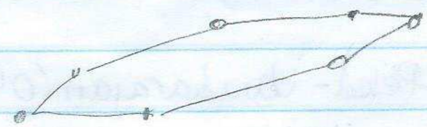
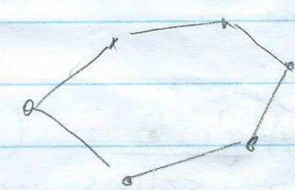
1. compute an ε-kernel S
2. return $(1+\varepsilon)$-approx of width of S ← time $O\left(\frac{1}{\varepsilon^{\frac{d-1}{2}}} \cdot \frac{1}{\varepsilon^{\frac{d-1}{2}}}\right) = O\left(\frac{1}{\varepsilon^{d}}\right)$

How to find ε-kernels?

<u>Def</u> P is <u>fat</u> iff width of $P \geq$ const $*$ diameter



not fat



fat

<u>Lemma1</u> $\exists$ affine transformation that makes P fat.

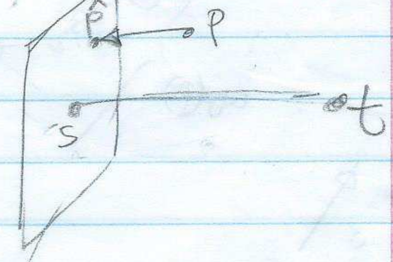
Pf2: (Barequet-Har-Peled '97)

transform$_d(P)$:

1. s = any pt of P

$O(n)$ time:

2. t = farthest pt from s
3. rotate & scale s.t. $s=(0,\dots,0)$, $t=(1,0,\dots,0)$
4. project each $p \in P$ to $\hat{p}$ along 1^{st} coord
5. transform$_{d-1}(\{\hat{p} : p \in P\})$



at the end, $P \subseteq [-1,1]^d$.

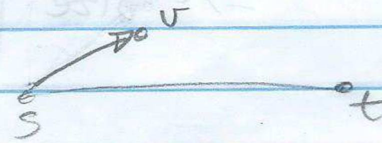
<u>Claim</u> $\forall$ unit vector v, $w_v(P) \geq \frac{1}{4d}$.

Pf: Let $v = (v_1, \dots, v_d)$.

Case 1. $|v_1| \geq \frac{1}{4^d}$,

$\Rightarrow \quad w_v(P) \geq |(t-s)\cdot v|$

$\geq \frac{1}{4^d}$.

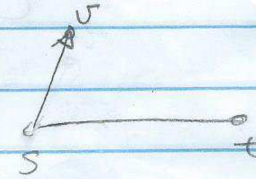


Case 2. $|v_1| < \frac{1}{4^d}$

By induction, $\exists\, p, q \in P$,

$(\hat{p}-\hat{q})\cdot\hat{v} \geq \frac{1}{4^{d-1}}\|\hat{v}\|$

$\geq \frac{1}{4^{d-1}}\left(1-\frac{1}{4^d}\right) \geq \frac{3}{4^d}$



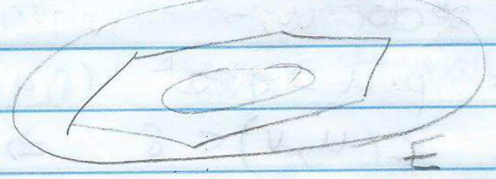
$(|p_1|, |q_1| \leq 1)$

$\Rightarrow \quad w_v(P) \geq (p-q)\cdot v = (\hat{p}-\hat{q})\cdot\hat{v} + (p_1-q_1)v_1$

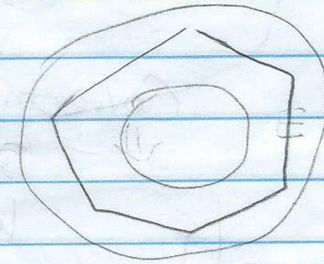
$\geq \frac{3}{4^d} - \frac{2}{4^d} = \frac{1}{4^d}$ □

Pf 1;
~~Rmk- alternative pf~~ by John ellipsoid E ('48)

$\frac{1}{d}E \subseteq CH(P) \subseteq E$



transform
E to ball
$\Longrightarrow$



Lemma 2 Let M be an affine transform
if $M(S)$ is ε-kernel of $M(P)$,
then S is ε-kernel of P (Pf: $(Mp)\cdot v = p\cdot M^T v$)

ε-Kernel Alg'm 1: idea - rounding pts

1. make P fat in $[-1,1]^d$
2. form uniform grid of side length ε
3. for each grid cell, put one pt in S.