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Pre-lab 7: Modeling Your Battery

Thevenin Equivalent Circuit

At this point we should all be familiar with solving circuits containing sources and resistors using KVL and KCL. It is beneficial to know that there are other very powerful tools to analyze circuits. One of these tools is the Thevenin Equivalent Circuit. The theory of Thevenin Equivalent Circuit starts with a black box with 2 terminals. **Black box** is a term used for *a device with unknown construction*. It refers neither to the color or enclosure! Assume this black box only contains ideal sources and resistors. However, you have no information on how many sources and resistor are present, nor do you know the values of these components. Thevenin's theory, can transform a circuit, as viewed through two terminals, into a Thevenin Equivalent Circuit. A Thevenin Equivalent circuit is an ideal voltage source in serials with a resistor. By applying the Thevenin's theory, you can actually find a simple *equivalent* circuit for the circuit composed by the black box. An equivalent circuit is a circuit that functions identically to the original circuit. Any circuit that performs mathematically-equivalently to another circuit is considered an *equivalent* to that circuit.

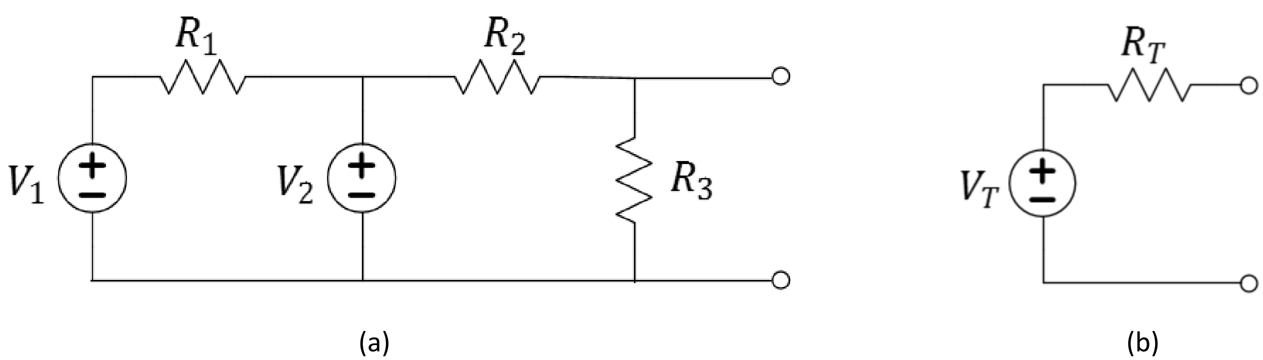


***Figure 1**: A complex DC circuit (a) and the Thevenin Equivalent Circuit (b) to which all DC circuits can be reduced.*

Thevenin's theory reduces the circuit into one equivalent voltage source V_T and one equivalent resistor R_T.

In order to find V_T, open the two terminals and measure the open-circuit voltage (connect a voltage meter to the two terminals).

In order to find R_T, short the two terminals and measure the short circuit current I_{sc}, then apply Ohms law. $R_T = \frac{V_T}{I_{sc}}$.

Notes:

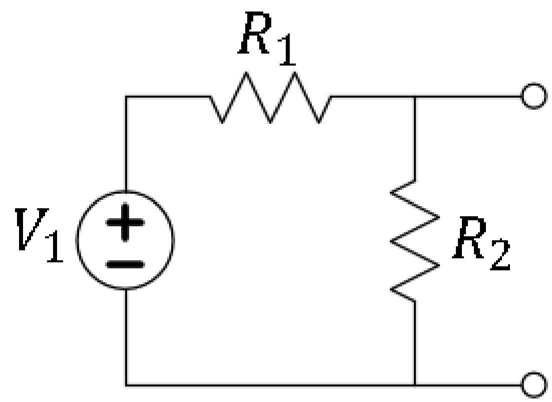
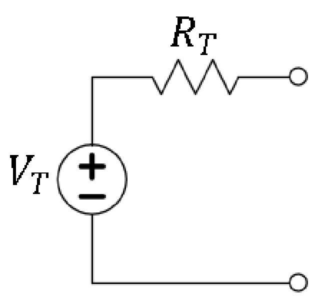


Figure 2: A practice circuit.

Question 1: For the practice circuit in Figure 2, find the Thevenin Equivalent Circuit when $V_1 = 5\ V,\ R_1 = 2.2\ k\Omega$, and $R_3 = 3.3\ k\Omega$.



$V_T =$ $R_T =$

Figure 3: The Thevenin equivalent of the practice circuit.

When given a circuit containing voltage sources, current sources, and resistors, it is possible to determine the Thevenin equivalent circuit using circuit analysis methods. Most real devices are not so simple, so we use empirical methods to determine the Thevenin equivalent circuit. In particular, we'll use graphical analysis of a devices IV curve.

Consider the IV curve of a resistor that we collected in Experiment #2. We found that the slope of the curve was equal to the inverse of the resistance. This very useful property of a *linear* IV curve is true for devices that are not resistors as well – the slope is the *Thevenin resistance*. In addition, we can find the value of the voltage of the Thevenin source by reading the voltage of the IV curve when the current (amps) is equal to zero. In many cases, the IV curve is not linear but we can still approximate the Thevenin circuit by conducting a linear curve-fit.

Notes:

Think about it: For a circuit consisting of a simple resistor with resistance R, the Thevenin resistance is $R_T = R$. What would be the Thevenin voltage for the equivalent circuit? What would be the Norton current for the Norton equivalent circuit?

Non-ideal Voltage Source

So far in this course, we have used the DC power supply as an ideal voltage source. For most practical applications, however, the DC power supply is too large and requires a wall outlet making it less-than-mobile. This week we'll learn about batteries and their non-ideal behaviors. Batteries present an engineering tradeoff. While they have poor behavior as an ideal voltage source, they can be made small and portable. What do we engineers do with this tradeoff? We figure out how to make the best of it by modeling the non-ideal behavior of the battery and accounting for it in our designs.

Let's begin by considering a very simple linear model of a battery, the Thevenin equivalent circuit. In this model, we assume that the battery is equivalent to an ideal voltage source (V_T) and source resistance (R_T) in series. In this prelab, we'll explore this model further by using DMM measurements (premeasured by your instructor) to determine the actual values of V_T and R_T.

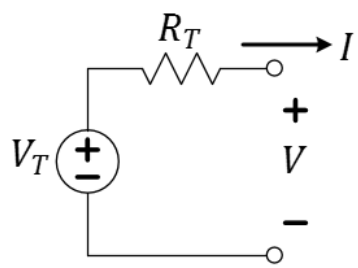


***Figure 4**: A first-order battery (Thevenin-equivalent) model.*

Below, we are providing you with a table of real-life measurements from the 6-pack of 1.2-volt, AA-sized, NiMH batteries used in the laboratory. It will be your task to plot the data and to estimate the internal resistance (according to a Thevenin model) in the region where the battery is not severely loaded (forced to source a lot of current).

Notes:

Current (mA)	Voltage (V)	Comments
0.0006	8.837	Using a potentiometer to control current via Ohm's Law. We did not record the resistance setting, but only the current and voltage. Voltmeter measures the voltage; Ammeter measures the current.
87.99	8.772	
93.60	8.759	
131.5	8.707	
148.9	8.688	
188.8	8.688	
215.2	8.627	
215.2	8.627	
258.4	8.586	
347.7	8.514	As the voltage dropped below 8 volts during the experiment, the current went to 800 mA, but then decreased wildly as the battery was forced well outside of its "linear" mathematical behavior!
534.9	8.375	
800	7.99	

***Table 2**: IV data collected for the battery by the instructors.*

Question 2: Plot the IV curve of a battery using the real-life data measured by your instructor. You should use MATLAB and fit a linear curve to the data (it is acceptable to use a pencil for the curve fit but MATLAB is preferred). Attach your graph to this assignment.

Question 3: Determine the Thevenin equivalent circuit of the battery using your linear curve-fit and draw the equivalent circuit below. Label it as Figure 5.

Question 4: *Suppose* we have a battery with an internal resistance of $0.5\ \Omega$ and we need its output voltage to stay within 10% of the ideal source when it is in operation (sourcing power to a load). What is the smallest resistive load we can put across our battery while keeping V within 10% of V_T (reference **Figure 4** and your answer from **Question 3**)?

Battery Data Sheet

We've seen some datasheets already this semester. Battery datasheets will give us an idea of the limitations of the battery and can be very useful for determining how to charge it properly.

The batteries we use in the ECE 110 Lab (a 6-pack of 1.2-volt, AA-sized, NiMH batteries, made in Japan and labeled by Amazon) do not have a specific datasheet, but we can estimate performance using a datasheet for a similar battery like the Duracell DX1500. Perform an Internet search on "Duracell DX1500 datasheet" and answer the question below.

Question 5: List at least one battery characteristic you would need to find on the datasheet that would help you determine an appropriate charger.

Power and Energy

Batteries are devices that store electrical energy typically by either exploiting a chemical process. In a circuit schematic, sources of electrical power can be distinguished from power sinks (or loads) by the relationship between the polarity of the voltage across the device and the polarity of the current flowing through it.

$$P = VI$$

Or, more specifically, if $V_{+\to-}$ is the voltage drop across a device in the direction of the current's polarity arrow:

$$P_{device} = V_{device} \times I_{device} = V_{+\to-} \times I_{\to} \text{ which is } \begin{cases} < 0\ if\ power\ source \\ > 0\ if\ power\ sink \end{cases}$$

Notes:

Think about it: In general, this battery model is very useful but it does not account for the behavior of the battery when it has mostly discharged (died). As the battery dies, its internal resistance grows and it deviates from the ideal voltage very quickly. In order to avoid this, it is important to charge our batteries regularly. The teaching assistants are making sure that the laboratory batteries remain charged.

Notes:

Power and energy are two related concepts often confused by the aspiring engineer. Often energy is first learned in the potential and kinetic forms. It is conserved in that it is neither created nor destroyed, but instead may incur transformations in form. It has units of joules (J). A charge-storing device, like a battery or a capacitor, is often referenced by the amount of energy it stores. A device that utilizes energy for long periods of time, like a light bulb, are often referenced by the amount of energy they utilize each second. Power is the amount of energy expended over time. It has units of Watts (1 Joule/sec). In this way, a 60-Watt incandescent bulb transforms 60 Joules each second into light and heat.

You have been supplied with a rechargeable 7.2-V (6 times 1.2V per battery) Nickel Metal Hydroxide (NiMH) battery with a 1900 milliamp-hour (mAh) rating. While this is a charge rating, Q, a little reasoning soon leads us to see that the battery might deliver 1900 milliamps at 7.2 volts while running continuously for 1 hour and thus it also provides an energy rating. Since milliamps are a measure of the maximum electron flow (coulombs/second), the battery might alternately deliver 1 milliamp for 1900 hours (79 days). Significant variations between batteries will occur due to deviations in manufacturing, the age, and current-supplying limitations imposed by the materials comprising the battery (in this case, NiMH). Later, we will find that the two motors used in our car project consume a large majority of the overall circuit power and we can use this fact to estimate the battery life when we reach full testing and demonstrations later in the semester.

Charging the Battery

It is important not to attempt to charge a battery faster than the chemical reaction can comfortably occur. For NiMH batteries, the suggested charging rate is from $0.5 \times C$ to $1.0 \times C$, where C is the energy rating designated on the battery in mAh. Since our battery is rated at 1900 mAh, a 1-amp maximum charging current should be a good, safe choice. Your TAs will keep the batteries charged using a maximum charging current of 1 A.

Question 6: Calculate how long the battery should charge if it were completely dead before charging continuously at 1.0 A. Assume that the battery has a charging efficiency of 66% (66% of the total current goes toward charging increasing the potential energy of the battery while the rest is wasted, perhaps as heat).

Notes:

Question 7: State the conservation of energy equation ($E_{input} = \Delta E_{state} + E_{waste}$) as it applies to charging the battery. Give numeric values in Joules for the energy input, change in state, and waste.

Using Circuit Models

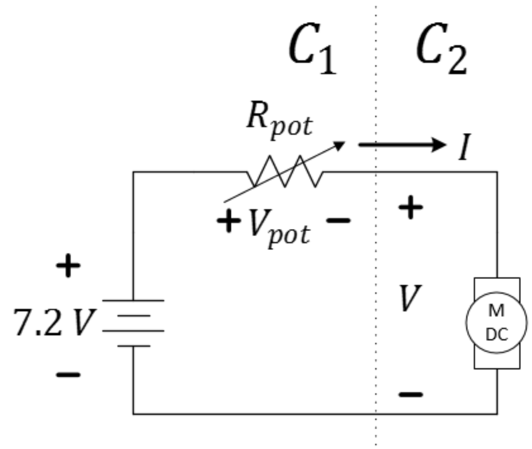


***Figure 6**: A motor in a simple voltage divider circuit.*

Question 8: Replace the battery of Figure 6 with the Thevenin model, V_T and R_T. Find the IV equation of circuit C_1 as a function of V_T, R_T, and R_{pot}.

$$I = mV + b = \qquad\qquad V +$$

Notes:

Question 9: Using your modeled values from **Question 3** for V_T and R_T, simplify your formula to give the IV equation as a function of R_{pot} only.

$$I = mV + b = \qquad\qquad V +$$

In lab, IV measurements will be made of the motor and Thevenin modelling applied.

Question 10: (**<u>This is an optional question</u>**…earn 1 point extra credit!) Complete the ***Explore More!*** **Module**: *Arduino as a Voltmeter* (you can turn this in for module credit if you haven't already). Now build the practice circuit of Figure 2 and the equivalent circuit of Figure 3. Note that both V_1 and V_T should be available from your microprocessor. Also, note that you can create R_T using the same-valued resistors as R_1 and R_2. Use the Arduino to record the integer values of the output voltage V for a load resistance of $R = 1\ k\Omega$ and for a load resistance of $R = 10\ k\Omega$ and one more resistance of your own choosing. Complete the table and show your circuits to your TA.

$R =$	Integer value	Voltage across R
$1\ k\Omega$		
$10\ k\Omega$		

TA signature if Question 10 is completed and circuits shown:_______________________________________