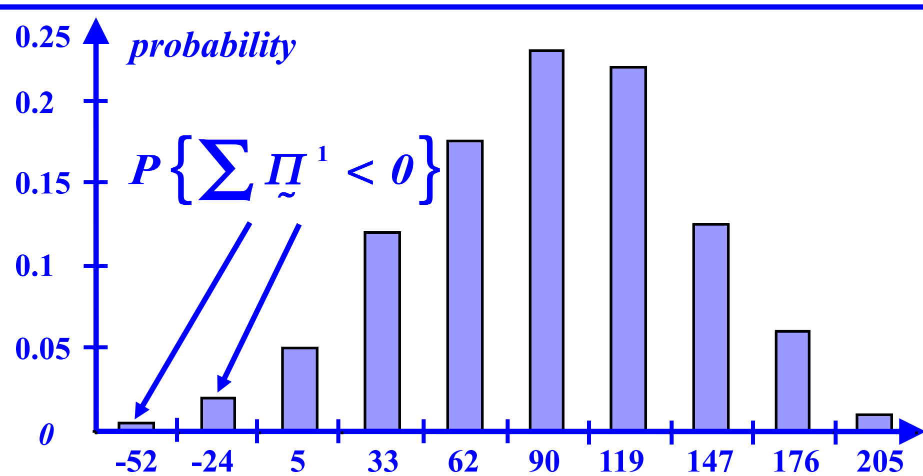
$$NPV^1 = E\left\{\underset{\sim}{\Pi}^1\right\} \quad NPV^2 = E\left\{\underset{\sim}{\Pi}^2\right\}$$

SIMULATION RESULTS

- For a 1,000 replications we obtain

process j	*mean (\$)*	*standard deviation (\$)*	$P\{\sum \underset{\sim}{\Pi}^{j} < 0\}$
1	91,160	46,970	0.029
2	110,150	72,300	0.046

SIMULATION RESULTS

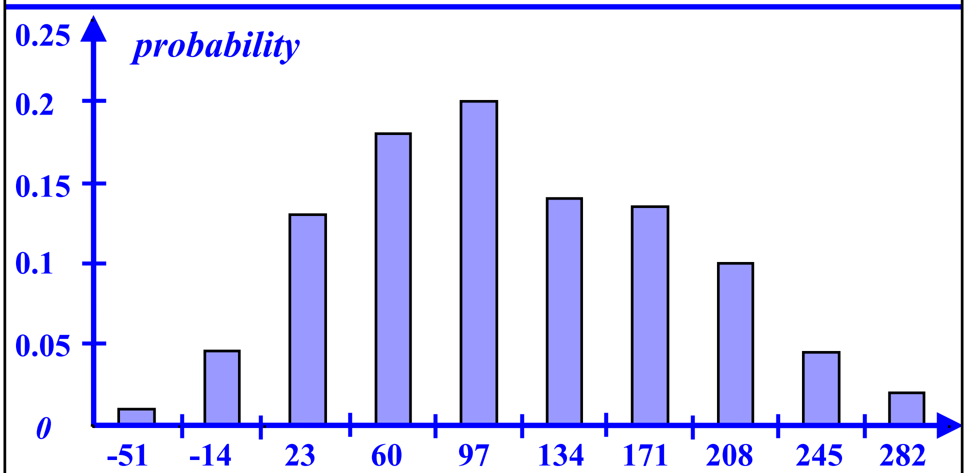


$NPV^{1} = 91,160$

$\sigma = 46,970$

$P\{\sum \underset{\sim}{\Pi}^{1} < 0\} = 0.029$

SIMULATION RESULTS



$\overline{NPV}^{\,2} = 110{,}150$

$\sigma = 72{,}300$

$P\left\{\sum \underset{\sim}{\Pi}^{2} < 0\right\} = 0.046$

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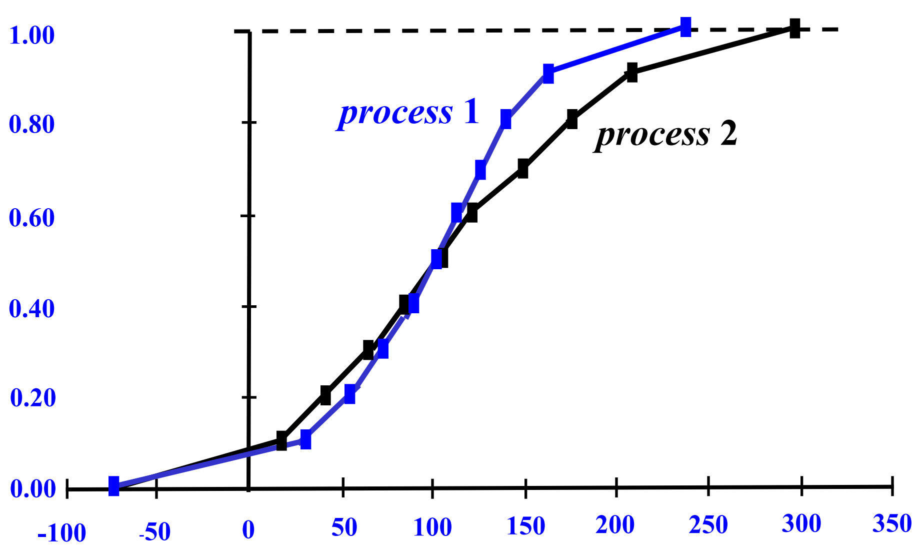
*c.d.f.*s OF THE TWO PROCESSES



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