CS 598TMC, Fall 2023
Timothy Chan

Homework 3 (due Nov 3 Friday 10am)

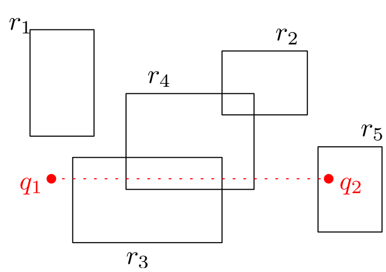
Instructions: see previous homework.

1. [*25 pts*] We are given a set S of n points in 2D, where each point is assigned a color. Present an efficient data structure so that a given query (axis-aligned) rectangle q, we can quickly decide whether all points inside q have the same color. (You may assume that q contains at least one point.)

   (Hint: modify the 2D range tree.)

2. [*20 pts*] Consider the following problem: store a set S of n (axis-aligned) rectangles in 2D so that for a given query horizontal line segment $\overline{q_1q_2}$, we can quickly count the number of rectangles $r \in S$ that $\overline{q_1q_2}$ completely cuts across (i.e., $\overline{q_1q_2}$ intersects both the left and right side of r). Design an efficient data structure for this problem.

   (Hint: directly reduce to orthogonal range searching. How many dimensions?)

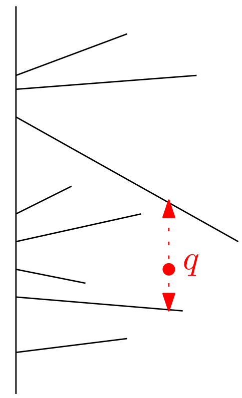


$\overline{q_1q_2}$ completely cuts across r_3 and r_4 (but not r_5)

3. [*30 pts*] In this question, you will explore a different method for solving the 2D planar point location problem.

   (a) [*15 pts*] Let $s_1, \ldots, s_n$ be a set of n disjoint line segments, such that the left endpoints all lie on a common vertical line ℓ. Give an $O(n)$-space data structure for this special case that can find the segment immediately above and the segment immediately below a query point in $O(\log n)$ time.

   Hint: sort $s_1, \ldots, s_n$ are sorted by the y-coordinates at ℓ. For $i = 1, \ldots, n/2$, let s'_i be the segment from $\{s_{2i-1}, s_{2i}\}$ whose right endpoint's x-coordinate is larger. Recursively build a data structure for $s'_1, \ldots, s'_{n/2}$.



(b) [*15 pts*] Using (a) and divide-and-conquer, present a data structure for 2D planar point location with $O(n)$ space and $O(\log^2 n)$ time.

4. [*25 pts*] We are given a set S of n horizontal line segments in 2D, with integer coordinates in $[U]$. We want a data structure to answer the following type of queries: given a vertical line segment q with integer coordinates in $[U]$, report all segments in S that intersects q. Describe a solution with $O(n \log\log U)$ space and $O(\log\log U \log\log n + k \log\log n)$ query time (or better), where k denotes the number of reported segments.

   (Hint: persistence, and vEB trees.)