ECE 307 – Techniques for Engineering Decisions

15. Value of Information

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While we cannot do away with uncertainty, there is always a desire to attempt to reduce the uncertainty about future outcomes. The reduction in uncertainty about future outcomes may provide us with choices that strongly improve the chances for a good outcome. We focus on the principles behind information valuation.
SIMPLE INVESTMENT EXAMPLE

stock investment entails a brokerage fee of $200
NOTION OF PERFECT INFORMATION

- We say that an expert’s information is perfect if it is always correct; we may view an expert as a clairvoyant.

- We can quantify the value of information in a decision problem by measuring the expected value of information (EVI).
NOTION OF PERFECT INFORMATION

- We consider the role of perfect information in the simple investment example.
- In this decision problem, the optimal policy is to invest in high-risk stock since it has the highest returns.
- Suppose an expert predicts that the market goes up: this implies the investor still chooses the high-risk stock investment and consequently the perfect information of the expert appears to be of no value.
On the other hand, suppose the expert predicts a market decrease or a flat market: under this information, the investor’s choice is the savings account and the *perfect information* has value because it leads to a *changed* outcome with better performance than would be the case otherwise.

In worst case conditions: regardless of the information, we take the same decision as...
without the information and consequently

\[ EVI = 0; \] the interpretation is that we are equally well off without the expert

Cases in which we have information and we change to a different optimal decision lead to

\[ EVI > 0 \] since we make a decision that improves the outcome using the available information
It follows that the value of information is always nonnegative, $EVI \geq 0$

Indeed, *perfect information* removes all uncertainty, and the *expected value of perfect information* $EVPI$ provides an upper bound for $EVI$

$$EVI \leq EVPI$$
INVESTMENT EXAMPLE: COMPUTATION OF *EVPI*

- Absent any expert information, a value – maximizing investor selects the high–risk stock option.

- The introduction of an expert or clairvoyant brings in *perfect information* since there is perfect *a priori* knowledge of what the market will do before the investor makes his decision and the investor’s decision is based on this information.
COMPUTATION OF EVPI

- We use a decision tree approach to compute EVPI by reversing the decision and uncertainty order:

we view the value of information in an a priori sense and define

\[
EVPI = E \{\text{decision with perfect information}\} - E \{\text{decision without additional information}\}
\]
COMPUTATION OF EVPI

- **High-risk stock**
  - **EMV = 580**
  - **Up (0.5):** 1500
  - **Flat (0.3):** 100
  - **Down (0.2):** -1000
  - **Low-risk stock**
  - **EMV = 540**
  - **Up (0.5):** 1000
  - **Flat (0.3):** 200
  - **Down (0.2):** -100
  - **Savings account**
    - **EMV = 500**

- **Consult clairvoyant**
  - **EMV = 1000**
  - **Market up (0.5):**
    - **High-risk stock:** 1500
    - **Low-risk stock:** 1000
    - **Savings account:** 500
  - **Market flat (0.3):**
    - **High-risk stock:** 100
    - **Low-risk stock:** 200
    - **Savings account:** 500
  - **Market down (0.2):**
    - **High-risk stock:** -1000
    - **Low-risk stock:** -100
    - **Savings account:** 500
COMPUTATION OF $EVPI$

- For the investment problem,

$$EVPI = 1,000 - 580 = 420$$

- We may view $EVPI$ to represent the maximum amount that the investor is willing to pay the expert for the perfect information resulting in the improved outcome.
EXPECTED VALUE OF IMPERFECT INFORMATION

- In practice, we cannot obtain perfect information; rather, the information is imperfect since there are no clairvoyants.

- We evaluate the expected value of imperfect information, EV\(\theta\).

- For example, we engage an economist to forecast the future stock market trends; the economist’s forecasts constitute imperfect information: the track record based on past performance is...
EXPECTED VALUE OF IMPERFECT INFORMATION

<table>
<thead>
<tr>
<th>economist’s prediction</th>
<th>up</th>
<th>flat</th>
<th>down</th>
</tr>
</thead>
<tbody>
<tr>
<td>“up”</td>
<td>0.8</td>
<td>0.15</td>
<td>0.2</td>
</tr>
<tr>
<td>“flat”</td>
<td>0.1</td>
<td>0.7</td>
<td>0.2</td>
</tr>
<tr>
<td>“down”</td>
<td>0.1</td>
<td>0.15</td>
<td>0.6</td>
</tr>
</tbody>
</table>

$P\{\text{“flat”} \mid \text{market is flat}\}$

conditioning event

$P\{\text{“flat”} \mid \text{market is flat}\}$

conditional probabilities
EVII ASSESSMENT

- We use the decision tree approach to compute $EVII$

- For the decision tree, we evaluate probabilities with Bayes’ theorem

- For the imperfect information, we define the r.v.

\[ M = \begin{cases} \text{market} \\
\text{performance} \end{cases} = \begin{cases} \text{up} & \text{with probability} & 0.5 \\
\text{flat} & \text{with probability} & 0.3 \\
\text{down} & \text{with probability} & 0.2 \end{cases} \]
EVII ASSESSMENT

and the forecast \( r.v. \)

\[
F = \begin{cases} 
"up" \\
"flat" \\
"down"
\end{cases}
\]

- We have no knowledge of the probabilities of the forecast \( r.v. \); all we know is the prior probabilities of \( F \) given \( M \)
In the decision tree, the economist's forecast is the starting point. The low-risk stock has an expected value (EV) of 580, while the high-risk stock has a lower EV. The savings account provides a guaranteed return.

1. **Economist's Forecast**:
   - **Market up**: 50% chance of a 1500 return, 30% chance of a 1000 return, 20% chance of a 200 return.
   - **Market flat**: 50% chance of a 100 return, 30% chance of a 200 return.
   - **Market down**: 50% chance of a -1000 return, 30% chance of a -100 return.

2. **High-Risk Stock**:
   - **Market up**: 50% chance of a 1500 return, 30% chance of a 1000 return, 20% chance of a 200 return.
   - **Market flat**: 50% chance of a 100 return, 30% chance of a 200 return.
   - **Market down**: 50% chance of a -1000 return, 30% chance of a -100 return.

3. **Low-Risk Stock**:
   - **Market up**: 50% chance of a 1500 return, 30% chance of a 1000 return, 20% chance of a 200 return.
   - **Market flat**: 50% chance of a 100 return, 30% chance of a 200 return.
   - **Market down**: 50% chance of a -1000 return, 30% chance of a -100 return.

**Expected Value (EMV)** for each decision point is calculated as follows:

- **High-Risk Stock**: EMV = 0.5 * 1500 + 0.3 * 1000 + 0.2 * 200 = 850
- **Low-Risk Stock**: EMV = 0.5 * 1000 + 0.3 * 200 + 0.2 * (-1000) = 300
- **Savings Account**: EMV = 0.5 * 100 + 0.3 * 200 + 0.2 * (-100) = 100

Given these calculations, the **EMV of a savings account** is 100, making it the most attractive option under expected utility maximization.
COMPUTATION OF REVERSE CONDITIONAL PROBABILITIES

\[ P \{ \bar{M} = \text{down} \mid \bar{F} = "up" \} = \]

\[ P \{ \bar{F} = "up" \mid \bar{M} = \text{down} \} P \{ \bar{M} = \text{down} \} \]

\[
\left[ P \{ \bar{F} = "up" \mid \bar{M} = \text{down} \} P \{ \bar{M} = \text{down} \} + 
P \{ \bar{F} = "up" \mid \bar{M} = \text{up} \} P \{ \bar{M} = \text{up} \} + 
P \{ \bar{F} = "up" \mid \bar{M} = \text{flat} \} P \{ \bar{M} = \text{flat} \} \right]
\]

\[ = \frac{0.2(0.2)}{0.2(0.2) + 0.15(0.3) + 0.8(0.5)} \]

We flip the probabilities in this way.
EVII COMPUTATION: FLIPPING THE PROBABILITY TREE

what we have

actual market performance

market flat (0.3)

market down (0.2)

what we need

economist’s forecast

market flat (0.70)

market down (0.15)

market up (0.20)

market flat (0.20)

market down (0.60)

conditional probabilities with the conditioning on the actual market performance

“market flat” (0.10)

“market up” (0.80)

“market flat” (0.10)

market up (0.15)

market flat (0.70)

market down (0.10)

market up (0.20)

market flat (0.20)

market down (0.60)

actual market performance

market flat (0.70)

market down (0.15)

market up (0.20)

market flat (0.20)

market down (0.60)

conditional probabilities with the conditioning on the economists’ forecast

“market flat” (0.10)

“market up” (0.80)

“market flat” (0.10)

market up (0.15)

market flat (0.70)

market down (0.10)

market up (0.20)

market flat (0.20)

market down (0.60)

what we have

actual market performance

market flat (0.3)

market down (0.2)

what we need

economist’s forecast

market flat (0.70)

market down (0.15)

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conditional probabilities with the conditioning on the actual market performance

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market up (0.15)

market flat (0.70)

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market up (0.20)

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actual market performance

market flat (0.3)

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<th>Market Flat</th>
<th>Market Down</th>
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</thead>
<tbody>
<tr>
<td>&quot;Up&quot;</td>
<td>0.8247</td>
<td>0.0928</td>
<td>0.0825</td>
</tr>
<tr>
<td>&quot;Flat&quot;</td>
<td>0.1667</td>
<td>0.7000</td>
<td>0.1333</td>
</tr>
<tr>
<td>&quot;Down&quot;</td>
<td>0.2325</td>
<td>0.2093</td>
<td>0.5581</td>
</tr>
</tbody>
</table>

**Posterior Probability for:**

- **Market Up**: 0.8247
- **Market Flat**: 0.0928
- **Market Down**: 0.0825

Conditional probabilities on economists forecast.
We use conditional probabilities in the table to build the posterior probabilities.

For example,

$$P \left\{ \text{market up}\mid \text{economist predicts } "\text{up}" \right\} = 0.8247$$

We then compute

$$P \left\{ F = "\text{up}" \right\} = 0.485$$

$$P \left\{ F = "\text{flat}" \right\} = 0.300$$

$$P \left\{ F = "\text{down}" \right\} = 0.215$$
EXPECTED VALUE OF IMPERFECT INFORMATION

- **High-risk stock**: EMV = 580
  - Market activity:
    - Up (0.5): 1500
    - Flat (0.3): 100
    - Down (0.2): -1000
  - Consult economist: EMV = 822

- **Low-risk stock**: EMV = 540
  - Market activity:
    - Up (0.5): 1000
    - Flat (0.3): 200
    - Down (0.2): -100
  - Consult economist: EMV = 293

- **Savings account**: EMV = 500
  - Market activity:
    - Up (0.1667): 500
    - Flat (0.7000): 1000
    - Down (0.1333): -1000

The economist says:

- "Market up" (0.485) for high-risk stock: EMV = 1164
- "Market flat" (0.300) for low-risk stock: EMV = 835
- "Market down" (0.215) for low-risk stock: EMV = 187

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EVII COMPUTATION

- The expected mean value for the decision made with the economist information is

  \[ EMV_{\text{economist}} = 1,164(0.485) + 500(0.515) = 822 \]

- The expected mean value without information is 580

- Consequently,

  \[ EVII = 822 - 580 = 242 \]

- This value represents the bound on the worth of the economist’s forecast
We consider the following decision tree with the events at $E$ and $F$ as independent.

We perform a number of valuations of $EVPI$ for this simple decision problem.
EVPI FOR $F$ ONLY

$EMV(A) = 3.0$

$EMV(B) = 3.2$

Perfect information about $F$

$EMV(\text{info. about } F) = 4.4$

$EVPI (\text{info. about } F) = EMV (\text{info. about } F) - EMV(B) = 4.4 - 3.2 = 1.2$
**EVPI FOR E ONLY**

- **EMV (A)** = 3.0
- **EMV (B)** = 3.2
- **EMV (info. about E)** = 6.24
- **EVPI (info. about E)** = **EMV (info about E)** – **EMV (B)** = 6.24 – 3.20 = **3.04**
**EVPI FOR BOTH E AND F**

- **EMV(A)** = 3.0
- **EMV(B)** = 3.2
- **EMV(info. about E and F)** = 6.42

**EVPI (info. about E and F)** = 
**EMV(info) − EMV(B) =** 
6.42 − 3.20 = 
3.22