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# Module 001: Understanding Resistance

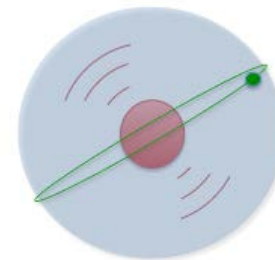
All materials have the ability to maintain currents of charged particles – the measure of this ability is termed *conductivity* and is typically designated by the Greek letter  $\sigma$ . The *resistivity* of a material is designated  $\rho$  and is inversely related to the conductivity  $\rho=1/\sigma$ . Some materials are very good at conducting current. Most of these are metals - elements like copper, silver, and gold. Some materials conduct current very poorly like glass, or some types of plastic.

So what makes an electronic device a 'resistor'? The name implies that the resistor is a device that controls the amount of current flowing through it. It does this by dissipating some of the electrical energy as heat. With the proper choice of material resistivity and fabrication geometry, resistors can be manufactured that control the amount of current flowing through the device. Resistors are circuit elements that follow Ohm's law  $V = RI$ . Many values of resistances are commercially available. For a given supply voltage, the amount of current delivered may be controlled with the choice of resistance.

Our common conception of a wire is in its role providing low loss connections between circuit elements. Wires are made of materials, most commonly metallic elements like copper with very high conductivity and are typically fabricated into long strands of varying thickness. Even the connectivity between components on most integrated circuit chips are made of the same material as the wiring in your house. The conductivity of these materials is many orders of magnitude greater than the conductivity of materials used to make most resistors but at a microscopic level the physics is the same.

Conductivity is a bulk property of materials that depends on parameters associated with the atoms/molecules of the material, and the geometry of how the atoms/molecules are bonded, as well as externally imposed parameters like temperature. Let's look at a *representation* of what takes place inside a solid material at a microscopic scale to see how the conductivity and current flow are related.

Several figures are used depicting a generic material with each individual atom represented by a totally not to scale rendering of the positively charged, relatively heavy, nucleus (red) vibrating with an energy proportional to the temperature of the material (red lines) and the orbiting negatively charged electrons (blue shell surrounding the nucleus). The main property that determines the conductivity of a material is the ease at which electrons can be removed from the influence of nucleus. If you look at a periodic table and look at all of the materials we call metals they all share the same property – the outermost shell contains only one or two electrons. It is shielded from the positive charge in the nucleus by all of the other electrons making it easy to

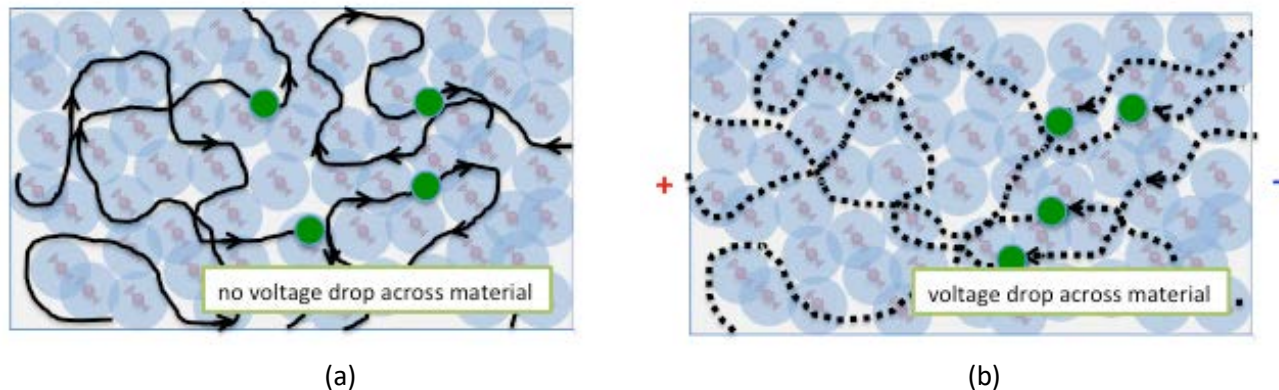


Example of single atom with one electron in the outermost shell.

Gold with an atomic number of 79 is such a good conductor because the positively charged nucleus is almost completely shielded by the 78 electrons in stable orbits.

dislodge at room temperatures. These escaped electrons are represented by the green dots in the figures below.

If you look inside a piece of this substance you will see a lattice of atoms or molecules bonded in a structure that depends on the material. Even at room temperature the free electrons have sufficient kinetic energy to escape the influence of the atoms in the lattice. A site missing an electron or two has a small region surrounding it that is positively charged. If a wandering electron comes close enough it can be re-captured. Through this minefield of attractors move the electrons that have broken free. When a voltage is applied across the material, the free electrons feel a force proportional to the voltage drop across the devices in the direction towards the terminal with the higher voltage. A very simplified view of the movement of these electrons in the absence of an electric field and in the presence of an electric field (voltage drop) is illustrated in the Figure 2(a).



Without a voltage drop, the free electrons can move through the empty spaces in the material at close to the speed of light, but most are not energetic enough to be totally free of the influence of the positively charged regions. As an electron approaches a positively charged site its trajectory is modified like a comet approaching the sun. If the encounter is too close to the positively charged nuclei, the electron can get re-captured or it can transfer some of its energy to a trapped electron knocking it free. Because the free electrons are influenced randomly by the thermal fluctuations in the vibrating lattice there is no bulk electric current.

Once a voltage is applied across the device the free electrons are still buffeted by the lattice, some are still captured by the lattice sites while others break free, but there is a net *trend* inside the material for the electrons to drift in a direction towards the higher voltage – Figure 2(b) illustrates this behavior. While each electron moves very quickly in the free space between lattice sites, the organized drift velocity is surprisingly slow – for a piece of copper (one of the better conducting materials) carrying a current of almost 50 A and sustaining a voltage drop of 1V an individual electron can move at  $\sim 0.5\%$  the speed of light, but the drift velocity

is at most millimeters per second. Unlike many of the software demonstrations of current flow, electric current is not a few free electrons racing around a circuit path, but instead current is countless electrons slowly drifting in the direction of (negative) current flow in the circuit path.

| Material                                | Resistivity $\rho$<br>(Ohm-m)            | Conductivity $\sigma$<br>(1/Ohm-m)   |
|---|--|--------------------------------------|
| Teflon                                  | $>10^{-23}$                              |                                      |
| Air                                     | $\sim 10^{16}$                           | $\sim 10$                            |
| Salt Water                              | .2                                       | 5                                    |
| Carbon Graphite – thin film<br>graphite | $3 \times 10^{-3}$<br>$5 \times 10^{-6}$ | $3.3 \times 10^2$<br>$3 \times 10^5$ |
| Carbon Amorphous - carbon<br>filled     | $5 \times 10^4$                          | $2 \times 10^5$                      |
| Nichrome - wirewound                    | $1 \times 10^{-6}$                       | $1 \times 10^6$                      |
| Aluminum                                |  | $3.5 \times 10^7$                    |
| Copper                                  |  | $6 \times 10^7$                      |
| Silver                                  |  | $6.3 \times 10^7$                    |

Table 1: Resistivity and conductivity of several common materials.

The table above shows the resistivity and its inverse the conductivity of common materials. As you can see the range of resistivity/conductivity spans a range of over  $10^{30}$ . The grey cells indicate materials commonly used to make resistors.

The main factors that determine the conductivity are:

- i) The ease at which the electrons can be dislodged – For a given volume of material the easier it is to remove electrons the more electrons there are to make up current.
- ii) The distance between the atoms in the lattice of the material – The more closely the spaced the atoms in the lattice the more often the free electrons are scattered or captured reducing their velocity and randomizing their direction of travel.
- iii) The temperature of the material – As the temperature increases the atoms in the lattice vibrate with greater amplitude lowering the mobility of the free electrons.

Electrical conductivity is often correlated with the thermal conductivity. Some of the same mechanisms that make a material a

good conductor of heat also make them good electrical conductors.

The units of resistance are Ohms. In SI units an Ohm is equal to *Volt/Amp* which, taking into account that  $\text{Amp} = \text{charge}/\text{time}$  is *Volt\*Sec/Coulombs*.

Notes:

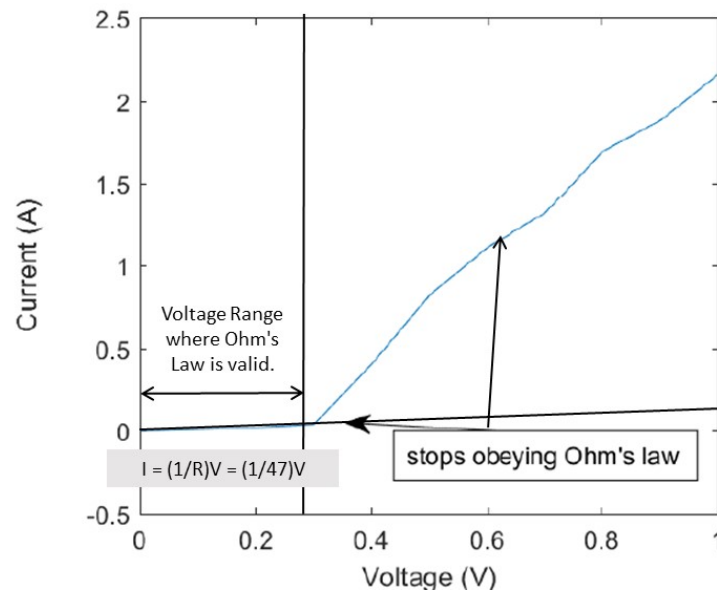
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# Procedures

## Modeling the behavior of a Resistor

Even though the physical behavior of current flowing through a resistor is enormously complicated if considered at scales much greater than quantum scales the bulk properties are surprisingly simple to quantify as Georg Ohm found in the early 1800's while putting together one of the first comprehensive books on connecting electromagnetics and circuit theory. This simple relationship between the voltage across a piece of material and the current flowing through the material  $V = RI$  was met with derision at first. This linear relationship was empirically derived but Ohm constructed the mathematical framework to show how it fit with the rather new subject of electromagnetics. It was apparently worthy of descriptions such as "web of naked fancies" by the then German minister of Education.

Today we use Ohm's law as our simplest model of the I-V characteristics of a resistor. The figure below is a graph of the behavior of a  $47\Omega$   $\frac{1}{4}$  W resistor as the voltage increases. Between 0 and  $\sim 0.3$  V the resistor follows the linear relationship expected by Ohm's Law. The resistor is from the same package that you have in your kits. It appears that the resistor fails by shorting the leads drawing more and more current until the resistor is destroyed in a puff of really bad smelling brown smoke. At this point no current can flow and you have a resistor of nearly infinite impedance.



**Errata:** While the idea is correct, a  $47\Omega$   $\frac{1}{4}$  W resistor will not deviate greatly from Ohm's Law, nor begin to fail at merely 0.3 V. The author has made a mistake in the presentation here. Do the math yourself and figure out how much power is dissipated at 0.3 V. Take a few (voltage, current) measurements to see for yourself. If you do get near the power rating, be careful! The resistor will get very hot and may be "pop" and send a shard of ceramic through the air. Wear goggles.

### Experimenting with Making a Resistor

The devices we call resistors are optimized through the choice of the most suitable **material and geometry**. With easy to use materials – graphite in pencil lead and aluminum wire – you will explore this relationship. **NOTE:** *The measurement device you will use during this lab– the Digital Multimeter - only measures resistance NOT resistivity.* As you work through the procedures it will become clear how resistance and resistivity are related.

- ✓ Set the Digital Multimeter in the mode used to measure resistance.

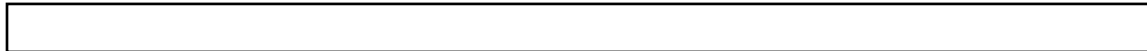
**Question 1:** All materials have the ability to conduct current. Conduction is the property associated with the resistivity. Measure the resistance of things around you – a pencil, your arm (don't pierce the skin), the bench material, etc. – but heed this warning: **the DMM ohmmeter uses a voltage source to inject a small amount of current to make the measurement.** Use good judgment – trying to find the resistance of your partner's eye is bad judgment. Write down the things you tried in Table 2 and whether they have high resistance (display reads OVLD), low resistance (display reads 0), or a measurable resistance (write down the value) according to the ohmmeter. Since you will be probing arbitrary surfaces the measurements process can be tricky since the banana cables were designed to interface with a device with conducting leads. For example, if I hold one connector between my thumb and index finger and the other between my ring and little finger the resistance is 5-15 M $\Omega$  depending on how tightly I squeeze the connectors.

| Material | Resistance | Configuration and Comments |
|----------|------------|----------------------------|
|          |            |                            |
|          |            |                            |
|          |            |                            |
|          |            |                            |
|          |            |                            |
|          |            |                            |
|          |            |                            |
|          |            |                            |

Table 2: Your measured resistances of various materials and structures.

**Question 2:** Did the results surprise you? Most people can guess which materials are good conductors – materials with a low resistivity/resistance - by knowing which materials are good thermal conductors. There is often a high correlation between thermal and electrical conductivity. What property makes people decent conductors?

- ✓ Using a graphite artist's (available in the lab) or a mechanical pencil color in the rectangle below. Make sure the line is fairly uniformly filled. Line dimensions: 6" x .25"



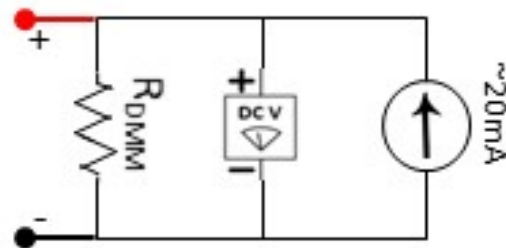
**Question 3:** Place either probe at one end of the line. With the other probe take measurements along the line at different points. Label each point at which you took your measurement and write the resistance as measured by the DMM next to your mark.

- ✓ Color the rectangle below – the width is the same but it is twice as high as the one used in Question 3. Try to make color this box with approximately the same darkness as the narrower one. Line dimensions: 6" x .5"



**Question 4:** Repeat Question 3 using the larger rectangle. Try to make the measurements at approximately the same points.

These resistance measurements also illustrate how the Digital Multimeter (DMM) makes the resistance measurement. This voltage is provided by a nearly ideal current source in parallel with a resistance of  $R_{DMM}=1M\Omega$  and an ideal voltmeter. A very small amount of current flows between the probes creating a voltage across  $R_{DMM}$ . The ideal voltmeter then measures the voltage across this internal resistor. The figure below shows a schematic of this simple model.



**Question 5:** When you were probing the line at different points along the length of the rectangle you were actually measuring the resistance of just that part – if you cut out the rectangle and snipped it to the length of one of your marks the resistance would be approximately equal to the resistance you measured. Why is this true – why does the resistance depend on where along the line you position the probe?



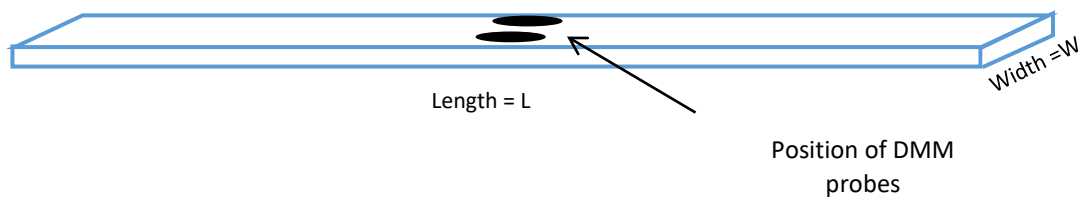
The resistance of a piece of material can be computed if the resistivity is known and the geometry of the piece of material is also known. Most resistors are made using materials that are formed into easy to use geometries – like a cylinder. For any shape whose cross-sectional area does not change along its length the resistance  $R$  can be computed as  $R = \frac{\rho l}{A} = \text{resistivity} \times \text{length} / (\text{cross-sectional area})$ .

Each of the rectangles that you colored in with graphite is a resistor. Since they are really thin you only varied the length and width. Since their cross-section is approximately rectangular the resistance relationship can be written as  $R = \frac{\rho l}{A} = \frac{\rho L}{W * H}$  with  $H$  being the height.



**Question 6:** Do your measurements agree with this formula?

- ✓ Probe the line you drew for question 4 differently, as shown in the figure below.



**Question 7:** Write down the result. Why is the value measured in this way different from that measured by probing the line at the ends.

Resistor or wire? Conducting materials are characterized by a bulk parameter – the conductivity  $\sigma$  measured in Siemens (SI units) or mhos. Conductors are most often used to carry current thereby transferring, with as little loss as possible, electrical kinetic energy from one point in the circuit to another. The geometry and materials of wires are optimized for this purpose.

- ✓ Cut a short piece of wire about the length as the rectangle below and tape it to the paper. Re-use any wire you find on the benches to conserve wire.

**Question 8:** Measure the resistance at the ends of the wire – did the DMM even measure a non-zero resistance? If so, write it down. Explain why a piece of wire this long might have trouble registering a non-zero resistance on the Ohmmeter.

**Question 9:** (Optional) Why are all wires made of copper? Some research should help you answer this question. One major consideration is the conductivity but there are others.