CS 598TMC, Fall 2019
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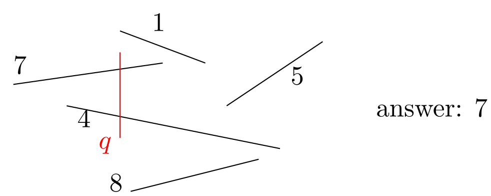
Homework 2 (due Oct 18 Friday (11am in class))

Instructions: You may work in groups of at most 2. Hand in one set of solutions per group. Acknowledge any discussions you have with other students and other sources you have consulted. Solutions must be written *in your own words*.

1. [*20 pts*] We are given a set S of n disjoint line segments in 2D, where each segment has a weight (a real number). We want a data structure to answer the following type of queries: given a vertical line segment q, find the maximum-weight segment $s \in S$ that intersects q. Describe a solution with $O(n \operatorname{polylog} n)$ space and $O(\log n)$ query time, by using persistence.



2. [*20 pts*] Recall the *cutting lemma* from class: given a set S of n disjoint line segments in 2D and $1 \le r \le n$, there exists a division of the plane into $O(r)$ vertical trapezoids, such that each trapezoid intersects at most $O(n/r)$ segments of S. The construction from class uses randomization. Describe a different construction that does not require randomization and takes $O(n \log n)$ time.

   Your construction should satisfy the following additional property: for each trapezoid Δ in the cutting[1], and for every vertical line ℓ, the number of segments of S intersecting $\Delta \cap \ell$ lies between n/r and $4n/r$.

   [Hint: sweep the plane from left to right, and decide when to "open" new trapezoids and "close" old trapezoids as we sweep...]

3. [*20 pts*] We are given a set S of n horizontal line segments in 2D, where coordinates lie in U. We want a data structure that can answer the following type of queries: given a vertical line segment q, report all segments of S intersecting q. Let k denote the output size (the number of reported segments).

   (a) [*5 pts*] First show that there is a data structure with $O(n)$ space and $O(\log\log U + k \log\log U)$ time, by directly applying a point location data structure from class.

   (b) [*15 pts*] Describe a better data structure with $O(n)$ space and $O(\log\log U + k)$ time. [Hint: use the cutting lemma, specifically, the version with the additional property stated in Problem 2, for an appropriate choice of r.]

[1] Except for trapezoids that are unbounded from below or above.