Pre-lab 6: Modeling Your Battery

Thevenin Equivalent Circuit
At this point we should all be familiar with solving circuits containing sources and resistors using KVL and KCL. It is beneficial to know that there are other very powerful tools to analyze circuits. One of these tools is the Thevenin Equivalent Circuit. The theory of Thevenin Equivalent Circuit starts with a black box with 2 terminals. **Black box** is a term used for a device with unknown construction. It refers neither to the color or enclosure! Assume this black box only contains sources and resistors. However, you have no information on how many sources and resistor are present, nor do you know any detail about these components. By applying the Thevenin’s theory, you can actually find a simple equivalent circuit for the circuit composed by the black box. An equivalent circuit is a circuit that functions identically to the original circuit. Any circuit that performs mathematically-equivalently to another circuit is considered an equivalent to that circuit.

Thevenin’s theory, as mentioned above, can transfer an equivalent circuit, containing sources and resistors, across 2 terminals into a Thevenin Equivalent Circuit. A Thevenin Equivalent circuit, in short, is in fact an ideal voltage source in serials with a resistor.

![A complex DC circuit (a) and the Thevenin Equivalent Circuit (b) to which all DC circuits can be reduced.](image)

*Figure 1: A complex DC circuit (a) and the Thevenin Equivalent Circuit (b) to which all DC circuits can be reduced.*

Thevenin’s theory reduces the circuit into one equivalent voltage source $V_T$ and one equivalent resistor $R_T$. 
In order to find $V_T$, open the 2 terminals and measure the open-circuit voltage (connect a voltage meter to the 2 terminal).

In order to find $R_T$, short the 2 terminals and measure the short circuit current $I_{sc}$, apply Ohms law. $R_T = \frac{V_T}{I_{sc}}$.

**Figure 2: A practice circuit.**

**Question 1:** For the practice circuit in Figure 2, find the Thevenin Equivalent Circuit when $V_1 = 5 \, V$, $R_1 = 3 \, k\Omega$, $R_2 = 1 \, k\Omega$, and $R_3 = 5 \, k\Omega$. Use both VDR and CDR in your solution.
When given a circuit containing voltage sources, current sources, and resistors, it is possible to determine the Thevenin equivalent circuit using circuit analysis methods. Most real devices are not so simple, so we use empirical methods to determine the Thevenin equivalent circuit. In particular, we’ll use graphical analysis of a devices IV curve.

Consider the IV curve of a resistor that we collected in Experiment #2. We found that the slope of the curve was equal to the inverse of the resistance. This very useful property of a linear IV curve is true for devices that are not resistors as well – the slope is the *Thevenin resistance*. In addition, we can find the Thevenin voltage source by reading the voltage of the IV curve when the current (amps) is equal to zero. In many cases, the IV curve is not linear but we can still approximate the Thevenin circuit by conducting a linear curve-fit.

**Question 2:** What is the Thevenin equivalent circuit for a 100 \( \Omega \) resistor. Draw it here.

Hint: It *must* include a voltage source to be a Thevenin equivalent circuit.
Non-ideal Voltage Source

So far in this course, we have used the DC power supply as an ideal voltage source. For most practical applications, however, the DC power supply is too large and requires a wall outlet. This week we’ll learn about batteries and their non-ideal behaviors. Batteries present an engineering tradeoff. While they have poor behavior as an ideal voltage source, they can be made small and portable. What do we engineers do with this tradeoff? We figure out how to make the best of it by modeling the non-ideal behavior of the battery and accounting for it in our designs.

Let’s begin by considering a very simple linear model of a battery, the Thevenin equivalent circuit. In this model, we assume that the battery is equivalent to an ideal voltage source (\(V_T\)) and source resistance (\(R_T\)) in series. In this week’s experiment, we’ll explore this model further by using DMM measurements determine the actual values of \(V_T\) and \(R_T\).

![Diagram of Thevenin equivalent circuit]

**Figure 3:** A simple battery (Thevenin-equivalent) model that includes ideal voltage source with a small series resistor.

Below, we are providing you with a table of real-life measurements from the 6-pack of 1.2-volt, AA-sized, NiMH batteries. It will be your task to plot the data and to estimate the internal resistance (according to a Thevenin model) in the region where the battery is not severely loaded (forced to source a lot of current).
### Table 2: IV data collected for the battery.

<table>
<thead>
<tr>
<th>Current (mA)</th>
<th>Voltage (V)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0006</td>
<td>8.837</td>
<td>Using a potentiometer to control current via Ohm's Law. Not recording the resistance setting, but only the current and voltage. Voltmeter measures the voltage; Ammeter measures the current.</td>
</tr>
<tr>
<td>87.99</td>
<td>8.772</td>
<td></td>
</tr>
<tr>
<td>93.60</td>
<td>8.759</td>
<td></td>
</tr>
<tr>
<td>131.5</td>
<td>8.707</td>
<td></td>
</tr>
<tr>
<td>148.9</td>
<td>8.688</td>
<td></td>
</tr>
<tr>
<td>188.8</td>
<td>8.688</td>
<td></td>
</tr>
<tr>
<td>215.2</td>
<td>8.627</td>
<td></td>
</tr>
<tr>
<td>215.2</td>
<td>8.627</td>
<td></td>
</tr>
<tr>
<td>258.4</td>
<td>8.586</td>
<td></td>
</tr>
<tr>
<td>347.7</td>
<td>8.514</td>
<td>As the voltage dropped below 8 volts during the experiment, the current went to 800 mA, but then decreased wildly as the battery was forced well outside of its “linear” mathematical behavior!</td>
</tr>
<tr>
<td>534.9</td>
<td>8.375</td>
<td></td>
</tr>
<tr>
<td>800</td>
<td>7.99</td>
<td></td>
</tr>
</tbody>
</table>

**Question 3:** Plot the IV curve of a battery using the real-life data measured by your instructor. You should use MATLAB and fit a linear curve to the data (it is acceptable to use a pencil for the curve fit but MATLAB is preferred). Attach your graph to this assignment.

**Question 4:** Determine the Thevenin equivalent circuit of the battery using your linear curve-fit and draw the equivalent circuit below.
**Question 5:** Suppose we have a battery with an internal resistance of 0.5 Ω and we need its output voltage to stay within 10% of the ideal source. What is the smallest resistive load we can put across our battery?

In general, this battery model is very useful but it does not account for the behavior of the battery when it has mostly discharged (died). As the battery dies, its internal resistance grows and it deviates from the ideal voltage very quickly. In order to avoid this, it is important to charge our batteries regularly.

**Battery Data Sheet**

We’ve seen some datasheets already this semester. Battery datasheets will give us an idea of the limitations of the battery and can be very useful for determining how to charge it properly.

The batteries we use in the ECE 110 Lab (a 6-pack of 1.2-volt, AA-sized, NiMH batteries, made in Japan and labeled by Amazon) do not have a specific datasheet, but we can estimate performance using a datasheet for a similar battery like the Duracell DX1500. Perform an Internet search on “Duracell DX1500 datasheet” and answer the question below.

**Question 6:** List at least one battery characteristic you would need to find on the datasheet that would help you determine an appropriate charger.
Power and Energy

Batteries are devices that store electrical energy typically by either exploiting a chemical process. In a circuit schematic, sources of electrical power can be distinguished from power sinks (or loads) by the relationship between the polarity of the voltage across the device and the polarity of the current flowing through it.

\[ P = VI \]

Or, more specifically, if \( V_{+\rightarrow-} \) is the voltage drop across a device in the direction of the current’s polarity arrow:

\[ P_{device} = V_{device} \times I_{device} = V_{+\rightarrow-} \times I_{\rightarrow} \text{ which is } \begin{cases} 0 & \text{if power source} \\ > 0 & \text{if power sink} \end{cases} \]

Power and energy are two related concepts often confused by the young aspiring engineer. Often energy is first learned in the potential and kinetic forms. It is conserved in that it is neither created nor destroyed, but instead may incur transformations in form. It has units of joules (J). A charge-storing device, like a battery or a capacitor, is often referenced by the amount of energy it stores. A device that utilizes energy for long periods of time, like a light bulb, are often referenced by the amount of energy they utilize each second. Power is the amount of energy expended over time. It has units of Watts (1 Joule/sec). In this way, a 60-Watt incandescent bulb transforms 60 Joules each second into light and heat.

You have been supplied with a rechargeable 7.2-V (6 times 1.2V per battery) Nickel Metal Hydroxide (NiMH) battery with a 1500 milliamp-hour (mAh) rating. While this does not immediately appear to be an energy rating, a little reasoning soon leads us to see that the battery might deliver 1500 milliamps while running continuously for 1 hour. Since milliamps are a measure of the maximum electron flow (coulombs/second), it might alternately deliver 1 milliamp for 1500 hours. Of course, while a circuit may draw 1 milliamp, other mechanisms will serve to discharge the battery over the span of 1500 hours (50 days). Also, significant variations will occur due to deviations in manufacturing, the age of the battery, and current-supplying limitations imposed by the materials comprising the battery (in this case, NiMH). Later, we will find that the two motors used in our car project consume a large majority of the overall circuit power and we can use this fact to estimate the battery life when we reach full testing and demonstrations later in the semester.

Charging the Battery

It is important not to attempt to charge a battery faster than the chemical reaction can comfortably occur. For NiMH batteries, the suggested charging rate is from 0.5 \( \times C \) to 1.0 \( \times C \), where \( C \) is the energy rating designated on the battery.
in mAh. Since our battery is rated at 1500 mAh, a 1-amp maximum charging current should be a good choice. Your TAs will keep the batteries charged using a maximum charging current of 1 A.

**Question 7:** Calculate how long the battery should charge if it were completely dead before charging continuously at 1.0 A. Assume that the battery has a charging efficiency of 66% (66% of the total current goes toward charging increasing the potential energy of the battery while the rest is wasted, perhaps as heat).

**Question 8:** State the conservation of energy equation \( E_{input} = \Delta E_{state} + E_{waste} \) as it applies to charging the battery. Give numeric values in Joules for the energy input, change in state, and waste.
Using Circuit Models

Let a battery be modeled by a Thevenin equivalent circuit with the values $V_T$ and $R_T$. Let a running motor be modeled by a Thevenin equivalent circuit with the values $V_{mT}$ and $R_{mT}$. The motor is placed in a voltage divider circuit with a resistor $R_{pot}$ as shown below.

![Figure 3: A motor in a simple voltage divider circuit.](image)

**Question 9:** Replace the battery and motor with their Thevenin models. Find the voltage across the motor as a function of $V_T$, $R_T$, $V_{mT}$, $R_{mT}$, and $R_{pot}$. HINT: It is not $V_{mT}$!

**Question 10:** Find the voltage across the motor when $R_S \ll R_{divide}$ such that it can be ignored. Do not ignore $R_{pot}$ or $R_{mT}$. 