We use the following notation for this problem –

- car A: outcome that the car is behind door A and analogous definitions for car B and car C

Then,

\[ P\{\text{car } A\} = P\{\text{car } B\} = P\{\text{car } C\} = \frac{1}{3} \]

which indicates that for the car to be behind any one of the 3 doors is equally likely

I pick door A and the host knows where the car is; the possible outcomes are:
PROBLEM 7.27

(i) car is behind door C

\[ P\left\{ \text{host picks door } B \mid \text{ car } C \right\} = 1 \]

(ii) car is behind door A that I picked as my choice

\[ P\left\{ \text{host picks door } B \mid \text{ car } A \right\} = \]

\[ P\left\{ \text{host picks door } C \mid \text{ car } A \right\} = \frac{1}{2} \]

(iii) car is behind door B

\[ P\left\{ \text{host picks door } B \mid \text{ car } B \right\} = 0 \]

Now

\[ P\left\{ \text{car } C \mid \text{ host picks door } B \right\} = \]

\[ \frac{P\left\{ \text{car } C \text{ and host picks door } B \right\}}{P\left\{ \text{host picks door } B \right\}} = \]
PROBLEM 7.27

\[ P\left( \text{host picks door B} \mid \text{car C} \right) P\left( \text{car C} \right) \]

\[ = \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{3} \]

\[ \quad = \frac{2}{3} \]

\[ P\left( \text{host picks door B} \mid \text{car A} \right) P\left( \text{car A} \right) \]

\[ = \frac{1}{2} \times \frac{1}{3} \]

\[ P\left( \text{host picks door B} \mid \text{car B} \right) P\left( \text{car B} \right) \]

\[ = \frac{1}{3} \times \frac{1}{3} \]

\[ P\left( \text{host picks door B} \mid \text{car C} \right) P\left( \text{car C} \right) \]

\[ = \frac{1}{3} \times \frac{1}{3} \]

Therefore, you should switch when the host reveals the goat.

PROBLEM 9 – 24

\[ P\left( \bar{Q} > 0.22 \right) = P\left( \bar{Q} < 0.08 \right) = 0.1 \]

and

\[ P\left( \bar{Q} > 0.14 \right) = P\left( \bar{Q} < 0.14 \right) = 0.5 \]
PROBLEM 9.24

Therefore

\[ P\{0.08 < Q < 0.14\} = P\{Q < 0.14\} - P\{Q < 0.08\} = 0.4 \]
\[ P\{0.14 < Q < 0.22\} = P\{Q > 0.14\} - P\{Q > 0.22\} \]
\[ = P\{Q < 0.22\} - P\{Q < 0.14\} \]
\[ = 0.4 \]

We pick \( n = 40, r = 6 \) and obtain beta distribution data from the tableau in the Appendix

\[ P_b\{Q \leq 0.0829 | n = 40, r = 6\} = 0.1 \]
\[ P_b\{Q \leq 0.2249 | n = 40, r = 6\} = 0.9 \]

Also,

\[ E\{Q\} = \frac{r}{n} = \frac{6}{40} = 0.15 \]

However,

\[ P_β\{Q \leq 0.1441 | n = 40, r = 6\} = 0.5 \]

and therefore,

\[ P_β\{Q \leq 0.15 | n = 40, r = 6\} > 0.5 \]
PROBLEM 9.26

- The inheritance can be invested entirely in \( \text{Mac} \) or in \( \text{USS} \) and we are given that
  
  \[ P\{\text{invested in Mac}\} = 0.8 \]

  and so

  \[ P\{\text{invested in USS}\} = 0.2 \]

- Each year return on investment is normal with
  
  \[ R_{\text{Mac}} \sim \mathcal{N}(14\%, 4\%) \]

  \[ R_{\text{USS}} \sim \mathcal{N}(12\%, 3\%) \]

  and the yearly returns are independent r.v.s.

PROBLEMS 9.26 (a)

- We compute then

  \[ P\{0.06 < R < 0.18 | \text{investment in Mac}\} \]

  \[ = P\left\{\frac{0.06 - 0.14}{0.04} < \frac{Z}{0.04} < \frac{0.18 - 0.14}{0.04}\right\} \]

  \[ = P\{-2 < Z < 1\} \]

  \[ = 0.8185 \]
PROBLEMS 9.26 (a)

Similarly

\[ P \{0.06 < R < 0.18 \mid \text{investment in USS} \} \]
\[ = P \left\{ \frac{6 - 12}{3} < Z < \frac{18 - 12}{3} \right\} \]
\[ = P \{ -2 < Z < 2 \} \]
\[ = 0.9544 \]

PROBLEM 9.26 (b)

Then, the unconditional probability is

\[ P \{6 < R < 18\} = P \{6 < R < 18 \mid \text{Mac}\} P \{\text{Mac}\} + \]
\[ P \{6 < R < 18 \mid \text{USS}\} P \{\text{USS}\} \]
\[ = 0.8185(0.8) + 0.9544(0.2) \]
\[ = 0.84568 \]
PROBLEM 9.26 (c)

- We are given \( P\{R > 12\} \) and wish to compute \( P\{\text{investment in Mac} \mid R > 12\} \).
- We compute

\[
P\{R > 12 \mid \text{Mac}\} = P\left\{ Z > \frac{12 - 14}{4} \right\} = P\left\{ Z > -0.5 \right\} = 0.6915
\]

and

\[
P\{R > 12\mid \text{USS}\} = P\left\{ Z > \frac{12 - 12}{3} \right\} = P\left\{ Z > 0 \right\} = 0.5
\]

Then \( P\left\{\text{Mac} \mid R > 12\right\} = \)

\[
\frac{P\{R > 12 \mid \text{Mac}\} P\{\text{Mac}\}}{P\{R > 12 \mid \text{Mac}\} P\{\text{Mac}\} + P\{R > 12\mid \text{USS}\} P\{\text{USS}\}}
\]

\[
= \frac{(0.6915)(0.8)}{(0.6915)(0.8) + (0.5)(0.2)}
\]

\[
= 0.847
\]
PROBLEM 9.26 (d)

- We are given that
  \[ P\{Mac\} = P\{USS\} = 0.5 \]
- Then,
  \[ E\{R\} = E\{R|Mac\} P\{Mac\} + E\{R|USS\} P\{USS\} \]
  \[ 0.13 = 0.5\{0.14 + 0.12\} \]
  and
  \[ \text{var}\{R\} = (0.5)^2 \text{var}\{R|Mac\} + (0.5)^2 \text{var}\{R|USS\} \]
  \[ = 0.25\{(0.04)^2 + (0.03)^2\} \]
  \[ 0.0625 = (0.5)^2 (0.5)^2 \Rightarrow \sigma_R = 0.25 \]

PROBLEM 9.31 (a)

- We know that the length r.v.
  \[ L \sim \mathcal{N}(5.9, 0.0365) \]
- We compute
  \[ P\{\text{not fit in a 6" envelope}\} = P\{L > 5.975\} \]
  \[ = P\left\{ Z > \frac{5.975 - 5.9}{0.0365} \right\} \]
  \[ = P\{Z > 2.055\} \]
  \[ = 0.02 \]
PROBLEM 9.31 (b)

- We have a box with \( n = 20 \) and a failure occurs whenever an envelope does not fit into a box:
  \[
P\{\text{no fit}\} = P\{L > 5.975\} = 0.02
\]

- From the binomial distribution for \( n = 20 \) with \( q = 0.02 \) we compute the \( P\{\text{2 or more no fits}\} \)

- The event of two or more no fits in a population of 20 is the event of 18 or less fits

\[
P\{\text{fit}\} = 1 - P\{\text{not fit}\} = 0.98
\]

\[
P\{\hat{R} \leq 18\} = P\{\hat{R} \geq 2\}
\]

number of no fits out of 20

\[
= 1 - P\{\hat{R} \leq 1\}
\]

binomial \((20; 0.02)\)

\[
= 1 - 0.94
\]

\[
= 0.06
\]
PROBLEM 9.31 (b)

- The interpretation of the .06 is as follows: we have the result that we expect, on average, that 6% of the boxes contain 2 or more cards that do not fit the envelopes.

9.34

- On average, 7.5 people arrive in 30 minutes since
  \[
  \frac{30 \text{ min}}{4 \text{ min/person}} = 7.5 \text{ persons}
  \]
  and so we have the number of arriving people \( X \) as an r.v. with \( X \sim \text{Poisson}(m = 7.5) \).

- A simplistic way to solve the problem is to view the individual 40% preference of each arriving...
9.34

person to be independent of the arrivals and then
treat the number of arriving persons who prefer
the new recipe as a r.v. \( \bar{P} \) with mean \((40\%)(7.5) = 3\)
and so

\[
\bar{P} \sim \text{Poisson}(m = 3)
\]

Table look up produces

\[
P\{P \geq 4\} = 0.353
\]

9.34

A more rigorous approach is to treat the perfor-
mance of each arrival as a binomial

\(X = \text{number of arrivals in 30 minutes} \sim \text{Poisson}(m = 7.5)\)

Each arrival \(i\) has a preference \(P_i\) for new recipe

with

\[
P_i \sim \text{binomial}(n = \bar{X}, \ p = 0.4)
\]
We need to compute \[ P \left\{ \sum_i P_i \geq 4 \right\} \]

We condition over the number of arrivals

\[
P \left\{ \sum_i P_i \geq 4 \right\} = \sum_{n=1}^{\infty} P \left\{ \sum_{i=1}^{n} P_i \geq 4 \mid X \geq n \right\} P \left\{ X = n \right\} \]

\[
= P \left\{ \sum_{i=1}^{4} P_i \geq 4 \mid X \geq 4 \right\} P \left\{ X = 4 \right\} + P \left\{ \sum_{i=1}^{5} P_i \geq 4 \mid X \geq 5 \right\} P \left\{ X = 5 \right\} + P \left\{ \sum_{i=1}^{6} P_i \geq 4 \mid X \geq 6 \right\} P \left\{ X = 6 \right\} + \ldots
\]

Note that \[ P \left\{ \sum_i P_i \geq 4 \right\} P \left\{ X = n \right\} \] is simply

the binomial distribution value with parameters \((n, 0.4)\) and \[ P \left\{ X = n \right\} \] is the Poisson distribution

value with \(m = 7.5\)

The sum has insignificant contributions for \(n > 16\)
10.12: PROBLEM FORMULATION

- This is a multi-period planning problem with a 7-month horizon.

- Define the following for use in backward regression:

  - **Stage**: A month in the planning period.
  - **State variable**: The number of crankcases $S_n$ left over from the stage $(n-1)$, $n = 1, 2, \ldots, N$.

  With $S_1 = 0$ (initial stage) and $S_0$ unspecified.

---

10.12: PROBLEM FORMULATION

- **Decision variables**: Purchase amount $d_n$ for stage $n$, $n = 1, 2, \ldots, 7$.

- **Transition function**: The relationship between the amount in inventory, purchase decision and demand in stages $n$ and $(n-1)$.

  $$ S_{n-1} = S_n + d_n - D_n, \quad n = 1, 2, \ldots, N $$

  Where,

  $$ D_n = \text{demand at stage } n, \quad n = 1, 2, \ldots, N $$
10.12: PROBLEM FORMULATION

Return function: costs of purchase in stage \( n \) plus the inventory holding costs, with the mathematical expression

\[
f_n^*(S_n) = C_n + (S_n + d_n - D_n)0.50 + f_{n-1}^*(S_{n-1})
\]

and

\[
f_0^*(S_0) = 0
\]

10.12: STAGE 1 SOLUTION

\( D_1 = 600 \)

\[
f_1^*(S_1) = \min_{d_1} \{ C_1 + (S_1 + d_1 - D_1)0.50 \}
\]

<table>
<thead>
<tr>
<th>( S_1 )</th>
<th>value of ( f_1^* ) for ( d_1 )</th>
<th>( f_1^*(S_1) )</th>
<th>( d_1^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>500</td>
<td>1000</td>
<td>1500</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>5200</td>
<td>7950</td>
</tr>
<tr>
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<td>3000</td>
<td>5250</td>
<td>8000</td>
</tr>
<tr>
<td>200</td>
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<td>5400</td>
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</tr>
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<td>600</td>
<td>0</td>
<td>3250</td>
<td>5500</td>
</tr>
</tbody>
</table>
## 10.12: STAGE 2 SOLUTION

\[ D_2 = 1200 \]

\[
f^*_2(S_2) = \min_{d_2} \left\{ C_2 + (S_2 + d_2 - D_2)0.50 + f^*_1(S_2 + d_2 - D_2) \right\}
\]

<table>
<thead>
<tr>
<th>( S_2 )</th>
<th>( f^*_2(S_2) ) for ( d_2 )</th>
<th>( f^*_1(0.50) )</th>
<th>( d^*_2 )</th>
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<td>500</td>
<td>8250</td>
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<td>1000</td>
</tr>
<tr>
<td>600</td>
<td>8350</td>
<td>8350</td>
<td>1000</td>
</tr>
</tbody>
</table>

## 10.12: STAGE 3 SOLUTION

\[ D_3 = 900 \]

\[
f^*_3(S_3) = \min_{d_3} \left\{ C_3 + (S_3 + d_3 - D_3)0.50 + f^*_2(S_3 + d_3 - D_3) \right\}
\]

<table>
<thead>
<tr>
<th>( S_3 )</th>
<th>( f^*_3(S_3) ) for ( d_3 )</th>
<th>( f^*_2(0.50) )</th>
<th>( d^*_3 )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>15900</td>
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</tr>
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<td>600</td>
<td>13300</td>
<td>13300</td>
<td>500</td>
</tr>
</tbody>
</table>
10.12: STAGE 4 SOLUTION

\[ D_4 = 400 \]

\[ f^*_4(S_4) = \min_{d_4} \left\{ C_4 + (S_4 + d_4 - D_4)0.50 + f^*_3(S_4 + d_4 - D_4) \right\} \]

<table>
<thead>
<tr>
<th>( S_4 )</th>
<th>value of ( f_4 ) for ( d_4 )</th>
<th>( f^*_4(S_4) )</th>
<th>( d^*_4 )</th>
</tr>
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</tr>
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</tr>
</tbody>
</table>

10.12: STAGE 5 SOLUTION

\[ D_5 = 800 \]

\[ f^*_5(S_5) = \min_{d_5} \left\{ C_5 + (S_5 + d_5 - D_5)0.50 + f^*_4(S_5 + d_5 - D_5) \right\} \]

<table>
<thead>
<tr>
<th>( S_5 )</th>
<th>value of ( f_5 ) for ( d_5 )</th>
<th>( f^*_5(S_5) )</th>
<th>( d^*_5 )</th>
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</thead>
<tbody>
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</table>
10.12: STAGE 6 SOLUTION

\[ D_6 = 1100 \]

\[ f^*_6(S_6) = \min_{d_6} \left\{ C_6 + (S_6 + d_6 - D_6) 0.50 + f^*_5(S_6 + d_6 - D_6) \right\} \]

<table>
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<th>value of ( f_6 ) for ( d_6 )</th>
<th>( f^*_6(S_6) )</th>
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</table>

10.12: STAGE 7 SOLUTION

- For stage 7, \( D_7 = 700 \) and

\[ f^*_7(S_7) = \min_{d_7} \left\{ C_7 + (S_7 + d_7 - D_7) 0.50 + f^*_6(S_7 + d_7 - D_7) \right\} \]

- Optimal total cost over 7 months = $31,400

obtained using the purchasing policy below

<table>
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<tr>
<th>month</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<tbody>
<tr>
<td>amount of material</td>
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<td>1000</td>
<td>1000</td>
<td>0</td>
<td>1000</td>
<td>1500</td>
<td>500</td>
</tr>
</tbody>
</table>
12.7: OIL WILDCATTING PROBLEM: DECISION TREE

Decision Tree:
- **EMV = $10k**
  - Drill: Strike oil (0.1) -> $190k
     - Dry hole (0.9) -> $0k
  - Don't drill: $0k

- **EMV = $10k**
  - Consult Clairvoyant:
    - Drill: Strike oil (0.1) -> $190k
      - Dry hole (0.9) -> $0k
    - Don't drill: $0k

12.7: BLOCK DIAGRAMS

Basic Model:
- **Strike oil?**
  - Drill -> Payoff

Perfect Information:
- **Strike oil?**
  - Drill -> Payoff
12.7 *EVPI AND EVII*

- We evaluate the expected value of the clairvoyant information
  
  \[ EVPI = \frac{EMV(\text{clairvoyant})}{\$19k} - \frac{EMV(\text{drill})}{\$10k} = \$9k \]

- We have the following conditional probabilities
  
  \[ P\left(\text{“good”} \mid \text{oil}\right) = 0.95 \quad \text{and} \quad P\left(\text{“poor”} \mid \text{dry}\right) = 0.85 \]

- We are also given that
  
  \[ P\left(\text{dry}\right) = 0.9 \quad \text{and} \quad P\left(\text{oil}\right) = 0.1 \]

- We can find \( P\left(\text{“good”}\right) \) and \( P\left(\text{“poor”}\right) \) with the law of total probability

\[
P\left(\text{“good”}\right) = P\left(\text{“good”} \mid \text{oil}\right) P\left(\text{oil}\right) + P\left(\text{“good”} \mid \text{dry}\right) P\left(\text{dry}\right) = (0.95)(0.1) + (0.15)(0.9) = 0.23
\]

\[
P\left(\text{“poor”}\right) = 1 - P\left(\text{“good”}\right) = 1 - 0.23 = 0.77
\]
12.7 EVPI AND EVII

Now we can find

\[
P\{\text{oil} | \text{"good"}\} = \frac{P\{\text{"good"} | \text{oil}\} P\{\text{oil}\}}{P\{\text{"good"} | \text{oil}\} P\{\text{oil}\} + P\{\text{"good"} | \text{dry}\} P\{\text{dry}\}}
\]

\[
= \frac{(0.95)(0.1)}{(0.95)(0.1) + (0.15)(0.9)}
\]

\[
= 0.41
\]

\[
P\{\text{dry} | \text{"good"}\} = 1 - P\{\text{oil} | \text{"good"}\} = 0.59
\]

and

\[
P\{\text{oil} | \text{"poor"}\} = \frac{P\{\text{"poor"} | \text{oil}\} P\{\text{oil}\}}{P\{\text{"poor"} | \text{oil}\} P\{\text{oil}\} + P\{\text{"poor"} | \text{dry}\} P\{\text{dry}\}}
\]

\[
= \frac{(0.05)(0.1)}{(0.05)(0.1) + (0.85)(0.9)}
\]

\[
= 0.0065
\]