Lecture 5 - 6: Circuit Analysis - KVL, Loop Analysis

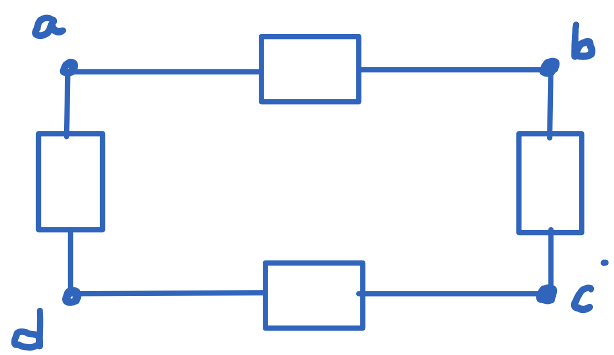
Learning Objectives:

1. **Define Kirchhoff's voltage law (KVL)**
2. **Compute currents in simple circuits using KVL and Ohm's law**
3. **Use loop analysis method to compute loop currents**
4. **Derive voltage division formula and analyze the limitations of of voltage divider**

In practice we can encounter circuits that may have several electrical elements. Ohm's law defines the relationship between current and voltage in a resistor. Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL) help us in defining relationships between current and voltages in more complex circuits. These relationships are useful is solving for unknown voltages and currents in a circuit.

1. Kirchhoff's Voltage Law(KVL)

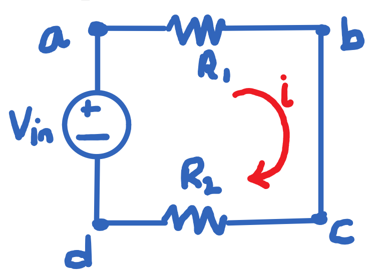
KVL states that the **algebraic sum of voltages** around a loop is zero. As we discussed earlier, Loop is a closed path in a circuit starting at a node, traversing through a series of nodes, and ending at the starting node without passing through the same node twice. Consider the circuit with one loop shown below. The sum of voltages while passing through the clockwise direction must sum to zero as shown.



KVL: $\mathbf{V_{ab} + V_{bc} + V_{cd} + V_{da} = 0}$

Fig 5.1. Illustration of KVL in a loop.

Example 1: Write the KVL equation for the circuit shown below.



KVL: $\mathbf{V_{ab} + V_{bc} + V_{cd} + V_{da} = 0}$

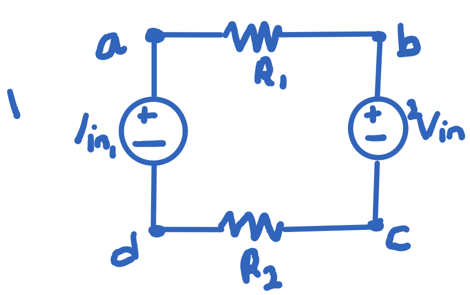
Ohm's Law: $\mathbf{i_{ab}R_1 + 0 + i_{cd}R_2 - V_{in} = 0}$

$$\Rightarrow \mathbf{iR_1 + iR_2 = V_{in}}$$

$$\Rightarrow \mathbf{\frac{V_{in}}{i} = R_1 + R_2.}$$

What does the quantity on the right hand side of the above equation represent?

Example 2:



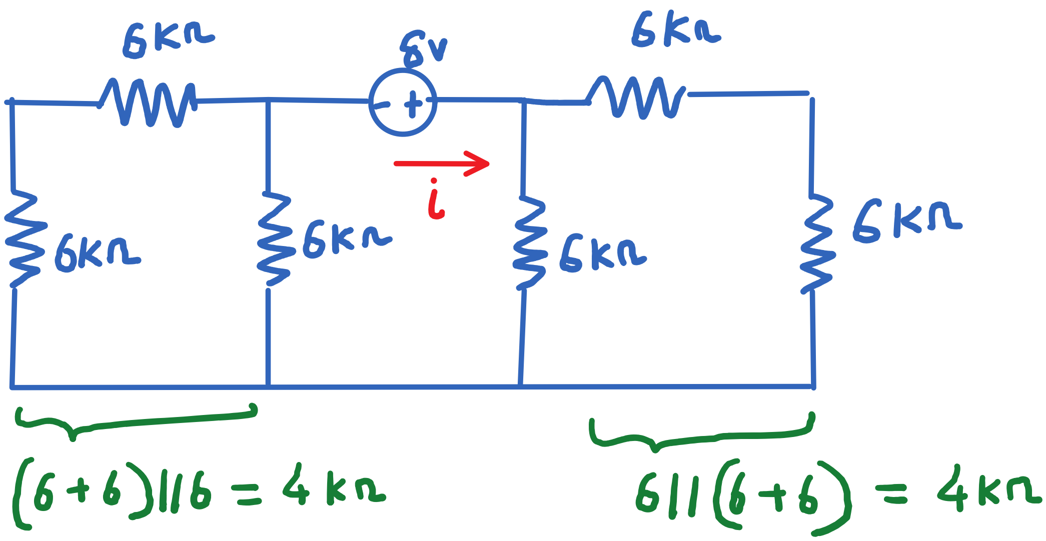
KVL: $\mathbf{V_{ab} + V_{bc} + V_{cd} + V_{da} = 0}$

Ohm's Law: $\mathbf{i_{ab}R_1 + V_{in2} + i_{cd}R_2 - V_{in1} = 0}$

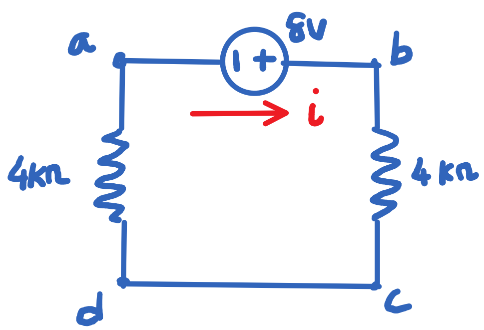
$$\Rightarrow \mathbf{iR_1 + iR_2 = V_{in1} - V_{in2}}$$

$$\Rightarrow \mathbf{\frac{V_{in1} - V_{in2}}{i} = R_1 + R_2.}$$

Example 3: Find current i in the circuit shown below.



Solution:
The circuit can be redrawn as shown below:



KVL: $\mathbf{V_{ab} + V_{bc} + V_{cd} + V_{da} = 0}$

Ohm's Law: $\mathbf{-8 + i_{bc} \times 4 + 0 + i_{da} \times 4 = 0}$

$\mathbf{\Rightarrow 4i + 4i = 8}$

$\mathbf{\Rightarrow i = 1mA.}$

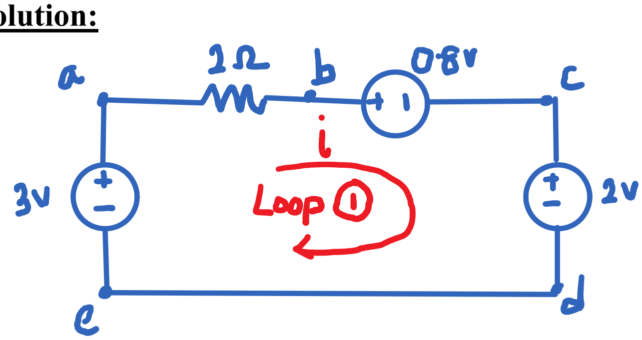
2. Loop analysis

We looked at Kirchhoff's voltage law and applied it for simple circuits containing one loop. Loop analysis is a systematic procedure based on KVL to solve for currents in more complex circuits. Loop current analysis involves the following steps:

- **Identify loops in a circuit.**
- **Pick currents in clockwise direction.**
- **Set up loop equations.**
- **Solve system of equations to obtain unknown currents.**

Example 2.1: Consider the single loop circuit shown below. Obtain the unknown current i.

Solution:

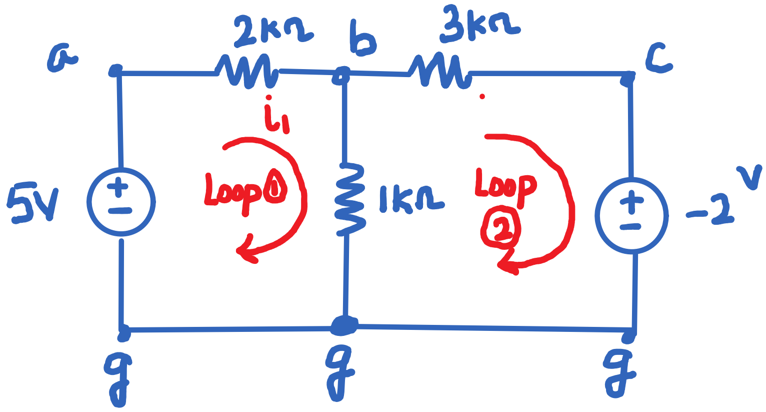


Solution:

KVL: $\mathbf{V_{ab} + V_{bc} + V_{cd} + V_{ea} = 0}$
Ohm's Law: $\mathbf{2i + 0.8 + 2 - 3 = 0}$

$\mathbf{\Rightarrow i = 0.1\ A}$

Example 2.2: Compute the currents i_1, i_2 in the circuit shown below.



Solution:

Loop 1(KVL): $V_{ab} + V_{bg} + V_{ga} = 0$

Ohm's Law: $2i_{ab} + \boxed{1i_{bg}} - 5 = 0$

$\Rightarrow 2i_1 + \boxed{1(i_1 - i_2)} - 5 = 0$

$\Rightarrow \boxed{3i_1 - i_2 = 5}$ (1)

Loop 2(KVL): $V_{bc} + V_{cg} + V_{gb} = 0$

Ohm's Law: $3i_{bc} - 2 + \boxed{1i_{gb}} = 0$

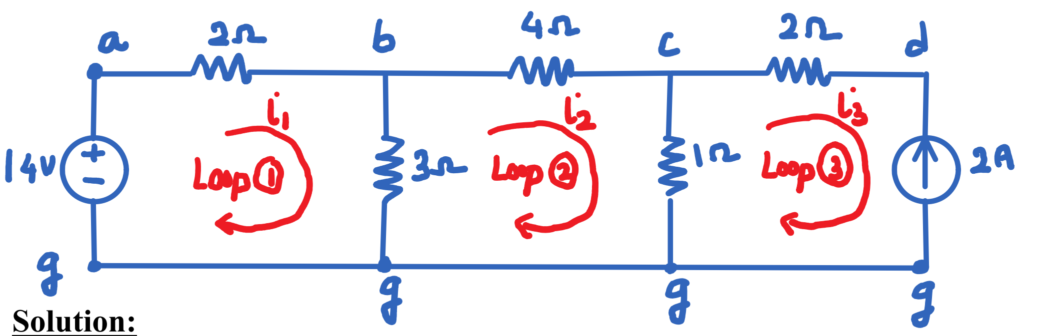
$\Rightarrow 3i_2 + \boxed{1(i_2 - i_1)} - 2 = 0$

$\Rightarrow \boxed{-i_1 + 4i_2 = 2}$ (2)

Solve (1) and (2): $\begin{bmatrix} 3 & -1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$

$$\boxed{\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -1 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \text{mA.}}$$

Example 2.3: Compute currents i_1, i_2, i_3, in the circuit shown below.



Solution:

By observation: $\boxed{i_3 = -2\text{ A}}$

Note: We cannot write a loop equation for loop 3. Why

Loop 1(KVL): $V_{ab} + V_{bg} + V_{ga} = 0$

Ohm's Law: $2i_{ab} + 3i_{bg} - 14 = 0$

$\Rightarrow 2i_1 + 1(i_1 - i_2) - 14 = 0$

$\Rightarrow \boxed{3i_1 - i_2 = 14 \quad (1)}$

Loop 2 (KVL): $V_{bc} + V_{cg} + V_{gb} = 0$

Ohm's Law: $4i_{ab} + 1i_{bg} + 3i_{gb} = 0$

$\Rightarrow 4i_2 + 1(i_2 - i_3) + 3(i_2 - i_1) = 0$

$\Rightarrow -3i_1 + 8i_2 - i_3 = 0$

$\Rightarrow \boxed{-3i_1 + 8i_2 = -2}$ (2)

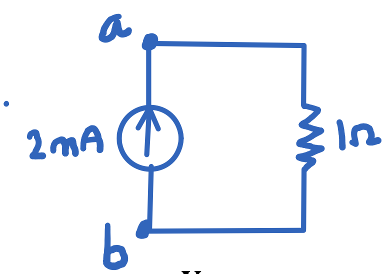
Solve (1) and (2): $\begin{bmatrix} 3 & -1 \\ -3 & 8 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 14 \\ -2 \end{bmatrix} \Rightarrow \boxed{\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -3 & 8 \end{bmatrix} \begin{bmatrix} 14 \\ -2 \end{bmatrix} = \begin{bmatrix} 5.42 \\ 2.28 \end{bmatrix} \text{A}}$

3. **Superloops**

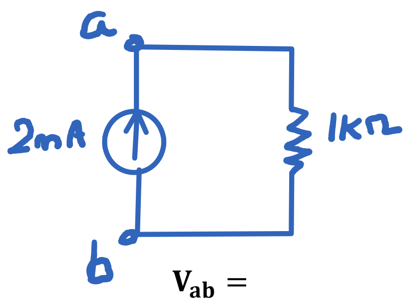
As we saw in the previous example, we cannot write a loop equation for loop that has a current source in a branch. This is because unlike a voltage source or a resistor, the voltage across a current source cannot be determined easily. Note that for a resistor, Ohm's law allows us to compute voltage easily. The difficulty in determining the voltage across a current source is illustrated by the following example.

Example 3.1:

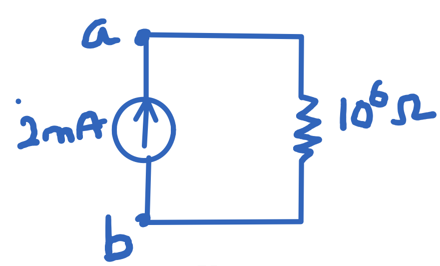
Find the voltage, $\mathbf{V_{ab}}$, between the the terminals of the current source for each of the cases shown below.



$\mathbf{V_{ab}}$ =



$\mathbf{V_{ab}}$ =

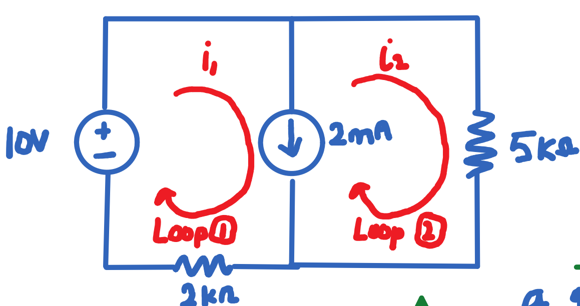


$\mathbf{V_{ab}}$ =

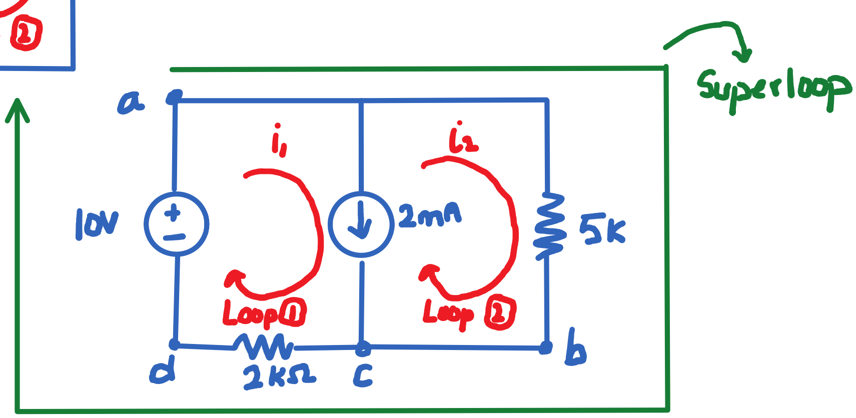
Solution: Solved in class.

In practice we may encounter situations when we have a current source in a branch. We will see an example of this when we discuss transistors. When there is a current source in a branch we will use the concept of **superloop** or a "bigger" loop to solve the problem. Note that since we still use a loop (just that it is bigger now), KVL holds! Let us look at the following example to get familiar with writing superloop equations.

Example 3.2: Compute currents $\mathbf{i_1}$ and $\mathbf{i_2}$ in the circuit shown below.



Solution: Note that in this problem we have a current source in the common branch between loops 1 and 2. We cannot write loop equations either loop 1 or loop 2. Let us form superloop by combining loops 1 and 2 as shown below.



Superloop Equation (KVL): $\mathbf{V_{ab} + V_{cd} + V_{da} = 0}$

Ohm's Law: $\mathbf{5i_2 + 2i_1 - 10} = 0$

$$\Rightarrow \mathbf{2i_1 + 5i_2 = 10} \qquad \textbf{(1)}$$

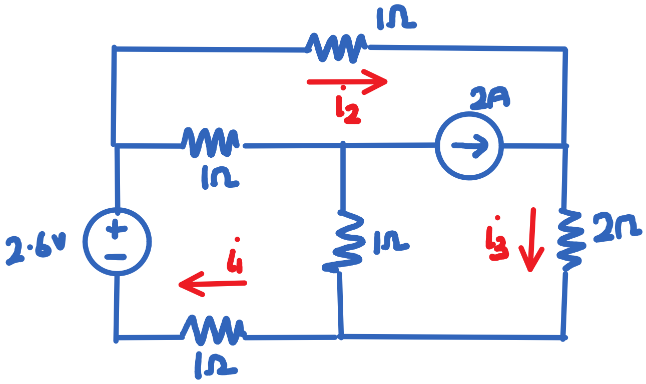
The second equation is obtained by using the current source,

$$\mathbf{i_1 - i_2 = 2} \qquad \textbf{(2)}$$

Solving equations (1) and (2),

$$\begin{bmatrix} 2 & 5 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 2 \end{bmatrix} \Rightarrow \boxed{\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 3.33 \\ 0.667 \end{bmatrix} \mathbf{mA}}$$

Example 3.3: Compute currents $\mathbf{i_1, i_2, i_3}$ in the circuit shown below.



Solution : **Solved in class as part of worksheet.**

4. An alternate notation for voltages.

We have so far used the notation shown in Fig. 4.1(a) to represent the voltage between two points A and B. Another common notation for voltage between two points is shown in Fig. 4.1(b). The relationship between the two notations is also described below.

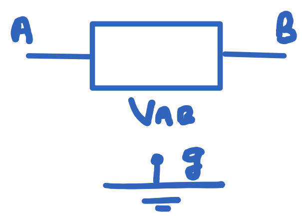


Fig. 4.1(a)

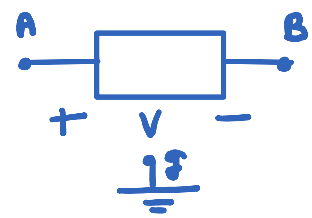
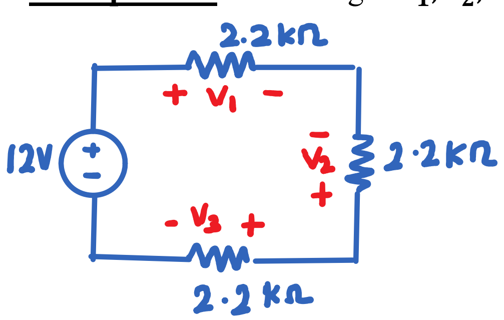


Fig. 4.1(b)

$$\boxed{\mathbf{V = V_{AB} = V_A - V_B,}}$$

where $\mathbf{V_A}$ and $\mathbf{V_B}$ represent voltage at nodes A and B with reference to ground

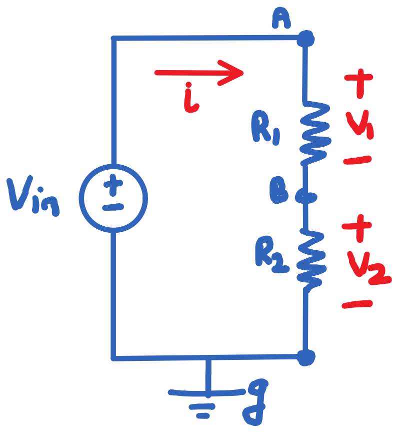
Example 4.1: Find voltages $\mathbf{V_1, V_2, V_3}$, in the circuit shown below



Solution: **Solved in class as part of worksheet.**

5. Voltage Dividers

A useful concept in circuit analysis and in applications is the voltage divider. A voltage divider circuit is shown below. We would like to find the voltages $\mathbf{V_1}, \mathbf{V_2}$ in the circuit shown below.



$$\mathbf{V_1} = \mathbf{V_{AB}} \text{ and } \mathbf{V_2} = \mathbf{V_{BG}}$$

$$\text{Current, } \mathbf{i} = \frac{\mathbf{V_{in}}}{\mathbf{R_1}+\mathbf{R_2}}$$

$$\mathbf{V_{AB}} = \mathbf{i_{AB}R_1} = \mathbf{iR_1} = \frac{\mathbf{V_{in}R_1}}{\mathbf{R_1}+\mathbf{R_2}}$$

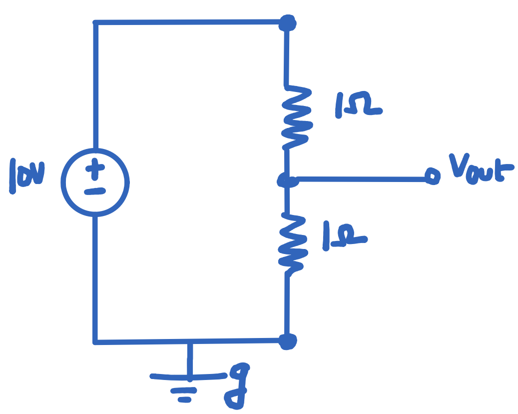
$$\Rightarrow \boxed{\mathbf{V_1} = \frac{\mathbf{V_{in}R_1}}{\mathbf{R_1}+\mathbf{R_2}}}$$

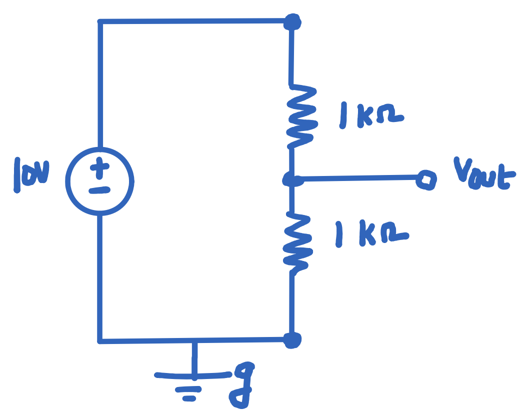
$$\mathbf{V_{BG}} = \mathbf{iR_2} = \frac{\mathbf{V_{in}R_1}}{\mathbf{R_1}+\mathbf{R_2}}$$

$$\Rightarrow \boxed{\mathbf{V_2} = \frac{\mathbf{V_{in}R_2}}{\mathbf{R_1}+\mathbf{R_2}}}$$

Example 5.1: Consider the two circuits shown below.

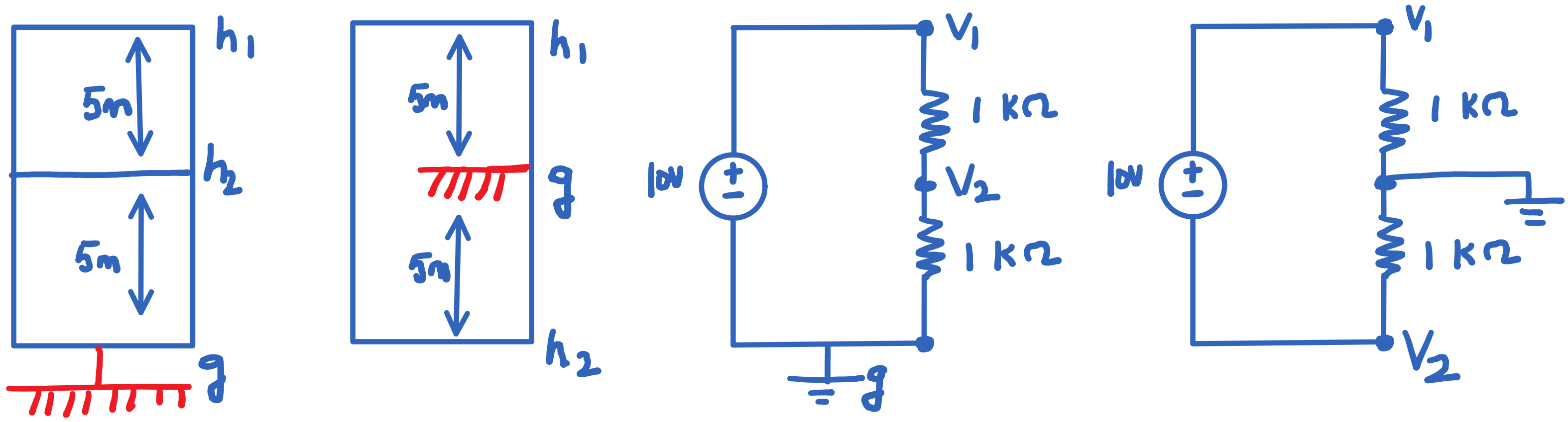
- Compute voltage V_{out} for both cases.
- What is the difference between the circuits?
- Is there a reason one circuit is preferred over other?





Solution: Solved in Class!

Example 3.2: Find voltages V_1 and V_2 in the two circuits shown below.



Case	h1	h2	V1	V2
1				
2				