

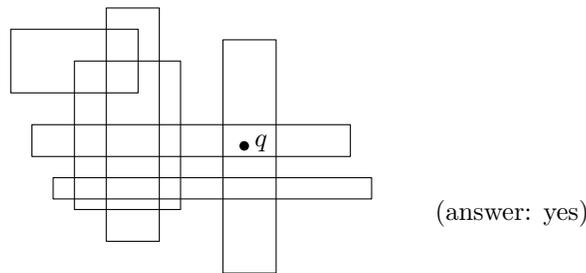
Homework 1 (due Sep 27 Friday (11am in class))

Instructions: You may work in groups of at most 2. Hand in one set of solutions per group. Acknowledge any discussions you have with other students and other sources you have consulted. Solutions must be written *in your own words*.

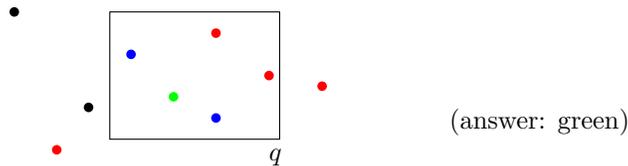
1. [10 pts] We want to store a set S of n (axis-aligned) rectangles in 2D so that for a given query point q , we can quickly decide whether there exists a rectangle of S containing q , and if true, return one such rectangle.

Show that there is a data structure with $O(n \log^c n)$ space and $O(\log^{c'} n)$ query time for this problem, for some constants c and c' .

[Hint: the “easiest” solution is to directly reduce the problem to orthogonal range searching in a constant dimension...]



2. [40 pts] We are given a set S of n points in 2D, where each point is assigned a color. We want to build a data structure so that for a given query (axis-aligned) rectangle q , we can quickly find a *conflict-free* color, i.e., a color that occurs just once inside q , or decide that such a color does not exist.



- (a) [20 pts] Show that in the case of dominance (2-sided) ranges, there is a solution with $O(n \log^c n)$ space and $O(\log^{c'} n)$ query time, for some constants c and c' .

[Hint: imagine drawing the locus of all points (x, y) for which a conflict-free color exists for the dominance range $(-\infty, x] \times (-\infty, y]$. Problem 1 may be useful...]

- (b) [20 pts] Show that the general (4-sided) case can be solved with $O(n \log^c n)$ space and sublinear query time, of the form $O(n^\alpha \log^{c'} n)$, for some constants $\alpha < 1$, and c and c' .

[Hint: form a grid where each row/column has n^α points. Consider two cases: 1. a conflict-free color appears in the row/column containing one of the 4 sides of the query rectangle, and 2. otherwise...]

3. [20 pts] We are given a set S of n points in 2D, where each point is assigned a color. We want to build a data structure so that for a given query (axis-aligned) rectangle q , we can quickly decide whether all points inside q have the same color. (You may assume that q contains at least one point.)

Describe the most efficient solution you can come up with, by modifying known methods from class. [Bonus points may be given if your solution has strictly better than $O(n \log n)$ space and better than logarithmic query time simultaneously.]