CS 598TMC, Fall 2018
Timothy Chan

Homework 3 (due Nov 9 Friday (11am in class))

Instructions: You may work in groups of at most 2. Hand in one set of solutions per group. Acknowledge any discussions you have with other students and other sources you have consulted. Solutions must be written *in your own words*.

1. [*15 pts*] Given two sets P and Q of n points each in $\mathbb{R}^d$ for a constant dimension d, we want to compute a $(1+\varepsilon)$-approximation to the k-th smallest distance among the n^2 distances $\{\|p-q\| : p \in P,\ q \in Q\}$. Describe an $O((1/\varepsilon^d)n \log n)$-time algorithm, by using WSPD.

2. [*15 pts*] Given a set P of n points in $\mathbb{R}^d$ for a constant dimension d, a *k-hop* $(1+\varepsilon)$*-spanner* is a subgraph H of the complete graph over P such that for every $p, q \in P$, there exists a path from p to q in H with at most $k+1$ edges and total length at most $(1+\varepsilon)\|p-q\|$. (The spanner constructions from class with $O((1/\varepsilon^d)n)$ edges do not guarantee small hops.) Describe a construction of a 1-hop $(1+\varepsilon)$-spanner with $O((1/\varepsilon^d)n \log n)$ edges, by using shifted, balanced quadtrees.

   [Hint: by a lemma from class, find a quadtree cell B with $|P \cap B|, |P - B| \le \frac{2^d}{2^d+1}|P|$. Pick $O(1/\varepsilon^d)$ points and join them to all points in P. Recurse...]

3. [*15 pts*] Given a set P of n points in $\mathbb{R}^2$ and an integer k, we want to find the smallest set S^* of unit disks in $\mathbb{R}^2$ such that the number of points of P that are covered by S^* is at least k. Describe a PTAS for the problem, by using the shifted grid technique and dynamic programming.

4. [*10 pts*] Given a polygon P with n edges contained in $[0, U]^2$, we would like to find the smallest number of (possibly rotated, possibly overlapping) unit squares whose union contains P. Give a simple constant-factor approximation algorithm that runs in time polynomial in n and U.