

$f(t)$ , period $T = \frac{2\pi}{\omega_o}$	Form	Coefficients
$\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_o t}$	Exponential	$F_n = \frac{1}{T} \int_T f(t) e^{-jn\omega_o t} dt$
$\frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_o t) + b_n \sin(n\omega_o t)$	Trigonometric	$a_n = F_n + F_{-n}$ $b_n = j(F_n - F_{-n})$
$\frac{c_o}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_o t + \theta_n)$	Compact for real $f(t)$	$c_n = 2 F_n $ $\theta_n = \angle F_n$

Table 1: Fourier series forms.

	Name:	Condition:	Property:
1	Scaling	Constant $K$	$K f(t) \leftrightarrow K F_n$
2	Addition	$f(t) \leftrightarrow F_n, g(t) \leftrightarrow G_n, \dots$	$f(t) + g(t) + \dots \leftrightarrow F_n + G_n + \dots$
3	Time shift	Delay $t_o$	$f(t - t_o) \leftrightarrow F_n e^{-jn\omega_o t_o}$
4	Derivative	Continuous $f(t)$	$\frac{df}{dt} \leftrightarrow jn\omega_o F_n$
5	Hermitian	Real $f(t)$	$F_{-n} = F_n^*$
6	Even function	$f(-t) = f(t)$	$f(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_o t)$
7	Odd function	$f(-t) = -f(t)$	$f(t) = \sum_{n=1}^{\infty} b_n \sin(n\omega_o t)$
8	Average power		$P \equiv \frac{1}{T} \int_T  f(t) ^2 dt = \sum_{n=-\infty}^{\infty}  F_n ^2$

Table 2: Fourier series properties

	Name:	Condition:	Property:
1	Amplitude scaling	$f(t) \leftrightarrow F(\omega)$ , constant $K$	$Kf(t) \leftrightarrow KF(\omega)$
2	Addition	$f(t) \leftrightarrow F(\omega)$ , $g(t) \leftrightarrow G(\omega)$ , $\dots$	$f(t) + g(t) + \dots \leftrightarrow F(\omega) + G(\omega) + \dots$
3	Hermitian	Real $f(t) \leftrightarrow F(\omega)$	$F(-\omega) = F^*(\omega)$
4	Even	Real and even $f(t)$	Real and even $F(\omega)$
5	Odd	Real and odd $f(t)$	Imaginary and odd $F(\omega)$
6	Symmetry	$f(t) \leftrightarrow F(\omega)$	$F(t) \leftrightarrow 2\pi f(-\omega)$
7	Time scaling	$f(t) \leftrightarrow F(\omega)$ , real $s$	$f(st) \leftrightarrow \frac{1}{ s } F(\frac{\omega}{s})$
8	Time shift	$f(t) \leftrightarrow F(\omega)$	$f(t - t_o) \leftrightarrow F(\omega)e^{-j\omega t_o}$
9	Frequency shift	$f(t) \leftrightarrow F(\omega)$	$f(t)e^{j\omega_o t} \leftrightarrow F(\omega - \omega_o)$
10	Modulation	$f(t) \leftrightarrow F(\omega)$	$f(t) \cos(\omega_o t) \leftrightarrow \frac{1}{2}F(\omega - \omega_o) + \frac{1}{2}F(\omega + \omega_o)$
11	Time derivative	Differentiable $f(t) \leftrightarrow F(\omega)$	$\frac{df}{dt} \leftrightarrow j\omega F(\omega)$
12	Freq derivative	$f(t) \leftrightarrow F(\omega)$	$-jtf(t) \leftrightarrow \frac{d}{d\omega} F(\omega)$
13	Time convolution	$f(t) \leftrightarrow F(\omega)$ , $g(t) \leftrightarrow G(\omega)$	$f(t) * g(t) \leftrightarrow F(\omega)G(\omega)$
14	Freq convolution	$f(t) \leftrightarrow F(\omega)$ , $g(t) \leftrightarrow G(\omega)$	$f(t)g(t) \leftrightarrow \frac{1}{2\pi} F(\omega) * G(\omega)$
15	Compact form	Real $f(t)$	$f(t) = \frac{1}{2\pi} \int_0^\infty 2 F(\omega)  \cos(\omega t + \angle F(\omega)) d\omega$
16	Parseval, Energy W	$f(t) \leftrightarrow F(\omega)$	$W \equiv \int_{-\infty}^\infty  f(t) ^2 dt = \frac{1}{2\pi} \int_{-\infty}^\infty  F(\omega) ^2 d\omega$

Table 3: Important properties of the Fourier transform.

	$f(t) \leftrightarrow F(\omega)$		
1	$e^{-at}u(t) \leftrightarrow \frac{1}{a+j\omega}$ , $a > 0$	14	$\delta(t) \leftrightarrow 1$
2	$e^{at}u(-t) \leftrightarrow \frac{1}{a-j\omega}$ , $a > 0$	15	$1 \leftrightarrow 2\pi\delta(\omega)$
3	$e^{-a t } \leftrightarrow \frac{2a}{a^2+\omega^2}$ , $a > 0$	16	$\delta(t - t_o) \leftrightarrow e^{-j\omega t_o}$
4	$\frac{a^2}{a^2+t^2} \leftrightarrow \pi a e^{-a \omega }$ , $a > 0$	17	$e^{j\omega_o t} \leftrightarrow 2\pi\delta(\omega - \omega_o)$
5	$te^{-at}u(t) \leftrightarrow \frac{1}{(a+j\omega)^2}$ , $a > 0$	18	$\cos(\omega_o t) \leftrightarrow \pi[\delta(\omega - \omega_o) + \delta(\omega + \omega_o)]$
6	$t^n e^{-at}u(t) \leftrightarrow \frac{n!}{(a+j\omega)^{n+1}}$ , $a > 0$	19	$\sin(\omega_o t) \leftrightarrow j\pi[\delta(\omega + \omega_o) - \delta(\omega - \omega_o)]$
7	$\text{rect}(\frac{t}{\tau}) \leftrightarrow \tau \text{sinc}(\frac{\omega\tau}{2})$	20	$\cos(\omega_o t)u(t) \leftrightarrow \frac{\pi}{2}[\delta(\omega - \omega_o) + \delta(\omega + \omega_o)] + \frac{j\omega}{\omega_o^2 - \omega^2}$
8	$\text{sinc}(Wt) \leftrightarrow \frac{\pi}{W} \text{rect}(\frac{\omega}{2W})$	21	$\sin(\omega_o t)u(t) \leftrightarrow j\frac{\pi}{2}[\delta(\omega + \omega_o) - \delta(\omega - \omega_o)] + \frac{\omega_o}{\omega_o^2 - \omega^2}$
9	$\Delta(\frac{t}{\tau}) \leftrightarrow \frac{\tau}{2} \text{sinc}^2(\frac{\omega\tau}{4})$	22	$\text{sgn}(t) \leftrightarrow \frac{2}{j\omega}$
10	$\text{sinc}^2(\frac{Wt}{2}) \leftrightarrow \frac{2\pi}{W} \Delta(\frac{\omega}{2W})$	23	$u(t) \leftrightarrow \pi\delta(\omega) + \frac{1}{j\omega}$
11	$e^{-at} \sin(\omega_o t)u(t) \leftrightarrow \frac{\omega_o}{(a+j\omega)^2 + \omega_o^2}$ , $a > 0$	24	$\sum_{n=-\infty}^\infty \delta(t - nT) \leftrightarrow \frac{2\pi}{T} \sum_{n=-\infty}^\infty \delta(\omega - n\frac{2\pi}{T})$
12	$e^{-at} \cos(\omega_o t)u(t) \leftrightarrow \frac{a+j\omega}{(a+j\omega)^2 + \omega_o^2}$ , $a > 0$	25	$\sum_{n=-\infty}^\infty f(t)\delta(t - nT) \leftrightarrow \sum_{n=-\infty}^\infty \frac{1}{T} F(\omega - n\frac{2\pi}{T})$
13	$e^{-\frac{t^2}{2\sigma^2}} \leftrightarrow \sigma\sqrt{2\pi} e^{-\frac{\sigma^2\omega^2}{2}}$		

Table 4: Important Fourier transform pairs. The left-hand column includes only “energy signals”  $f(t)$ , while the right-hand column includes “power signals” and distributions.

Name	Property
Commutative	$h(t) * f(t) = f(t) * h(t)$
Distributive	$f(t) * (g(t) + h(t)) = f(t) * g(t) + f(t) * h(t)$
Associative	$f(t) * (g(t) * h(t)) = (f(t) * g(t)) * h(t)$
Shift	$h(t) * f(t) = y(t) \Rightarrow h(t - t_0) * f(t) = h(t) * f(t - t_0) = y(t - t_0)$
Derivative	$h(t) * f(t) = y(t) \Rightarrow (\frac{d}{dt}h(t)) * f(t) = h(t) * (\frac{d}{dt}f(t)) = \frac{d}{dt}y(t)$
Reversal	$f(t) * h(t) = y(t) \Rightarrow h(-t) * f(-t) = y(-t)$
Start-point	If $h(t) = 0$ for $t < t_{sh}$ and $f(t) = 0$ for $t < t_{sf}$ then $y(t) = h(t) * f(t) = 0$ for $t < t_{sy} = t_{sh} + t_{sf}$ .
End-point	If $h(t) = 0$ for $t > t_{eh}$ and $f(t) = 0$ for $t > t_{ef}$ then $y(t) = h(t) * f(t) = 0$ for $t > t_{ey} = t_{eh} + t_{ef}$ .
Width	$h(t) * f(t) = y(t) \Rightarrow T_y = T_h + T_f$ where $T_h$ , $T_f$ , and $T_y$ denote the widths of $h(t)$ , $f(t)$ , and $y(t)$ .

Table 5: Convolution properties.

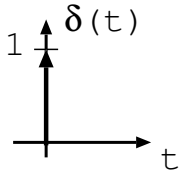
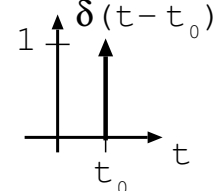
Name	Impulse properties	Shifted-impulse properties
Convolution	$\delta(t) * f(t) = f(t)$	$\delta(t - t_0) * f(t) = f(t - t_0)$
Sifting	$\int_{-\infty}^{\infty} \delta(t)f(t)dt = f(0)$ and $\int_a^b \delta(t)f(t)dt = \begin{cases} f(0) & \text{if } a < 0 < b \\ 0, & \text{otherwise} \end{cases}$	$\int_{-\infty}^{\infty} \delta(t - t_0)f(t)dt = f(t_0)$ and $\int_a^b \delta(t - t_0)f(t)dt = \begin{cases} f(t_0) & \text{if } a < t_0 < b \\ 0, & \text{otherwise} \end{cases}$
Sampling	$f(t)\delta(t) = f(0)\delta(t)$	$f(t)\delta(t - t_0) = f(t_0)\delta(t - t_0)$
Symmetry	$\delta(-t) = \delta(t)$	$\delta(t_0 - t) = \delta(t - t_0)$
Scaling	$\delta(at) = \frac{1}{ a }\delta(t)$ , $a \neq 0$	$\delta(a(t - t_0)) = \frac{1}{ a }\delta(t - t_0)$ , $a \neq 0$
Area	$\int_{-\infty}^{\infty} \delta(t)dt = 1$ and $\int_a^b \delta(t)dt = \begin{cases} 1 & \text{if } a < 0 < b \\ 0, & \text{otherwise} \end{cases}$	$\int_{-\infty}^{\infty} \delta(t - t_0)dt = 1$ and $\int_a^b \delta(t - t_0)dt = \begin{cases} 1 & \text{if } a < t_0 < b \\ 0, & \text{otherwise} \end{cases}$
Definite integral	$\int_{-\infty}^t \delta(\tau)d\tau = u(t)$	$\int_{-\infty}^t \delta(\tau - t_0)d\tau = u(t - t_0)$
Unit-step derivative	$\frac{d}{dt}u(t) = \delta(t)$	$\frac{d}{dt}u(t - t_0) = \delta(t - t_0)$
Derivative	$(\frac{d}{dt}\delta(t)) * f(t) = \frac{d}{dt}f(t)$	$(\frac{d}{dt}\delta(t - t_0)) * f(t) = \frac{d}{dt}f(t - t_0)$
Fourier transform	$\delta(t) \leftrightarrow 1$	$\delta(t - t_0) \leftrightarrow e^{-j\omega t_0}$
Graphical symbol		

Table 6: Properties of the impulse and shifted impulse.

1	$\delta(t) \leftrightarrow 1$	7	$\delta'(t) \leftrightarrow s$
2	$e^{pt}u(t) \leftrightarrow \frac{1}{s-p}$	8	$u(t) \leftrightarrow \frac{1}{s}$
3	$te^{pt}u(t) \leftrightarrow \frac{1}{(s-p)^2}$	9	$tu(t) \leftrightarrow \frac{1}{s^2}$
4	$t^n e^{pt}u(t) \leftrightarrow \frac{n!}{(s-p)^{n+1}}$	10	$t^n u(t) \leftrightarrow \frac{n!}{s^{n+1}}$
5	$\cos(\omega_o t)u(t) \leftrightarrow \frac{s}{s^2+\omega_o^2}$	11	$\sin(\omega_o t)u(t) \leftrightarrow \frac{\omega_o}{s^2+\omega_o^2}$
6	$e^{-\alpha t} \cos(\omega_d t)u(t) \leftrightarrow \frac{s+\alpha}{(s+\alpha)^2+\omega_d^2}$	12	$e^{-\alpha t} \sin(\omega_d t)u(t) \leftrightarrow \frac{\omega_d}{(s+\alpha)^2+\omega_d^2}$

Table 7: Laplace transforms pairs  $h(t) \leftrightarrow \hat{H}(s)$  involving frequently encountered causal signals —  $\alpha$ ,  $\omega_o$ , and  $\omega_d$  stand for arbitrary real constants,  $n$  for non-negative integers, and  $p$  denotes for an arbitrary complex constant.

	Name:	Condition:	Property:
1	Multiplication	$f(t) \rightarrow \hat{F}(s)$ , constant $K$	$Kf(t) \rightarrow K\hat{F}(s)$
2	Addition	$f(t) \rightarrow \hat{F}(s)$ , $g(t) \rightarrow \hat{G}(s) \dots$	$f(t) + g(t) + \dots \rightarrow \hat{F}(s) + \hat{G}(s) + \dots$
3	Time scaling	$f(t) \rightarrow \hat{F}(s)$ , real $a > 0$	$f(at) \rightarrow \frac{1}{a}\hat{F}(\frac{s}{a})$
4*	Time delay	$f(t) \leftrightarrow \hat{F}(s)$ , $t_o \geq 0$	$f(t - t_o) \leftrightarrow \hat{F}(s)e^{-st_o}$
5	Frequency shift	$f(t) \rightarrow \hat{F}(s)$	$f(t)e^{s_o t} \rightarrow \hat{F}(s - s_o)$
6	Time derivative	Differentiable $f(t) \rightarrow \hat{F}(s)$	$f'(t) \rightarrow s\hat{F}(s) - f(0^-)$ $f''(t) \rightarrow s^2\hat{F}(s) - sf(0^-) - f'(0^-)$ $\dots$ $f^{(n)}(t) \rightarrow s^n\hat{F}(s) - \dots - f^{(n-1)}(0^-)$
7	Time integration	$f(t) \rightarrow \hat{F}(s)$	$\int_{0^-}^t f(\tau)d\tau \rightarrow \frac{1}{s}\hat{F}(s)$
8	Freq. derivative	$f(t) \rightarrow \hat{F}(s)$	$-tf(t) \rightarrow \frac{d}{ds}\hat{F}(s)$
9*	Time convolution	$h(t) \leftrightarrow \hat{H}(s)$ , $f(t) \leftrightarrow \hat{F}(s)$	$h(t) * f(t) \leftrightarrow \hat{H}(s)\hat{F}(s)$
10	Freq. convolution	$f(t) \rightarrow \hat{F}(s)$ , $g(t) \rightarrow \hat{G}(s)$	$f(t)g(t) \rightarrow \frac{1}{2\pi j}\hat{F}(s) * \hat{G}(s)$
11	Poles	$f(t) \rightarrow \hat{F}(s)$	Values of $s$ such that $ \hat{F}(s)  = \infty$
12	ROC	$f(t) \rightarrow \hat{F}(s)$	Portion of $s$ - plane to the right of rightmost pole $\neq \infty$
13*	Fourier transform	$f(t) \leftrightarrow \hat{F}(s)$	$F(\omega) = \hat{F}(j\omega)$ if and only if ROC includes $s = j\omega$
14	Final value	Poles of $s\hat{F}(s)$ in LHP	$f(\infty) = \lim_{s \rightarrow 0} s\hat{F}(s)$
15	Initial value	Existence of the limit	$f(0^+) = \lim_{s \rightarrow \infty} s\hat{F}(s)$

Table 8: Important properties of the *one-sided* Laplace transform. Properties marked by \* in the first column hold only for causal signals.