ECE 220 Computer Systems & Programming

Lecture 13 – Recursive sorting and Recursion with Backtracking

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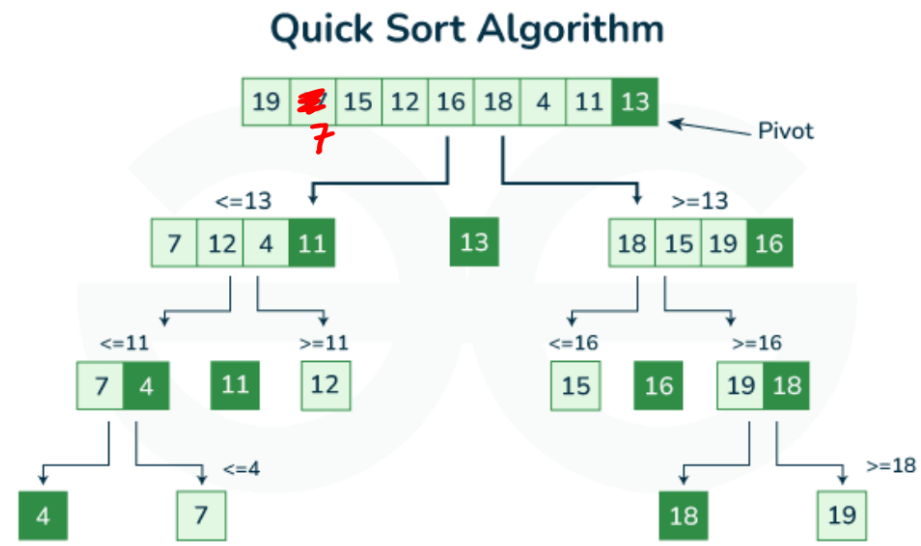
Binary Search (recursive)

```c
// C program to implement binary search using recursion
#include <stdio.h>

// A recursive binary search function. It returns location
// of x in given array arr[l..r] if present, otherwise -1
int binarySearch(int arr[], int l, int r, int x)
{
    // checking if there are elements in the subarray
    if (r >= l) {

        // calculating mid point
        int mid = (l + r) / 2;

        // If the element is present at the middle itself
        if (arr[mid] == x)
            return mid;

        // If element is smaller than mid, then it can only
        // be present in left subarray
        if (arr[mid] > x) {
            return binarySearch(arr, l, mid - 1, x);
        }

        // Else the element can only be present in right
        // subarray
        return binarySearch(arr, mid + 1, r, x);
    }

    // We reach here when element is not present in array
    return -1;
}
```

Recursive Quick Sort

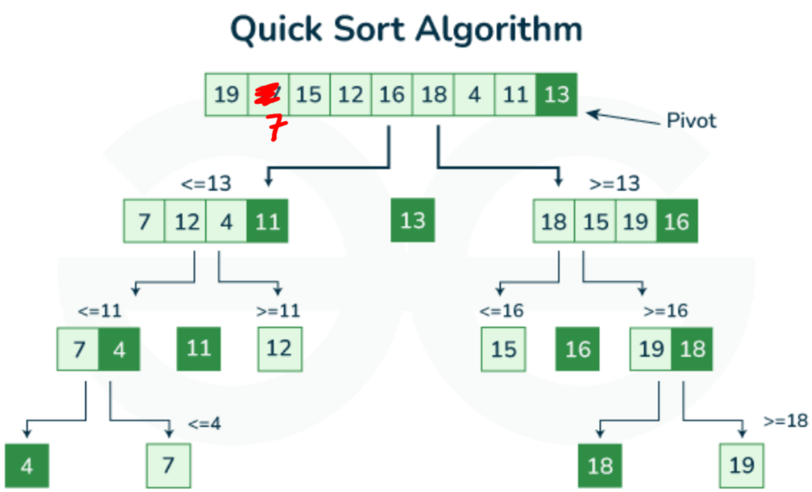
```c
void swap(int *a, int *b)
{
    int tmp;
    tmp=*a;
    *a=*b;
    *b=tmp;
}

void print(int array[], int size)
{
    int i;
    for(i=0;i<size;i++)
    printf("%d ", array[i]);
        printf("\n");
}

int main()
{
    int size=8;
    int l=0;
    int h=size-1;
    int array[8]={10, 20, 80, 30,100, 90, 15, 40};
    quickSort(array,l,h);
    print(array, size);
    return 0;
```



https://www.geeksforgeeks.org/quick-sort-in-c/

Recursive Quick Sort (cont.)

```c
void quickSort(int array[], int l, int h)
{
    if (l<h){
    // Call partion() to get the partition index
    int p=partition(array, l, h);
    // Call partion() to partition the left side
    quickSort(array, l, p-1);
    // Call partion() to partition the right side
    quickSort(array, p+1, h);
    }

}
```




```c
int partition(int array[], int l, int h)
{
int i=l-1;
int x=array[h];
int j;
for (j=l;j<=h-1;j++){
    if (array[j]<=x)
    {
        i++;
        swap(&array[i], &array[j]);

    }

}
swap(&array[i+1], &array[h]);
return (i+1);

}
```

Recursive Bubble Sort

```c
#include <stdio.h>
#define SIZE 8
void bubble_recursion(int *array, int n);

int main()
{
    int n = SIZE-1;
    int array[] = {6,5,3,1,8,7,2,4};
int i;
bubble_recursion(array, n);

    printf("sorted array: \n");
    for(i=0;i<SIZE;i++){
        printf("%d ", array[i]);
    }
    printf("\n");

    return 0;
}
```

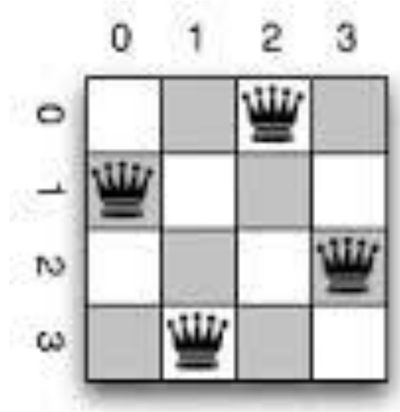
```c
void bubble_recursion(int *array, int n)
{
if(n==0)
return;
int i, temp, swap = 0;

    //sort number in ascending order

        swap = 0;
        for(i=0;i<n;i++)
        {
            //swap the two numbers if order is incorrect
            if(array[i]>array[i+1])
            {
                temp = array[i];
                array[i] = array[i+1];
                array[i+1] = temp;
                //set the swap flag
                swap = 1;
            }
        }
n--;
if (swap==0)
return;
bubble_recursion(array, n);
}
```

Recursive Backtracking:

Backtracking is an algorithmic-technique for solving problems recursively by trying to build a solution incrementally, one piece at a time, removing those solutions that fail to satisfy the constraints of the problem statement.

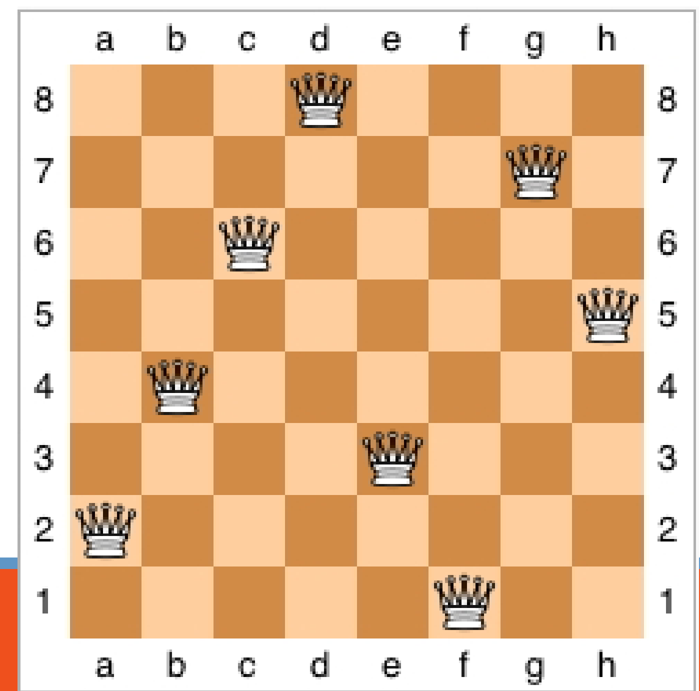
N queens problem using recursive Backtracking

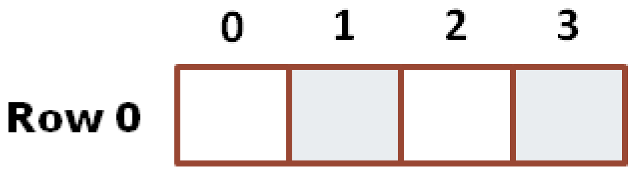
- Place N queens on an NxN chessboard so that none of the queens are under attack;

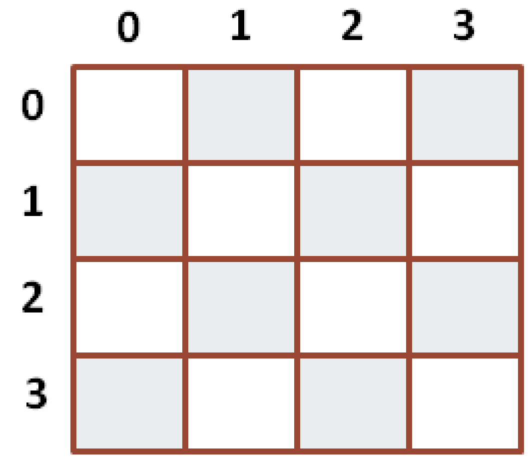


0	0	1	0
1	0	0	0
0	0	0	1
0	1	0	0

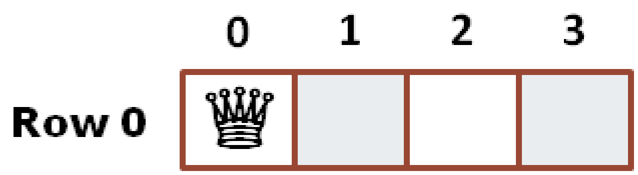
- Brute force: total number of possible placements:
- ~N^2 Choose N ~ 4.4 B (N=8)

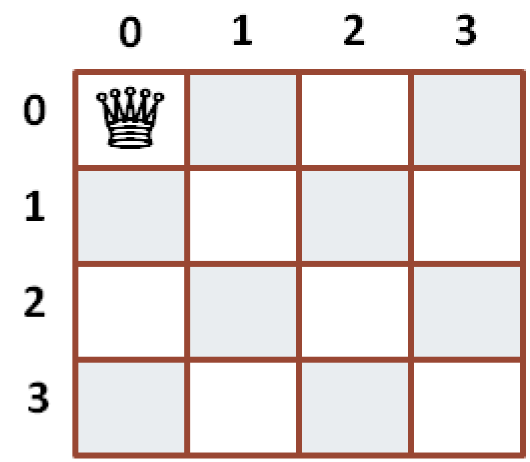






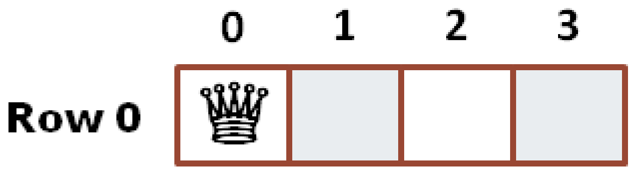
QueenPosition[ ]

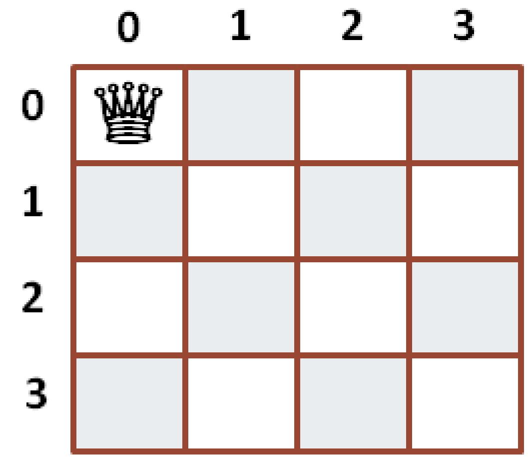




QueenPosition[ ]

Place **0**th Queen on the **0**th Column of **0**th Row

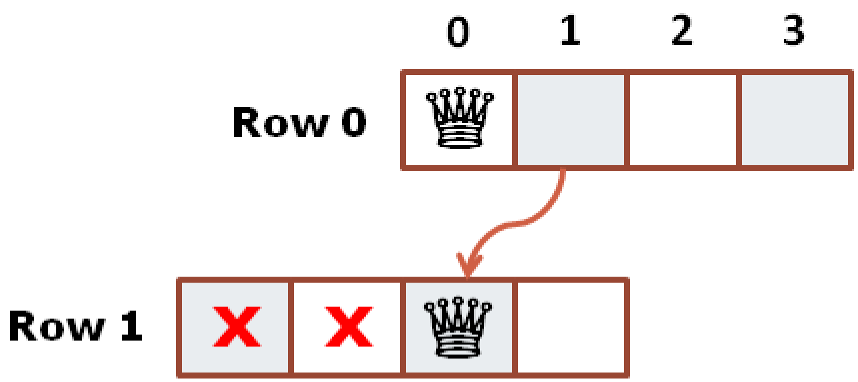


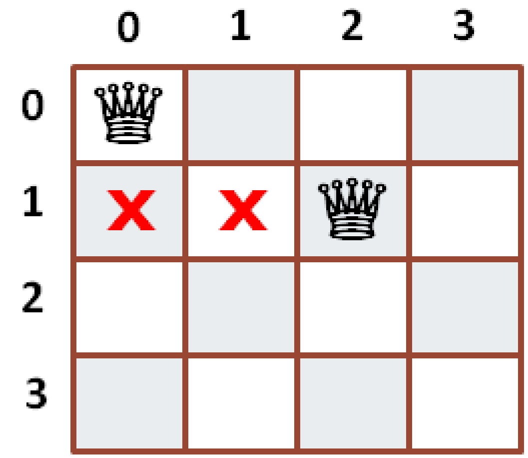


QueenPosition[ ]

Row 0 (0,0)

Add **0**th Queen's position to position array



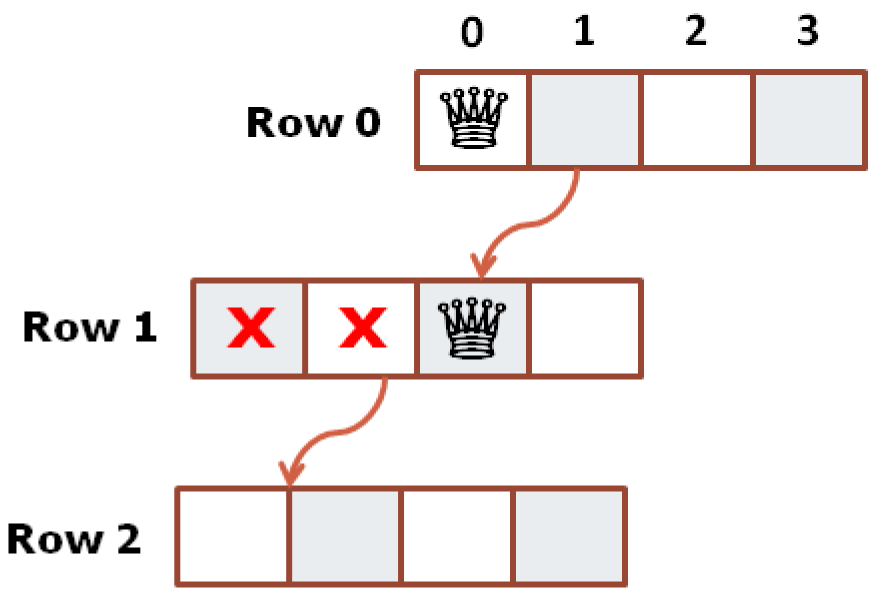


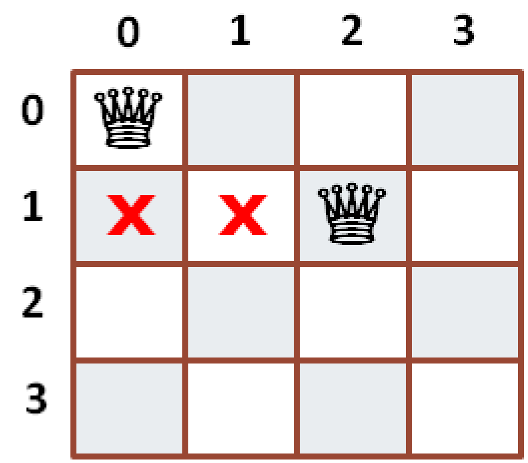
QueenPosition[ ]

Row 0	(0,0)
Row 1	(1,2)

Go to the next level of recursion.

Place the **1**st queen on the **1**st row such that she does not attack the **0**th queen and add that to Positions.

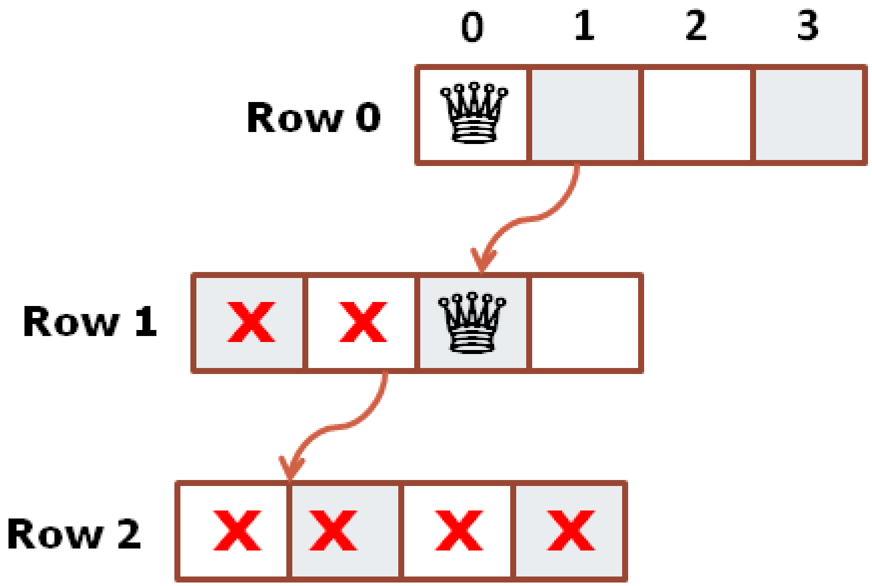


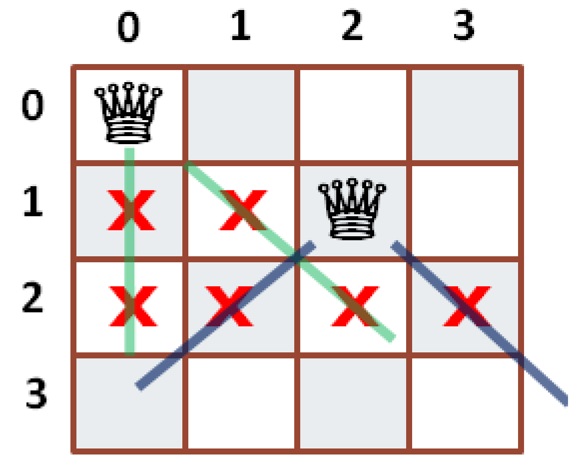


QueenPosition[ ]

Row 0	(0,0)
Row 1	(1,2)

In the next level of recursion, find the cell on **2**nd row
such that it is not under attack from any of the available queens.



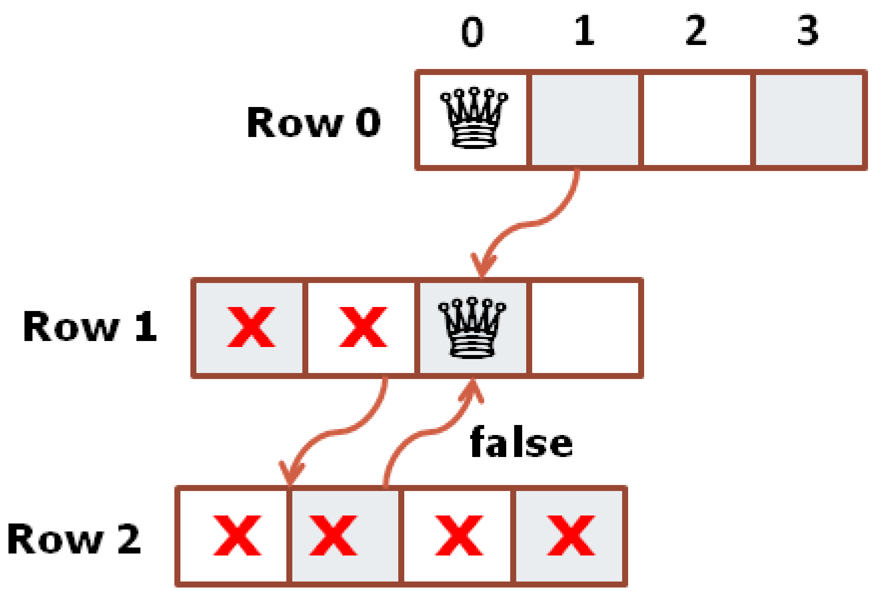


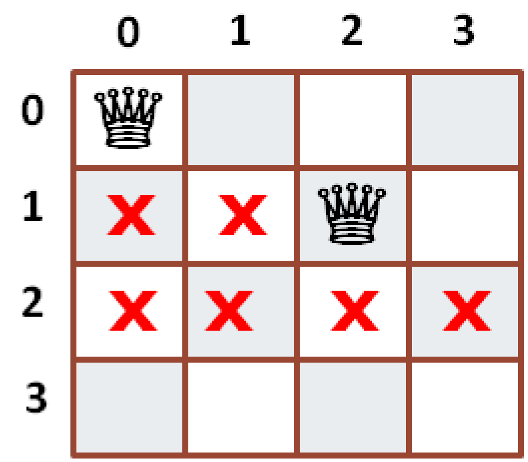
QueenPosition[ ]

Row 0 (0,0)

Row 1 (1,2)

But cell **(2,0)** and **(2,2)** are under attack from **0**th queen and cell **(2,1)** and **(2,3)** are under attack from **1**st queen.

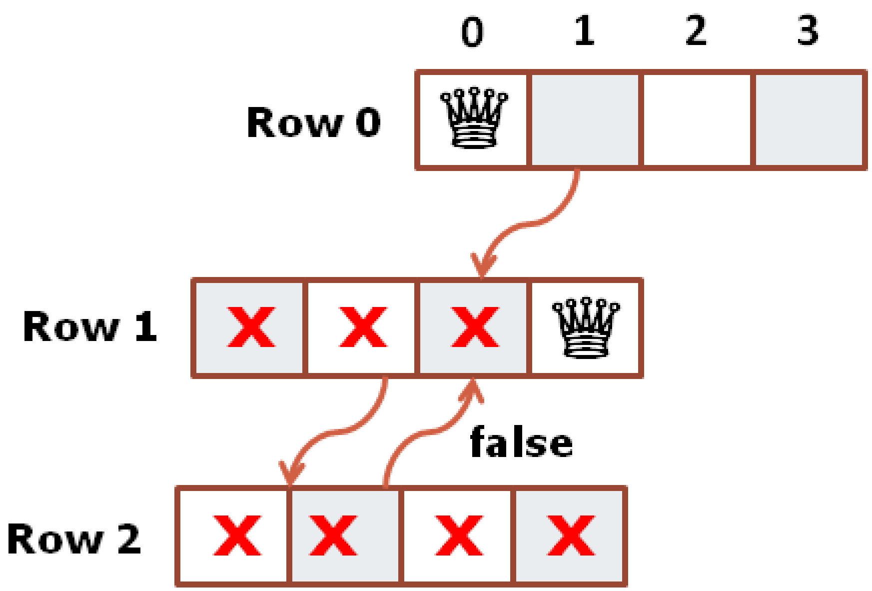


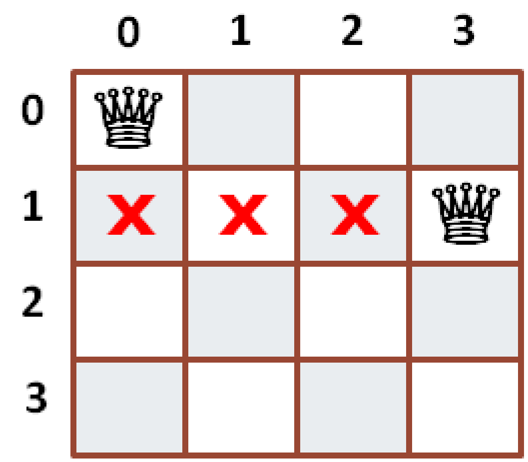


QueenPosition[ ]

Row 0	(0,0)
Row 1	(1,2)

So function will return false to the calling function.

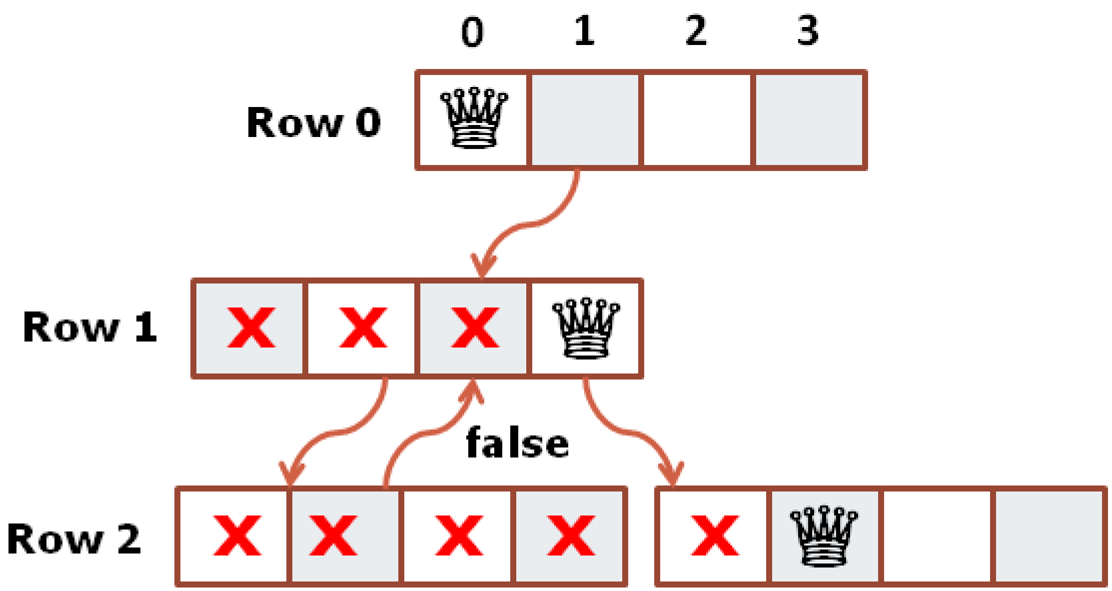


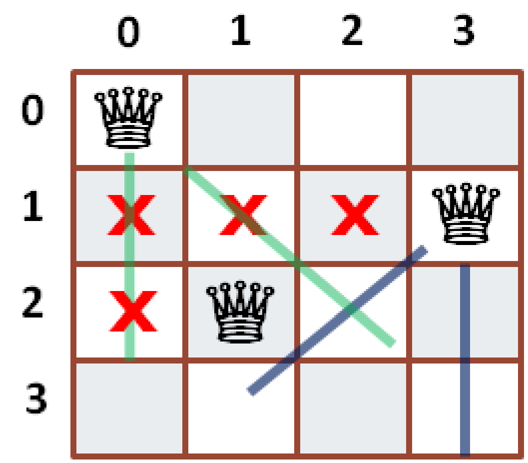


QueenPosition[ ]

Row 0 (0,0)
~~**Row 1** (1,2)~~
Row 1 (1,3)

Calling function will try to find next possible place for the **1**st queen on **1**st row and update the queen position in position array.



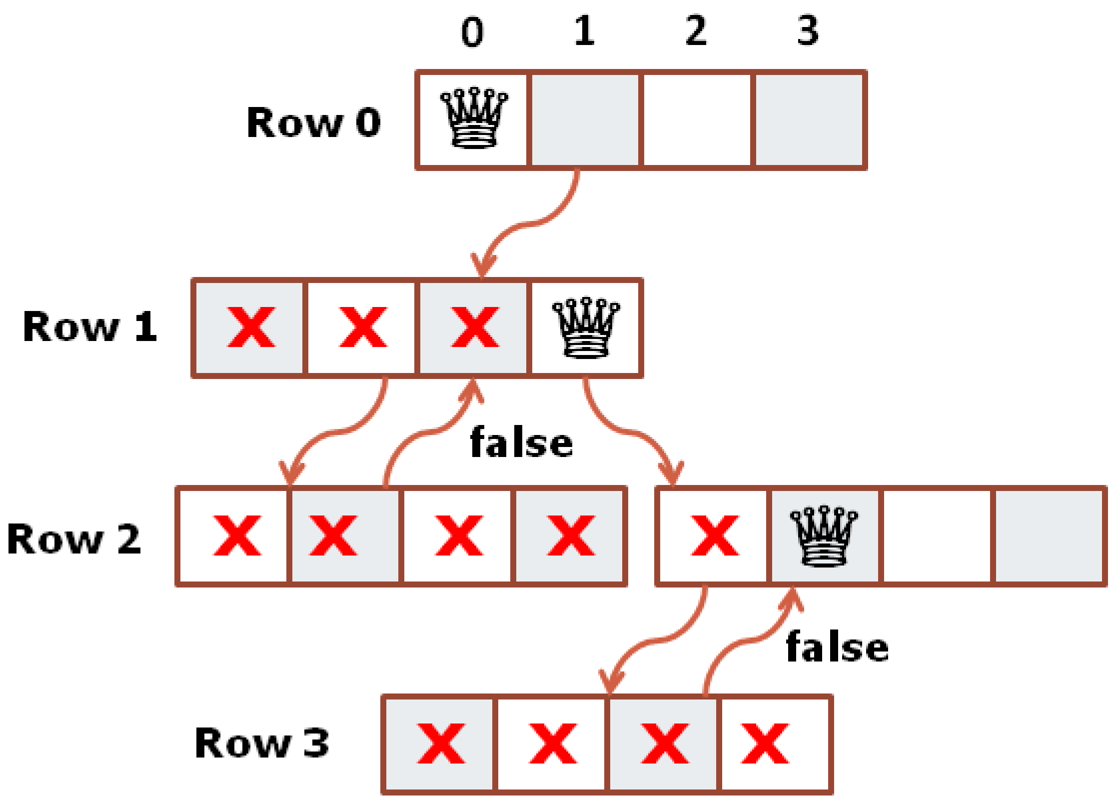


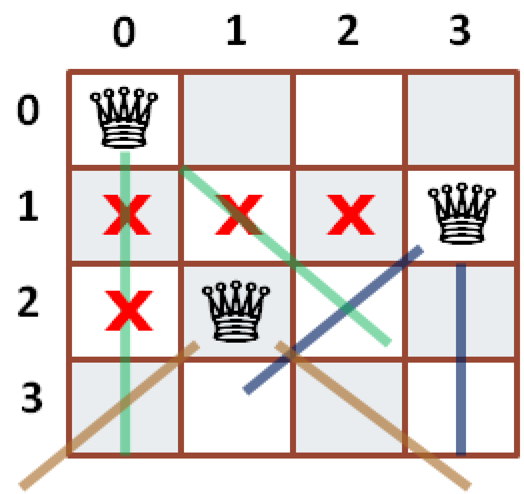
QueenPosition[ ]

Row 0	(0,0)
Row 1	(1,3)
Row 1	(2,1)

Again find the cell on **2**nd row such that
it is not under attack from any of the available queens.

Placing the queen in cell **(2,1)** as it is not
under attack from any of the queen.

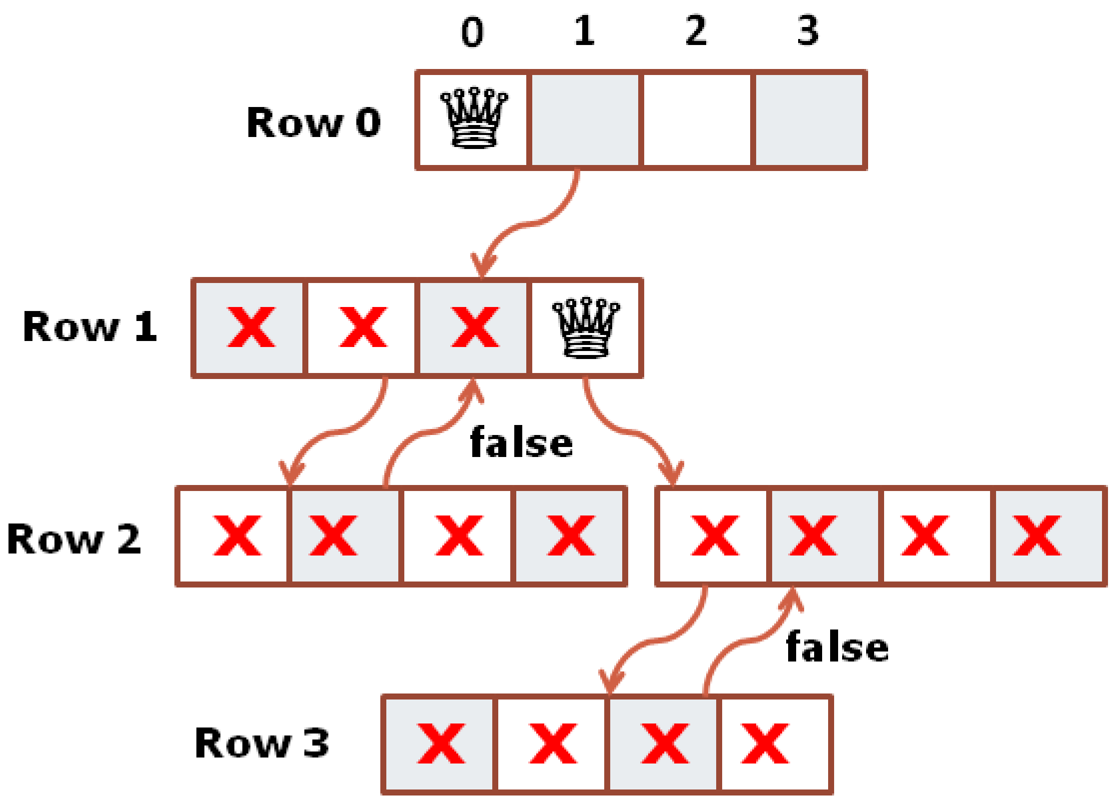


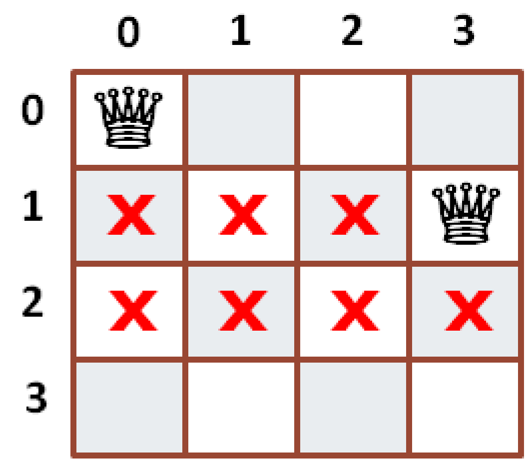


QueenPosition[ ]

Row 0	(0,0)
Row 1	(1,3)
Row 1	(2,1)

For **3**rd queen, no safe cell is available on **3**rd row.
So function will return false to calling function.

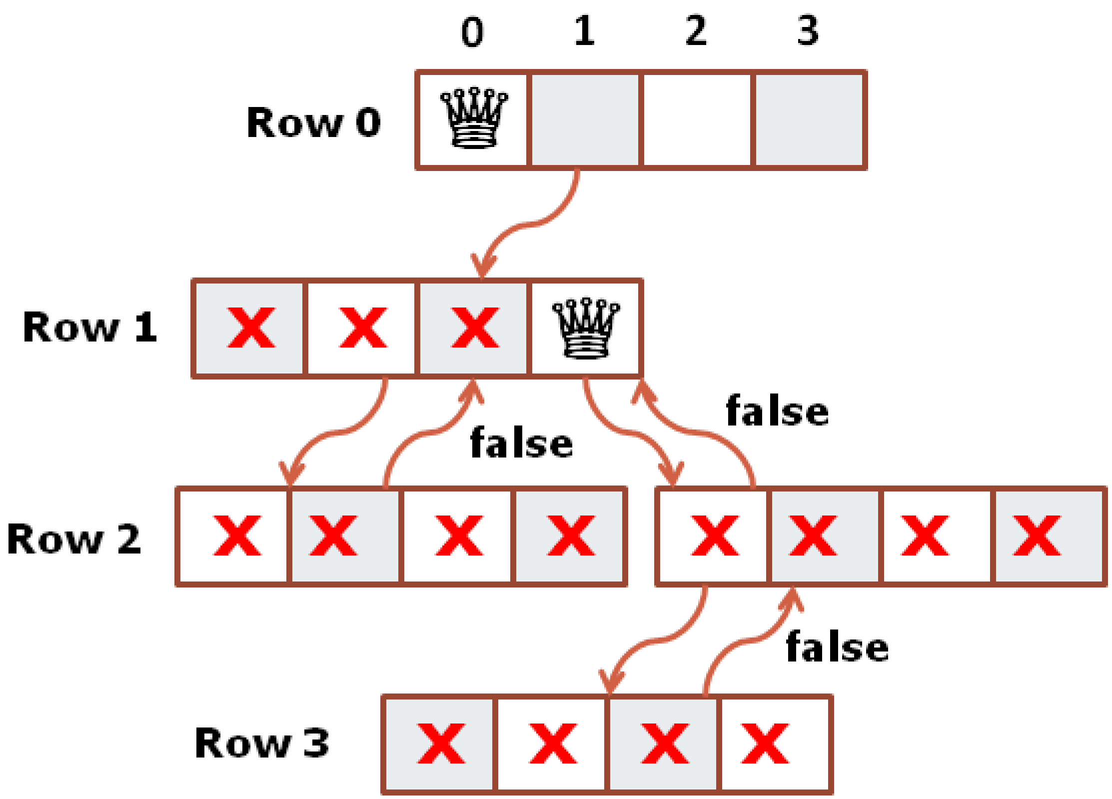


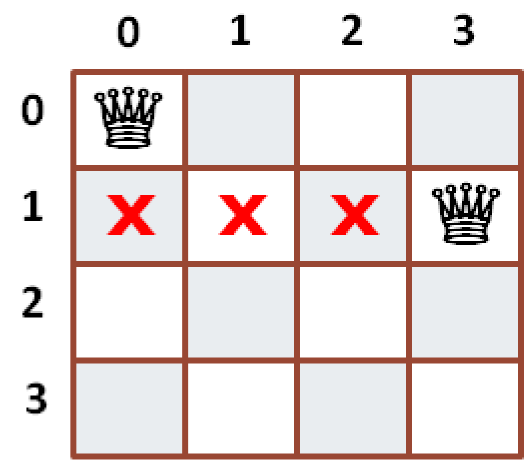


QueenPosition[ ]

Row 0 (0,0)
Row 1 (1,3)
~~**Row 1** (2,1)~~

Queen at the **2**nd row tries to find next safe cell.

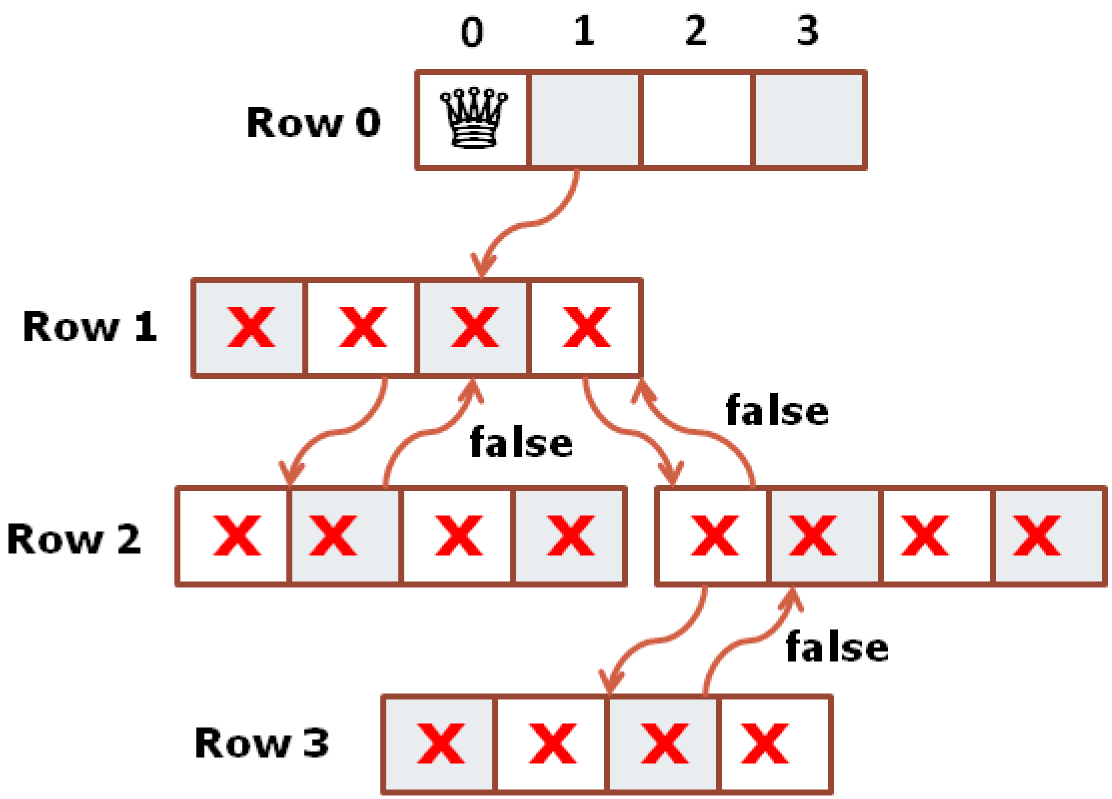


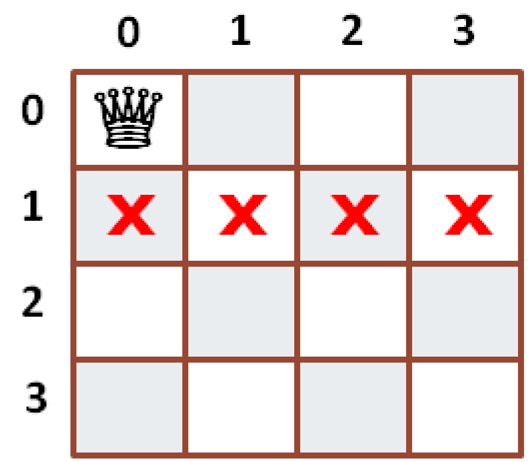


QueenPosition[ ]

Row 0	(0,0)
Row 1	(1,3)

But as both remaining cells are under attack from other queens, this function also returns false to its calling function.



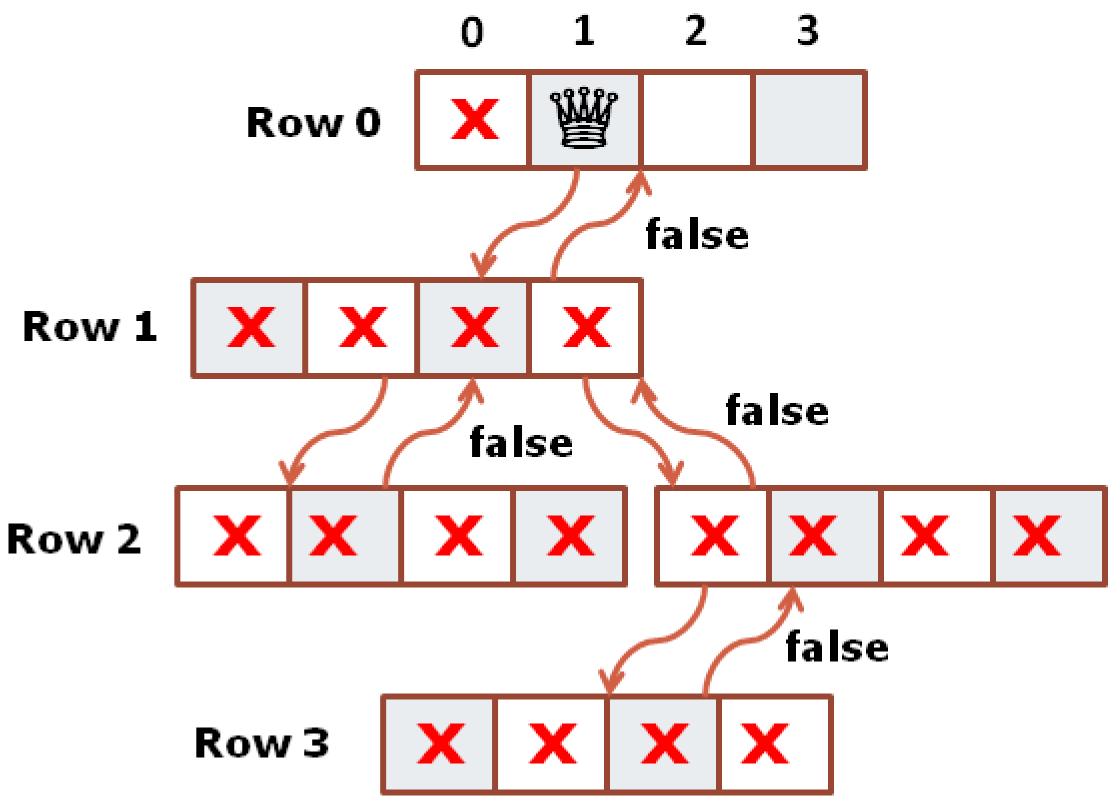


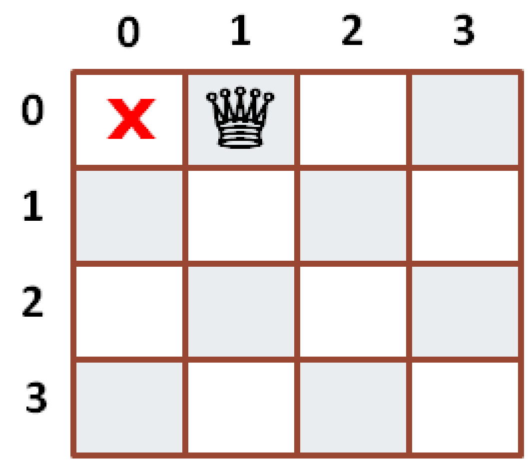
QueenPosition[ ]

Row 0 (0,0)

~~**Row 1** (1,3)~~

Queen at the **1**st row tries to find next safe cell.
But as queen is in the last cell, it will retuen false to
Its calling function.

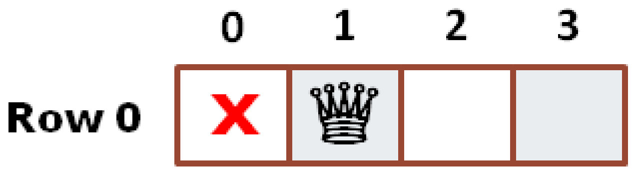


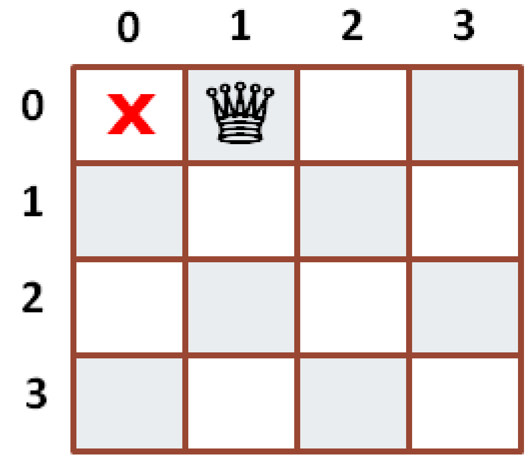


QueenPosition[ ]

~~Row 0 (0,0)~~
Row 0 (0,1)

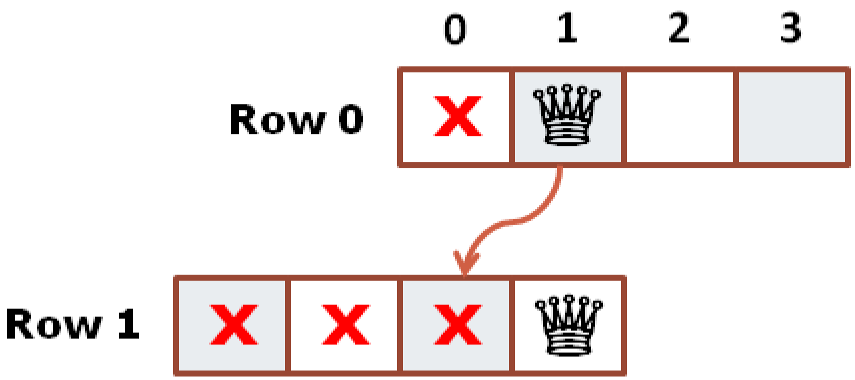
Queen at the **1**st row tries to find next safe cell.
Let us remove these failed recursion calls from the screen.

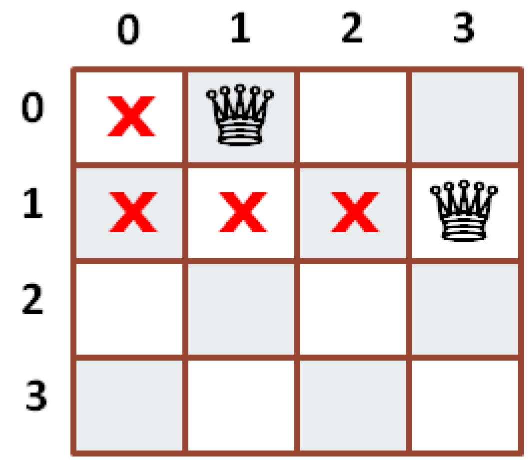




QueenPosition[ ]

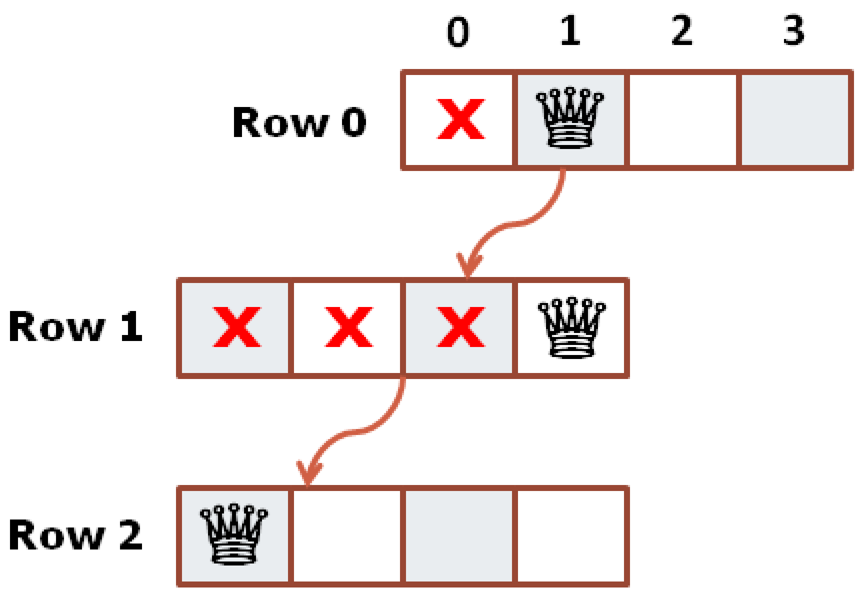
Row 0 (0,1)

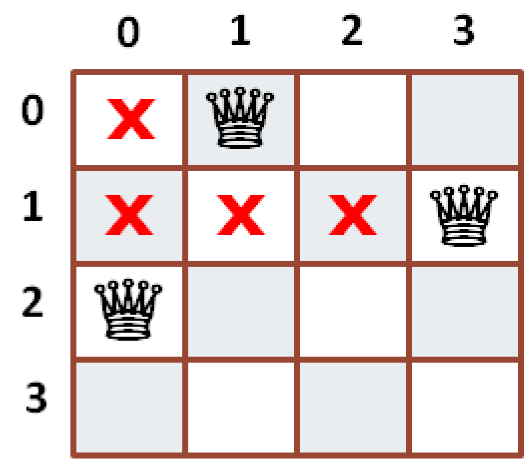




QueenPosition[ ]

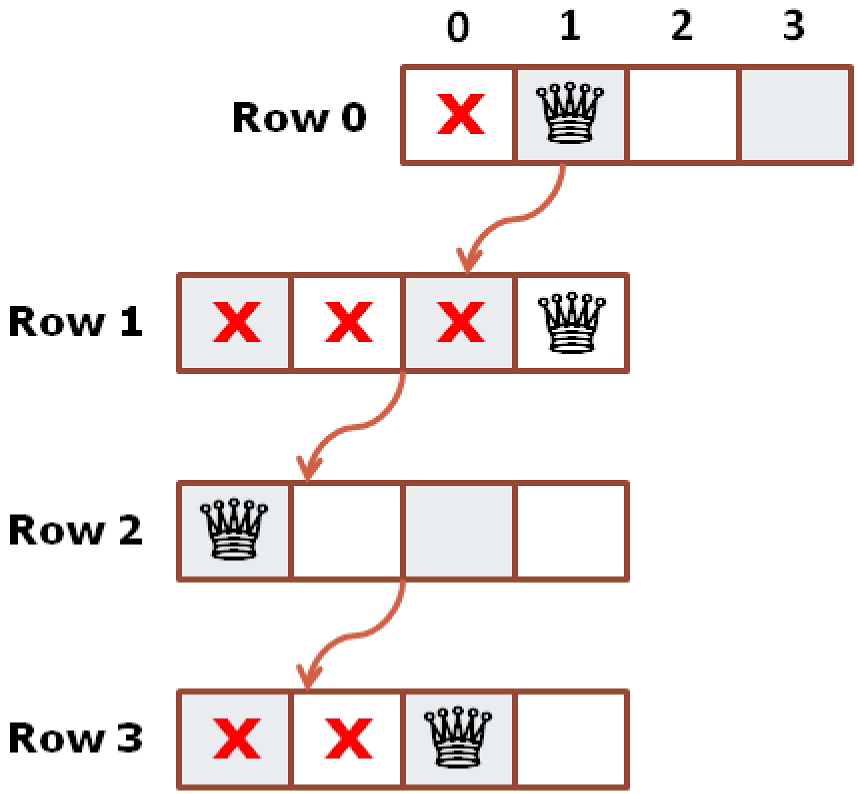
Row 0	(0,1)
Row 1	(1,3)

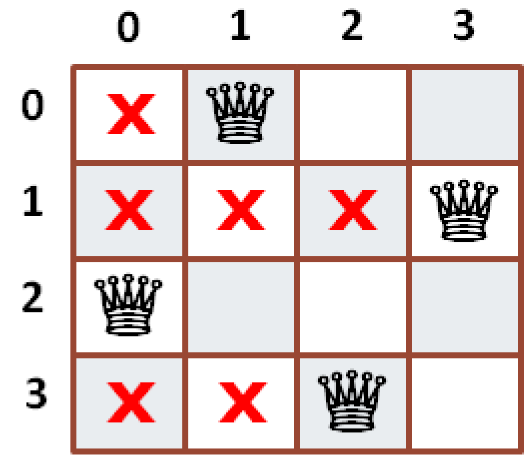




QueenPosition[ ]

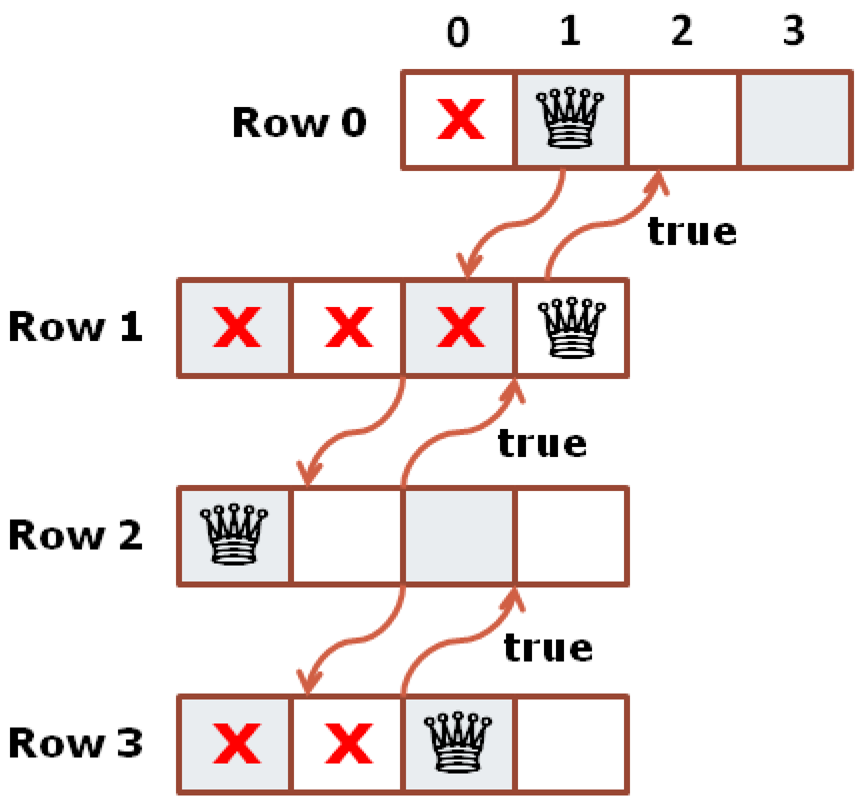
Row 0	(0,1)
Row 1	(1,3)
Row 2	(2,0)

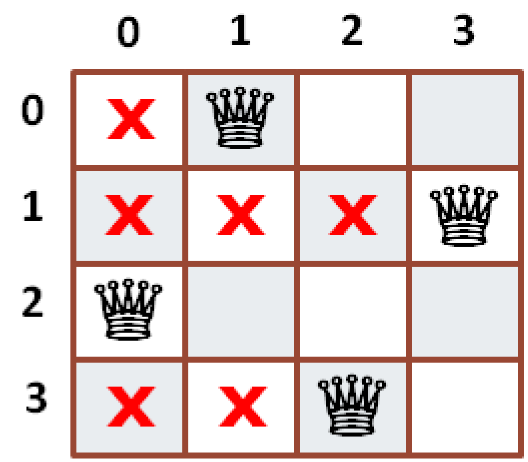




QueenPosition[ ]

Row 0 (0,1)
Row 1 (1,3)
Row 2 (2,0)
Row 3 (3,2)





QueenPosition[ ]

Row 0	(0,1)
Row 1	(1,3)
Row 2	(2,0)
Row 3	(3,2)

All functions will return true to their calling function.
It means all queens are placed on the board such that they are not attacking each other.

N Queens with backtracking

- int board[N][N] represents placement of queens
  - board[i][j] = 0: no queen at row i column j
  - board[i][j] = 1:queen at row i column j
- Initialize, for all i,j board[i][j] = 0
- Functions
  - PrintBoard(board): Prints board on the screen
  - IsSafe(borad, row, col): returns 1 iff new queen can be placed at (row,col) in board
  - Solve(board, row: recursively attempts to place (N-row) queens; returns 0 iff it failes

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

Initial board

Solve(board,3) returns 0

1	0	0	0
0	0	0	0
0	1	0	0
0	0	0	0

Recursive with Backtracking

- N-Queen Problem by Backtracking
  1. **Decision**
     Place a queen at a safe place.
  2. **Recursion**
     Explore the solution for the next row.
  3. **Backtrack (Undo)**
     Remove the queen if no solution for the next row.
  4. **Base case**
     Reach the goal.

N-Queen (4x4) Backtracking – CODE (Main function)

```c
#include <stdio.h>

//Solve 4x4 n Queen problem using recursion with backtracking

#define N 4
#define true 1
#define false 0

void printSolution(int board[N][N]);
int Solve(int board[N][N], int col);
int isSafe(int board[N][N], int row, int col);

int main()
{
    int board[N][N] = {{0,0,0,0},{0,0,0,0},{0,0,0,0},{0,0,0,0}};

    //game started at row 0
    if(Solve(board,0) == false)
    {
        printf("Solution does not exist.\n");
        return 1;
    }

    printf("Solution: \n");
    printSolution(board);
    return 0;
}
```

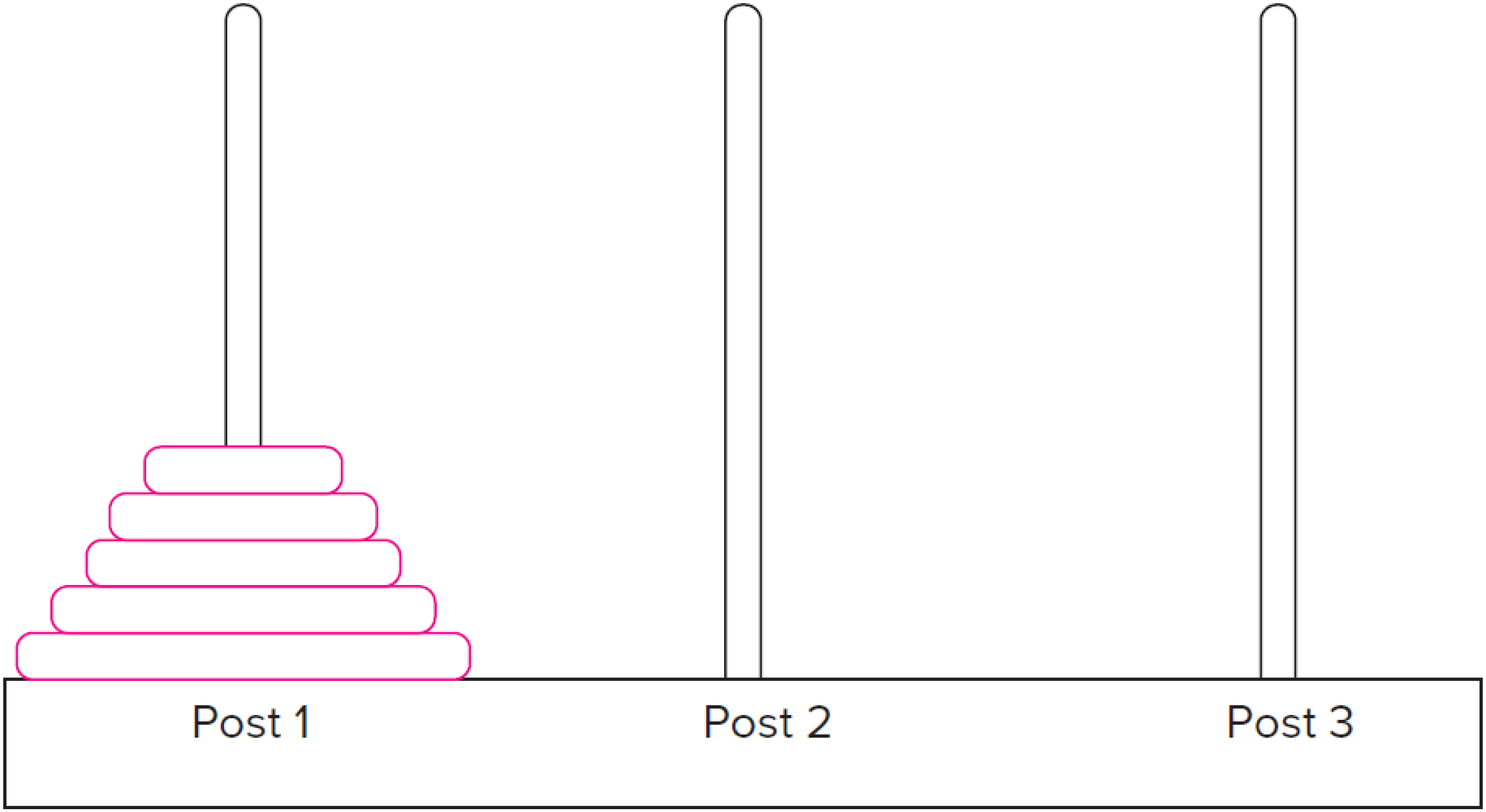
N-Queen (4x4) Backtracking – CODE (Solve function)

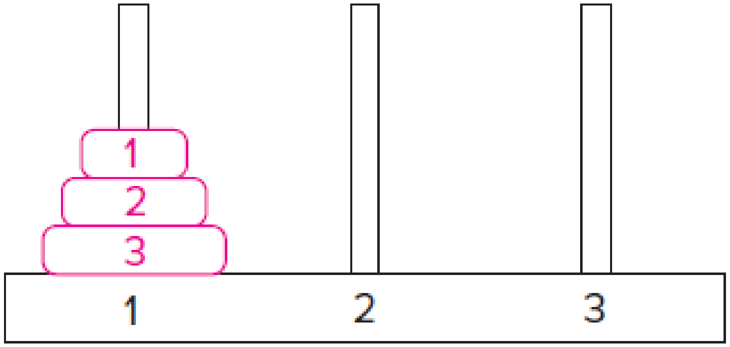
```c
int Solve(int board[N][N], int row)
{
    //base case
    if(row>=N)
        return true;

        //find a safe column(j) to place queen
    int j;
    for(j=0;j<N;j++)
    {
        //column j is safe, place queen here
        if(isSafe(board, row, j) == true)
        {
            board[row][j]=1;
            printf("Current Play: \n");
                printSolution(board);

            //increment row to place the next queen
            if(Solve(board, row+1) == true)
                return true;
            //attempt to place queen at row+1 failed,-
            //backtrack to row and remove queen
            board[row][j]=0;
            printf("Backtrack: \n");
            printSolution(board);
        }
    }
    return false;
}
```

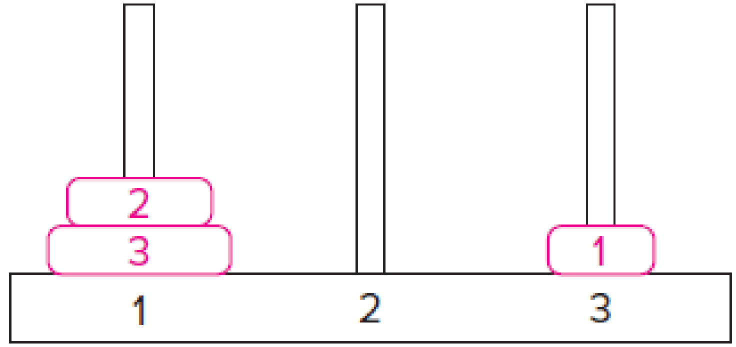
N-Queen (4x4) Backtracking – CODE (isSafe & PrintSolution functions)

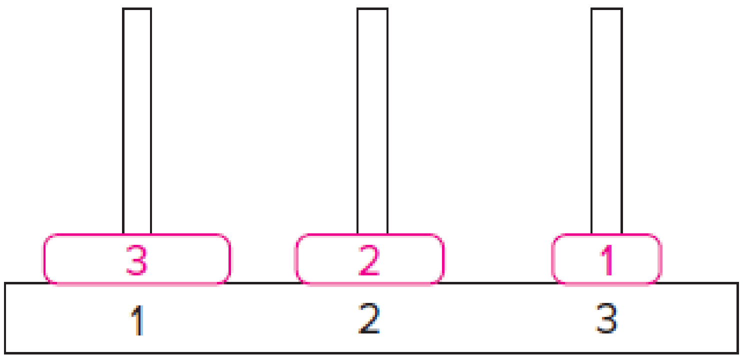
```c
int isSafe(int board[N][N], int row, int col)
{
    int i, j;
    for(i=0;i<row;i++)
    {
        for(j=0;j<N;j++)
        {
            //check whether there's a queen at the same column or the 2 diagonals
            if(((j==col) || (i-j == row-col) || (i+j == row + col)) && (board[i][j]==1))
                return false;
        }
    }
    return true;
}


void printSolution(int board[N][N])
{
    int i,j;
    for(i=0;i<N;i++)
    {
        for(j=0;j<N;j++)
            printf(" %d ", board[i][j]);
        printf("\n");
    }
}
```

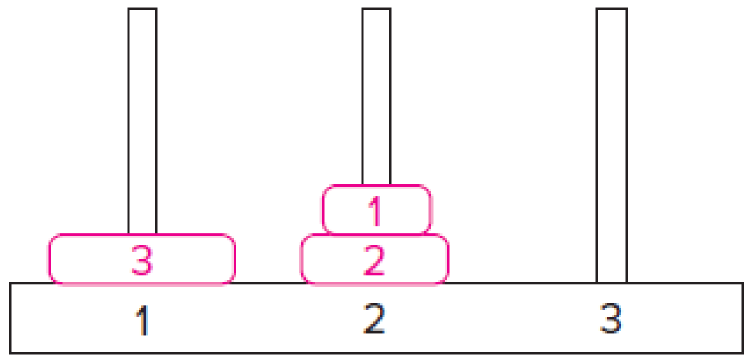
Towers of Hanoi - 17.4

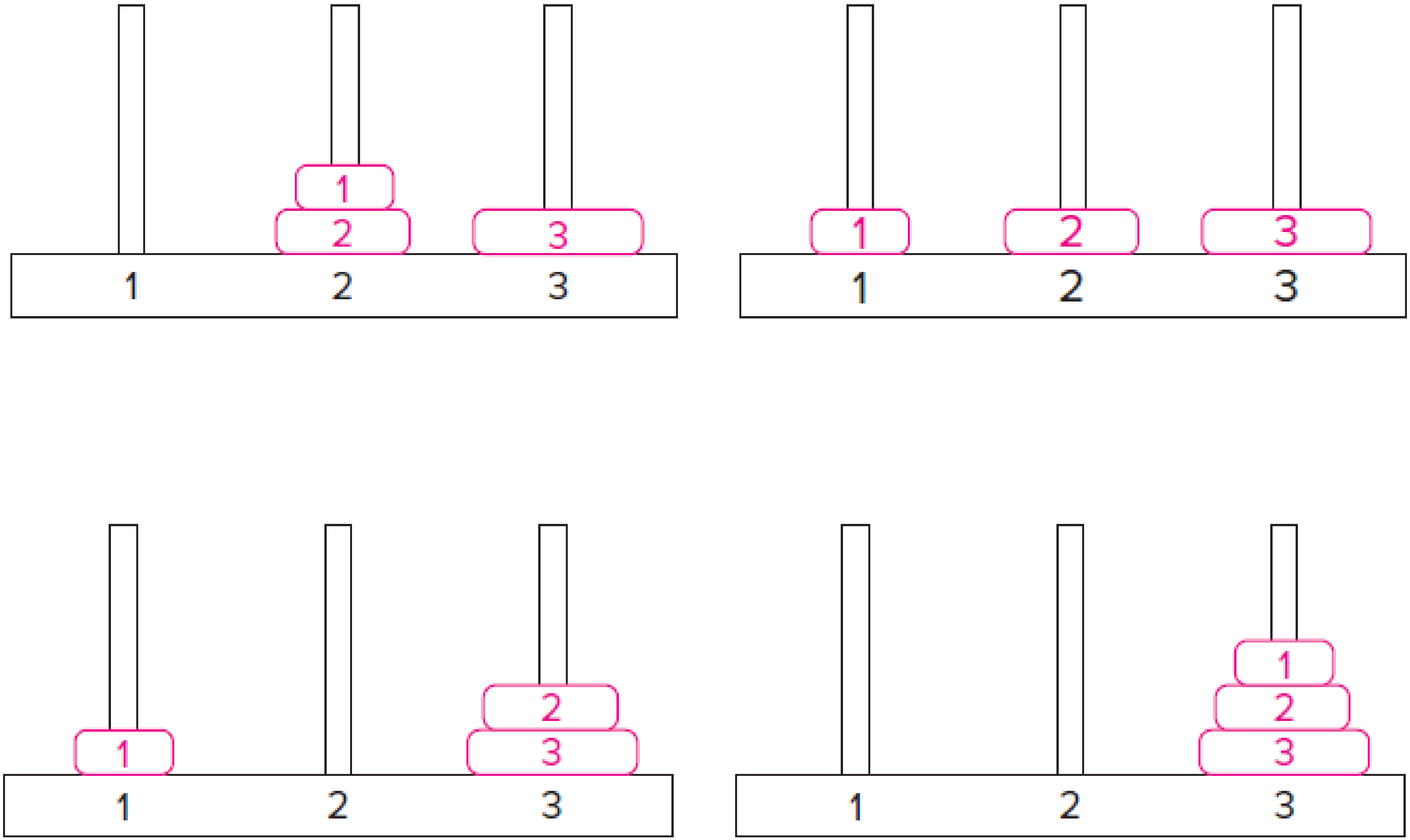



1
2
3
1
2
3


2
3
1
1
2
3


3
2
1
1
2
3


1
3
2
1
2
3


1
2
1 2 3
1 2 3
1 2 3
1 2 3
2
1 3
1 2 3
1
2
3
1 2 3

```c
// diskNumber is the disk to be moved (disk1 is smallest)
// startPost is the post the disk is currently on
// endPost is the post we want the disk to end on
// midPost is the intermediate post
void MoveDisk(int diskNumber, int startPost, int endPost, int midPost)
{
   if (diskNumber > 1) {
      // Move n-1 disks off the current disk on
      // startPost and put them on the midPost
      MoveDisk(diskNumber-1, startPost, midPost, endPost);

      printf("Move disk %d from post %d to post %d.\n",
              diskNumber, startPost, endPost);

      // Move all n-1 disks from midPost onto endPost
      MoveDisk(diskNumber-1, midPost, endPost, startPost);
   }
   else
      printf("Move disk 1 from post %d to post %d.\n",
              startPost, endPost);
}
```