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## Module 400 (previously 2A): Oscilloscope’s XY mode

## Module Outline

In this module you will use the oscilloscope in what is called the xy mode. In lecture, we will often trace out the transfer characteristics of many different devices (called I-V curves). Previously, you used the oscilloscope to plot voltage vs. time. The oscilloscope can also plot voltage vs. voltage. But, if we measure the voltage across a resistor, Ohm's Law tells us that we will have a voltage value that is proportional to the current.

For this experiment, you will need to cooperate with one of your neighbors as you will need a second function generator.
A fun way to experiment with the $x y$ mode is by generating Lissajous figures. To do so, we'll the sine-wave output of one function generator to channel 1 of the oscilloscope and the sine-wave output of another function generator to channel 2 of the oscilloscope. By letting channel 1 drive the horizontal and channel 2 the vertical, a shape will be traced out on the scope's display.

In this mode the variations in $x$ and $y$ are time-varying. For example: consider the two equations $x(t)=\sin (t)$ and $y=\cos (t)$. We could find the shape that is drawn out by reducing the two equations to a single equation $-x^{2}(t)+y^{2}(t)=1$ which is the equation of a circle. The set of equations, where the variables, in this case $x$ and $y$, are written in terms of another variable, in this case $t$, are called parametric equations. Often it is not simple to find an equation for the shape of the resulting curve so let us look at a graphical approach.


By plotting $x(t)$ along the vertical axis and $y(t)$ along the horizontal axis the resulting curve can be drawn by plotting $(x(0), y(0)),(x(0.5), y(0.5)),(x(1), y(1)), \ldots$ using $x(t)$, and $y(t)$ and a ruler and pencil. The frequencies of the sine waveforms depicted in the graph are not simple multiples of each other so it will take many cycles to repeat the curve. Only the time interval $t=0-8 s$ has been plotted.

Wikipedia and YouTube have a wealth of information about these curves. It is about the most fun you can have with an oscilloscope (unless it secretly plays Tetris as did our old scopes). Laser light shows often use complex parametric equations to draw interesting figures with light.

To understand some of the questions in this module consider the following set of parametric equations and the shape of the resulting figure.

Set 1 - identical sine waves
$x(t)=\sin (2000 \pi t)$
$y(t)=\sin (2000 \pi t)$

The resulting figure plotted on the $x-y$ plane can be found by eliminating the variable $t$. Since $x(t)=y(t)=\sin (2000 \pi t)$ the equation describing the figure is $x=y$ for $|x| \leq 1$ and $|y| \leq 1$. If the frequencies were not the same it would be considerably more difficult to find a closed form solution.

Set 2 - sine waves with the same frequency but out of phase by $\pi / 2$ or $90^{\circ}$
$x(t)=\sin (2000 \pi t)$
$y(t)=\sin (2000 \pi t+\pi / 2)$

The resulting figure in this case can be found by eliminating $t$ but in a less straight-forward way.
Step 1: Rewrite $y(t)=\sin (2000 \pi t+\pi / 2)$ as $y(t)=\sin (2000 \pi t) \cos \left(\frac{\pi}{2}\right)+\cos (2000 \pi t) \sin \left(\frac{\pi}{2}\right)$. Since $\cos \left(\frac{\pi}{2}\right)=0$ then $y(t)=\cos (2000 \pi t)$. The variable $t$ is eliminated by squaring both equations and adding to get $x^{2}+y^{2}=1$.

Set 3 - sine waves with the same frequency with a phase difference that varies with time
$x(t)=\sin (2000 \pi t)$
$y(t)=\sin (2000 \pi t+\phi(t))$
Question 1: What shape is the figure given in set 3?

## Procedures

Using the output from 2 function generators, generate Lissajous figures.
$\checkmark$ Connect the output of your function generator to channel 1 of the oscilloscope. Have the function generator output a sine wave with a peak-to-peak amplitude of 2 V , a frequency of 1 kHz , and a OV offset.
$\checkmark$ Connect the output of your neighbor's function generator to channel 2. Have the function generator output a sine wave with a peak-to-peak amplitude of 2 V , a frequency of 1 kHz , and a 0 V offset. You can use the function generator at the same time as your neighbors if you use a special connector - a T connector - that allows 2 BNC connectors to connect to the output of the function generator. Please see your TA if you do not have one.
$\checkmark$ At this point you should see two sine waves on the display of the oscilloscope. Push the Horiz button on the oscilloscope in the set of buttons controlling the horizontal sweep and put the display in XY mode. The oscilloscope is now displaying the same information except now one of the sine waves is driving the $x$-axis deviations and the other the $y$-axis.
$\checkmark$ Find a recording of "Dark side of the Moon" by Pink Floyd and listen while doing module.

Question 2: Plot the figure using MATLAB.

Question 3: Since the outputs of the both signal generators are sine waves of identical amplitudes and frequency, is the resulting figure what you expected? Explain.

Question 4: When using 2 signal generators, while the amplitude and frequencies may be the same the relative phases are not the same and they often drift slowly with time. Is your figure time-varying? Describe how the figure is changing.

Question 5: Change the frequency of one of the function generators to 2Khz. Plot the result.

Question 6: The following chart shows you some of the possible curves you can draw by varying the frequency and phase of the sinusoidal functions. Unfortunately, the phase of the signal generator outputs is fixed, though it often drifts slowly with time. Play with all sorts of different frequency combinations and see how many you can create. Plot your favorites.


Typical Lissajous shapes. Source: http://en.wikipedia.org/wiki/Lissajous_curve. CC BY-SA 3.0.

Oscilloscope's xy mode

This is only the beginning. More fun things to try...
$\checkmark$ Because the important parameter is the proportional values of the frequencies of the two signals you can watch the figure being drawn. Reduce the frequencies of the signals to 10 Hz and 20 Hz . You should see a dot tracing out the figure you got in question 4.
$\checkmark$ There are more functions that you can try - the signal generator can provide a sine wave, a square wave, a triangle wave, a sawtooth wave, a cardiac signal, a signal that sweeps frequencies, and many, many more. Play around with them to see what patterns you can generate.

Question 7: Plot some interesting combinations along with the two signals used to get them.

