Last Time:
triangle listing $\mathcal{L}(m^{4/3})$. aplln's to dynamic graph DSs.
e.g. dynamic graph connectivity with vertex deletes/ re-inserts.

\[ \gamma(m^{2/3}) \]

C.- Patrascu-Ruditty '11: $\delta(m^{2/3})$.

\[ \mathcal{L}(m^{13/5}) \]

Jumbled Text Indexing
build data structure for a text string $t = t_1 \ldots t_n \in \Sigma^*$, $\Sigma = \{\sigma\}$, $\sigma = \text{const}$.

st. given any query pattern $p = p_1 \ldots p_k$,
can determine if \exists i s.t.
$p_1 \ldots p_k$ matches $t_{i+k-i}$ up to permutation!

e.g. $t = \text{"algorithm is fun"}$ $t = 011010110$

Reduction: Convolution - \( \text{Convolve} \) for \( \text{3SUM} \) is \( \Omega(n^2) \) time.

Assuming \( \text{3SUM} \) hard, no algorithm for \( \text{3SUM} \) queries for \( \text{Junction} \) matching has \( \Omega(n^{2 - \epsilon}) \) time. The \( \text{Junction} \) matching case takes \( \Omega(n^{2 - \epsilon}) \) time for \( \mathcal{O}(n) \) queries.

\( \text{Lewenstein} \) \( \gamma = 2 \)  
\( \mathcal{O}(n^{2 - 2/13}) \) time for \( \mathcal{O}(n) \) queries.

Then, \( \text{Lewenstein} \) \( \gamma = 3 \)  
\( \mathcal{O}(n^{2 - 2/9}) \) time.

\( \text{3SUM} \) for \( \alpha \)-dimensional vectors reduces to standard \( \text{3SUM} \) ... like \( \text{3SUM} \) for \( \alpha \)-dimensional vectors reduces to standard \( \text{3SUM} \).  

Offline version reduces to a special case of \( \text{3SUM} \) if \( \gamma - \alpha = \epsilon \).

For each \( \alpha \in \mathbb{Z} \), let \( \alpha(x) \) = frequency of \( \alpha \). Assume \( \alpha \) is known.

Offline version: queries are given in advance. \( t = \text{algorithm} \) \( \text{3SUM} \), \( t = \text{algorithm} \) \( \text{3SUM} \) for \( \mathcal{O}(n) \) queries.
Reduction: \( \text{Conv3SUM} \rightarrow \text{Conv3SUM in } \mathbb{U} \text{ with } U = \mathcal{O}(n^2) \)

by hashing
\[
\begin{align*}
\text{Let } h(x) &= x \mod p \text{ for rand. prime } p \in \mathbb{N} \\setminus \{0\} \\
\text{err prob } &\leq \mathcal{O}(n^2 \cdot \frac{1}{R})
\end{align*}
\]

Reduction: \( \text{Conv3SUM in } \mathbb{U} \rightarrow d\text{-dim Conv3SUM in } \mathbb{U}^{1/d} \)

map number to d-dim vector by
just writing each number in base \( B = U^{1/d} \)
with \( d \) "digits"

\[
a_i = a_i(d-1) \ldots a_i(0) \\
c_k = c_k(d-1) \ldots c_k(0)
\]

\[\begin{array}{c}
\text{e.g.} \\
B=10 \\
738 \\
\hline
214 \\
524
\end{array} \quad \begin{array}{c}
738 \\
\hline
256 \\
482 \\
\downarrow \downarrow \\
5, -2, 2
\end{array}
\]

Guess carry bits \( \Delta_0, \Delta_1, \ldots, \Delta_{d-1} \in \{0,1\} \)

let \( c_k(x) = \begin{cases} 
\ ch_k(x) + \Delta_k & \text{if } \Delta_k = 0 \\
\ ch_k(x) - B + \Delta_k & \text{if } \Delta_k = 1
\end{cases} \)

\( \Rightarrow \) 2\(^d\) instances

Reduction: \( d\text{-dim Conv3SUM in } \mathbb{U}^{1/d} \)
Reduction: d-dim Convolution in \( \Omega \geq f \) \rightarrow Jumbled Indexing

Suppose Jumbled Indexing could be solved in
\[ T(n) = O(n^{2-4\sigma-8}) \] time.

Given d-dim vectors \( a_1, \ldots, a_n, c_1, \ldots, c_n \),

Let \( \Sigma = \{d\} \cup \{\$, %\} \), \( \sigma = d+2 \)
define text string

\[ t = \%\%\%f_1 \%\%\%f_2 \%\%\% \cdots \%\%\%f_n \%\%\% \]

where

\[ f_i = \begin{cases} a_i(1) - a_i(1) & \text{if } i = 1 \\ a_i(2) - a_i(2) & \text{if } 2 \leq i \leq d \\ a_i(n) - a_i(n) & \text{if } i = d+1 \end{cases} \]

\[ \beta = \Omega^{1/d} \]

for each \( ck \),
find pattern with \( C_k(\alpha) + 8_k \) occurrences of \( \alpha \)
for \( \alpha \in \{d\} \}

\[ \begin{cases} k+1 \%\%\%s \\ 2k \%\%\%s \end{cases} \]

Claim: for each \( k \), pattern exists
\( \iff \exists i,j \), with \( a_j - a_i = c_k \) and \( j-i = k \).

Proof: \((\Leftarrow) \)
take substring

\[ \%\%\%f_{i+1} \%\%\% \cdots \%\%\%f_j \%\%\% \]

\# occurrences of \( \alpha = a_i(1) - a_i(1) + a_i(2) - a_i(2) + \cdots + a_j(\alpha) - a_j(\alpha) + a_j(n) - a_j(n) = C_k(\alpha) \)

\# $s's = i-j-1 + 2 = b+1
\# $^j$'s = \( j-i-1 + 2 = k+1 \)
\# $^\circ$'s = \( 2(j-i-1) + 2 = 2k \).

(\( \Rightarrow \))

pattern must be of the form
\[ \begin{array}{c}
\hline
\hspace{0.5cm} \hline
\hline
\hline
\end{array} \]
\[ \begin{array}{c}
\hline
\hline
\hline
\hline
\end{array} \]
\[ \begin{array}{c}
\hline
\hline
\hline
\hline
\end{array} \]

Since \( \# $^j$'s = k+1 \), \( \# $^\circ$'s = 2k \), for some \( i,j \) with \( j-i = k \).
So, \( \chi_\alpha(x) = a_j(x) - a_i(x) \) where \( \alpha \in [d] \).

\[
\text{text length} = \mathcal{O}(n \cup \nu^d) = \tilde{\mathcal{O}}(n^{1+2/d}) = \tilde{\mathcal{O}}(n^{\frac{d+2}{d}}) = \tilde{\mathcal{O}}(n^{\frac{d+2}{d}}) = \tilde{\mathcal{O}}(n^{\frac{d+2}{d}}).
\]

= time \( \tilde{\mathcal{O}}(n^{\frac{2 \sigma - 4}{\sigma - \delta}}) \)
\[
= \tilde{\mathcal{O}}(n^{\frac{2 \sigma - 4}{\sigma - \delta}}) \]
\[
= \tilde{\mathcal{O}}(n^{2 - \delta'}) \]

Final Ranks on Conditional LBs
- other conjectures:
  \( k\text{SUM for larger } k \) \( (\text{no } n^{k/2 - \delta'} \text{ alg}) \)
  \( 1\text{-line} \) \( (\text{no } n^{k-\delta} \text{ alg}) \)
mini-wt k-clique (no n^{k-o} alg'm)

- OMv: online matrix-vector mult.