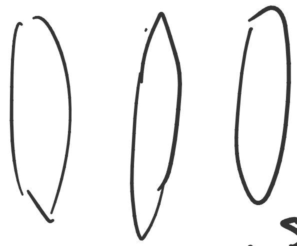
CONDITIONAL LOWER BDS

APSP - Related Problems

APSP

min-wt triangle

min-wt cycle

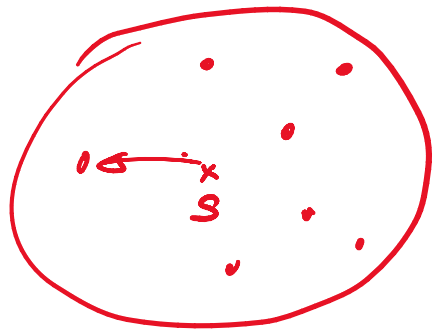


shortest path diam

graph diameter $\max_{s,t \in V} d(s,t)$

graph radius $\min_{s \in V} \max_{t \in V} d(s,t)$

center



⋮

APSP Conjecture No $O(n^{3-\delta})$ alg'm for APSP for arbitrary real edge-wt'ed graphs for any const $\delta > 0$.

Int APSP Conjecture No $\tilde{O}(n^{3-\delta})$ alg'm for APSP for int wts in $[U]$ ($\tilde{O}$ hides polylog n & polylog U factors)

Conditional to conjecture, Could we prove lower bds for other problems?

Thm (Vassilevska W. & Williams '13)
(Abboud, Grandoni, Vassilevska W. '15)

(Abboud, Gra…

For int wts:

APSP has an $\tilde{O}(n^{3-\delta})$ alg'm for some $\delta > 0$

$\Leftarrow$ $\Leftrightarrow$ min-wt triangle has an $\tilde{O}(n^{3-\delta'})$ alg'm for some $\delta' > 0$

$\Leftrightarrow$ radius has an $\tilde{O}(n^{3-\delta''})$ alg'm for some $\delta'' > 0$

$\vdots$

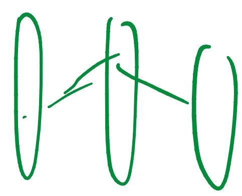
basic approach - reductions! (like in NP-completeness, but fine-grained)

obviously - min-wt triangle, radius, ... reduce to APSP

will reduce in other dir.

Reduction 0: APSP $\to$ (min,+)-MM

already known



Problem: (min,+)-MM Decision

Given $n \times n$ matrices A, B, D, for each i, j,

decide whether $\underbrace{\min_k (a_{ik} + b_{kj})}$ $\leq d_{ij}$

i.e. decide $\exists k,\ a_{ik} + b_{kj} \leq d_{ij}$.

Reduction 1: (min,+)-MM $\to$ (min,+)-MM Decision

Reduction 1: (min,+)-MM → (min,+)-MM Decision

(assume ints)

Suppose (min,+)-MM Decision can be solved in $T(n) \leq O(n^{3-\delta})$ time.

To solve (min,+)-MM:

by n^2 binary search! simultaneously

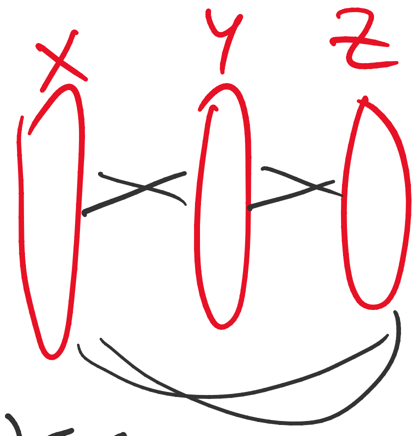
with $O(\log U)$ calls to decis alg'm

$\Rightarrow$ total time $O(T(n) \log U)$. $\square$

Problem: Tripartite All-Pairs Negative Triangle (APNT).

Given tripartite weighted graph $G = (X \cup Y \cup Z, E)$

$E \subseteq ((X \times Y) \cup (Y \times Z) \cup (X \times Z))$



for each $x \in X$, $z \in Z$,

decide $\exists y$ s.t.

$w(xy) + w(yz) + w(xz) < 0$.

Note: (min,+)-MM Decis $\equiv$ Tripartite APNT

Problem: Tripartite Find-One Neg. Triangle (1NT)

find one $(x,y,z) \in X \times Y \times Z$ s.t.

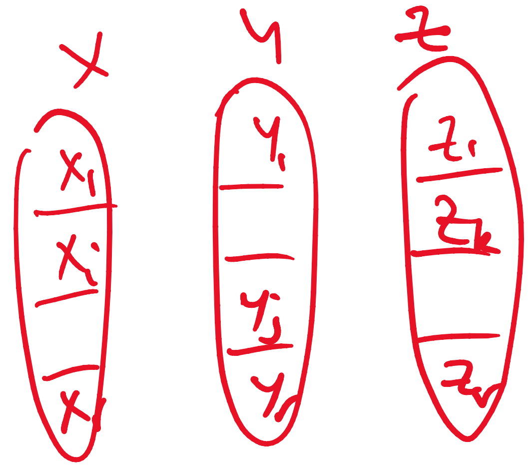
$w(xy) + w(yz) + w(xz) \leq 0$

or declare none exists.

Reduction 2: Tripartite APNT → Tripartite 1NT

Suppose 1NT ~~can~~ could be solved in $T(n) \leq O(n^{3-\delta})$ time.

To solve APNT:



divide X into subsets $X_1, \ldots, X_r$ of size n/r

Y " " $Y_1, \ldots, Y_r$

Z into " $Z_1, \ldots, Z_r$

for each $i, j, k \in [r]$.

repeat {

$T(\frac{n}{r}) \to$ find one neg triangle $(x,y,z) \in X_i \times Y_j \times Z_k$

if exist

set answer for (x,z) to true

remove (x,z) from E

}

Total time: $O\left(\left(r^3 \boxed{+K}\right) \cdot T\left(\frac{n}{r}\right)\right)$

where K = # pairs (x,z) with true answer

$\leq n^2$

$$= O\left(\left(r^3 \boxed{+n^2}\right) T\left(\frac{n}{r}\right)\right)$$

Set $r = n^{2/3}$

$$= O\left(n^2 T\left(n^{1/3}\right)\right)$$

$$= O\left(n^2 \left(n^{1/3}\right)^{3-\delta}\right)$$

$$= \boxed{O\left(n^{3-\delta/3}\right)}$$

$$= \boxed{O\left(n^{3-\delta/3}\right)}. \quad \square$$

Note: 1NT → min-wt triangle, obviously

$$\left(\text{APSP} \to (\min,+)\text{-MM} \to \underbrace{(\min,+)\text{-MM Decis}}_{=\ \text{APNT}} \to \underline{1\text{NT}} \to \text{min-wt } \Delta \right)$$

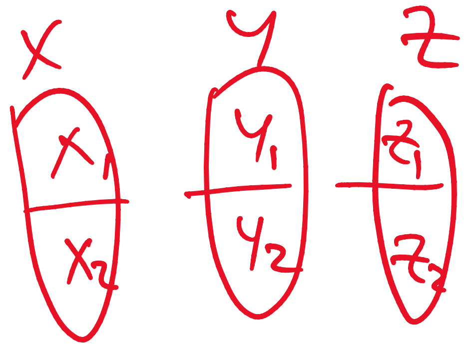
Problem: Tripartite Negative Triangle Detection (NT)

test if $\exists\, x,y,z \in X \times Y \times Z$
s.t. $w(xy) + w(yz) + w(xz) \le 0$.

Reduction 3: Triparite 1NT → Tripartite NT

Suppose NT can be solved in $T(n) = O(n^{3-\delta})$ time.

To solve $\underline{1\text{NT}}$:



Divide X into X_1, X_2 of size $n/2$
Y Y_1, Y_2
Z Z_1, Z_2

for each $i,j,k \in \{1,2\}$,
test if $\exists$ neg triangle in $X_i \times Y_j \times Z_k$
If true, $\underline{\text{recurse}}$ in X_i, Y_j, Z_k & exit

(self-reducibility)

Runtime:

$$T'(n) \le 1\, T'\left(\tfrac{n}{2}\right) + 8\, T\left(\tfrac{n}{2}\right)$$

$$T'(n) \le \frac{1}{\sqcup} T'\left(\frac{n}{2}\right) + \gamma T\left(\frac{n}{2}\right)$$

$$\Rightarrow \quad T'(n) \le T'\left(\frac{n}{2}\right) + O(n^{3-\delta})$$

$$\Rightarrow \quad O(n^{3-\delta}) \text{ time. } \square$$

Rmk: similarly,

Boolean MM in subcubic time reduces to triangle finding in subcubic time

for "~~combinatorial~~ algms"

Problem: Zero Triangle Detection

test if $\exists\, x, y, z \in X \times Y \times Z$

s.t. $w(xy) + w(yz) + w(xz) = 0$.