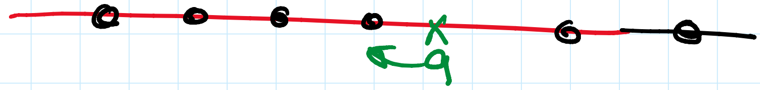
August 30, 2023 10:46 AM

Problem (Dynamic Predecessor Search)

build data structure for n numbers $S = \{a_1, \ldots, a_n\}$
to support:

query { – pred search: given q, find largest $a_i \in S$ less than q



(succ similar)

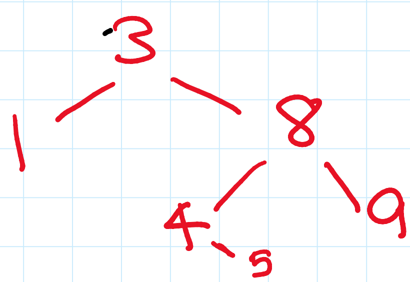
update { – insert an elem to S
– delete " " from S

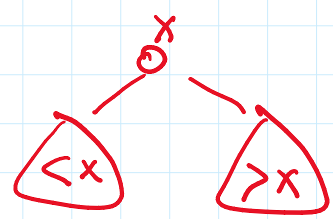
Method 0 sorted array

query time $O(\log n)$ by binary search
space $O(n)$
preproc time $O(n \log n)$

but insert/delete $O(n)$ ← bad

Method 1 binary search tree (BST)



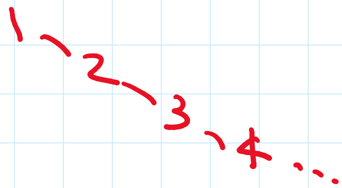


query: can still use binary search
insert: straightforward
delete: replace with pred/succ

insert: straightforward
delete: replace with pred/succ

time $O(\text{tree height})$

but height may be $\Omega(n)$ in worst case!

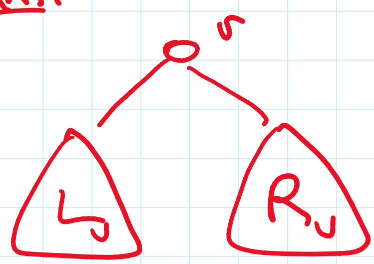


$\Rightarrow$ need <u>balanced</u> search trees!

<u>Method 2: AVL Tree</u> (Adelson-Velsky, Landis '62)

(sketch only)

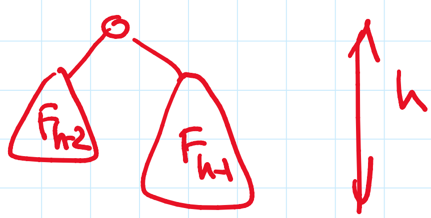
<u>Invariant:</u>



at every node v,
$|\text{height}(L_v) - \text{height}(R_v)| \le 1.$

$\Rightarrow$ height $\lesssim \log_\phi n$ $\quad \phi = \frac{1+\sqrt{5}}{2}.$

F_h =



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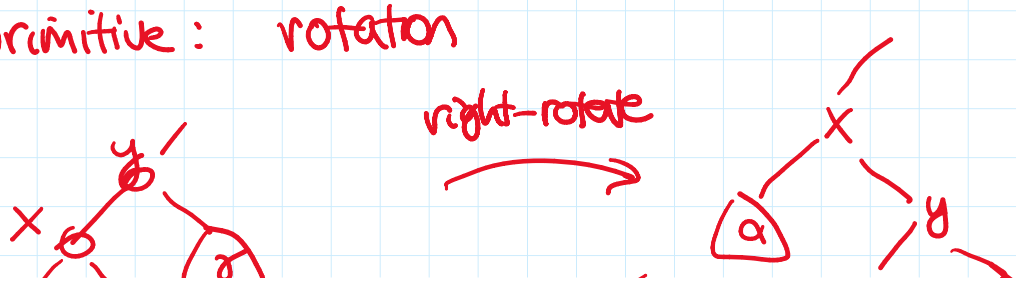
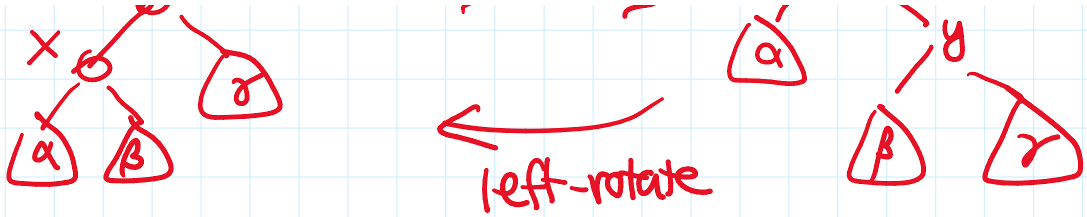
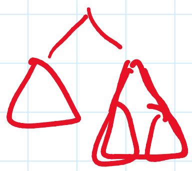
basic primitive: rotation







→ insert: 4 cases … (messy)

delete: 4 cases …

all ops $\boxed{O(\log n)}$ time

Other balanced BSTs:

- red-black tree (Guibas-Sedgewick '78)
- AA tree (Andersson '?) (shortest code)
- treap (simple but randomized)

Method 3: 2-3 Trees or 2-3-4 Trees (Hopcroft '70)

(easiest to understand conceptually)

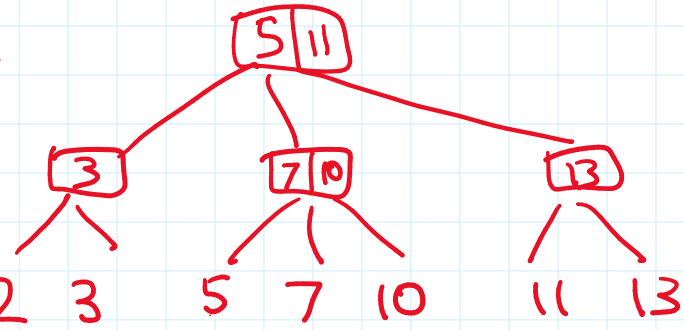
be more flexible with degree

Invariants:

1. $\forall$ node v, $a \le \deg(v) \le b$ (except at root), for params a, b with $a = \lceil b/2 \rceil$

2. all leaves at same depth

e.g. $b=3$
$a=2$



elems stored at leaves

$n \geq a^h$

$\Rightarrow$ height $\lesssim \log_a n$

query: still $O(\log n)$

insert: whenever $\deg(v) = b+1$ for some v:

split v into 2 nodes of deg $\sim \frac{b+1}{2}$

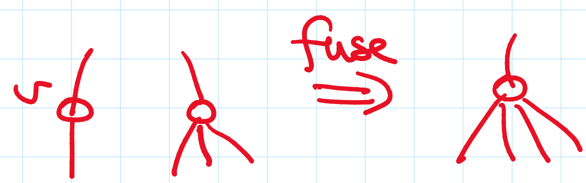
$\Rightarrow$ $O(\log n)$ splits (up a path)

$\Rightarrow$ $\boxed{O(\log n)}$ time.

delete: whenever $\deg(v) = a-1$ for some v:

fuse v with sibling

& split if $\deg \geq b$



$\Rightarrow$ $O(\log n)$ fuses

$\Rightarrow$ $\boxed{O(\log n)}$ time