CS 598TMC, Fall 2018
Timothy Chan

Homework 4 (due Dec 5 Wednesday (11am in class))

Instructions: You may work in groups of at most 2. Hand in one set of solutions per group. Acknowledge any discussions you have with other students and other sources you have consulted. Solutions must be written *in your own words.*

1. [*10 pts*] Consider a set of m points and a set of n (unweighted, axis-aligned) rectangles in the plane that have only 10 distinct aspect ratios (width-to-height ratios). Describe how to obtain a polynomial-time constant-factor approximation algorithm for the set cover problem in this case, using techniques from class.

2. [*10 pts*] Consider a weighted set H of n horizontal line segments and a set V of m vertical line segments. We want to find the minimum-weight subset of H that hits all line segments of V. Describe how to obtain a polynomial-time constant-factor approximation algorithm, using techniques from class.

   Hint: it suffices to bound the "shallow-cell complexity" (the number of combinatorial distinct vertical line segments that hit exactly k segments of H).

3. [*15 pts*] Recall the independent set problem for a set of n pseudo-disks in the plane. In class, we have given a constant-factor approximation algorithm using linear programming. In this problem, you will explore an alternative approach by the multiplicative weights update technique.

   Let T^* be an optimal solution. Assume we are given a value ε with $|T^*| \leq \frac{c}{\varepsilon} \leq 2|T^*|$ for some constant c. Let M be a sufficiently large power of 2. Consider the following algorithm:

```
initialize a multiset \widehat{S} ← S, where each element has multiplicity M
repeat {
    if there exists a point p whose depth in \widehat{S} is > ε|\widehat{S}| then
        for each pseudo-disk s covering p do
            halve the multiplicity of s in \widehat{S}
    if no such point p exists then
        return an independent set of size Ω(1) · |T*| by using some result from class
}
```

   Here, $|\widehat{S}|$ denotes the size of the multiset $\widehat{S}$, i.e., the sum of the multiplicities of all its elements. with multiplicities included.

   Show that for an appropriate choice of c, the above algorithm always terminates and runs in polynomial time.