Graph Theoretic Connectivity Control of Mobile Robot Networks

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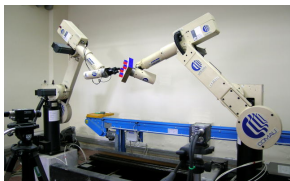April 20, 2015

Background

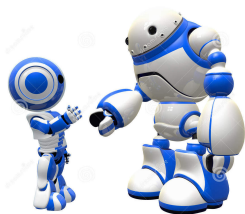Figure 1: Interaction of robots

Figure 2: Communication between robots

Research I am doing

- Task space control of robots
- Dynamics of the interaction of the robot networks
- Robustness of robot control

One issue comes out: Since the efficiency of robot network depends on the information exchange, i.e. communication, the robots must stay close to each other in order to be able to have a reliable communication.

Today's presentation

- Connectivity control of robot networks
- Applications of connectivity control

Connectivity in Mobile Robot Networks
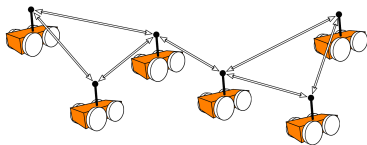
Figure 3: Network of robots

Consider n points robots in R^d.

- Single integrator model,

$$\dot{x}_i(t) = u_i(t), \qquad (1)$$

where $x_i(t) \in \mathrm{R}^d$ is the position of robot i, and $u_i(t) \in \mathrm{R}^d$ is the control input to robot i at time t.

- Double integrator models,

$$\dot{x}_i(t) = v_i(t),$$
$$\dot{v}_i(t) = u_i(t), \qquad (2)$$

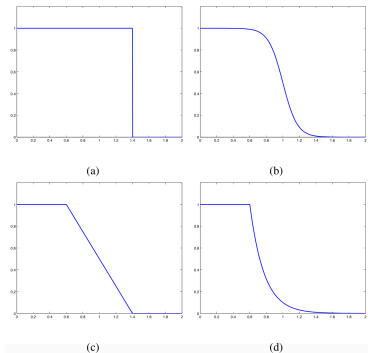where $v_i(t) \in \mathrm{R}^d$ is the velocity of robot i.

Figure 4: Different choices for the function f.

Denote (i, j) a communication link between robots i and j. Define the weight function of the link (i, j),

$$w : \mathrm{R}^d \times \mathrm{R}^d \to \mathrm{R}_+$$

such that

$$w_{ij}(t) = w(x_i(t), x_j(t)) = f(\|x_{ij}(t)\|_2), \tag{3}$$

for some $f : \mathrm{R}_+ \to \mathrm{R}_+$, where $x_{ij}(t) = x_i(t) - x_j(t)$. Here f is chosen to be a function of the inter robot distance $\|x_{ij}(t)\|_2$, such that

$$1 - \epsilon < f(\|x_{ij}(t)\|_2) \leq 1, if \|x_{ij}(t)\|_2 < \rho_1$$

and

$$0 < f(\|x_{ij}(t)\|_2) \leq \epsilon, \|x_{ij}(t)\|_2 > \rho_2$$

The weighted state-dependent graph

$$\mathbb{G} = (\mathbb{V}, \mathbb{W}),$$

where $\mathbb{V} = \{1, \ldots, n\}$ denotes the set of nodes indexed by the set of robots, and $\mathbb{W} : \mathbb{V} \times \mathbb{V} \times R_+ \to R_+$ denotes the set of edge weights, such that

$$w_{ij}(t) = \mathbb{W}(i, j, t)$$

for $i, j \in \mathbb{V}$ and with $w_{ij}(t)$ defined as $f(\|x_{ij}(t)\|_2)$.

Assumption

- $\mathbb{G}$ is symmetric weighted, i.e. $w_{ij}(t) = w_{ji}(t)$;
- $\mathbb{G}$ has no loop, i.e. $w_{ii}(t) = 0, \forall i \in \mathbb{V}$.

Definition (Graph connectivity)

An undirected graph $\mathbb{G}$ is connected if for every pair of nodes there exists a path starting at one node and ending at the other.

Definition (Adjacency Matrix)

Define the adjacency matrix $\mathcal{A}(t) \in \mathrm{R}_+^{n \times n}$ of the weighted graph $\mathbb{G}$ with entries

$$[\mathcal{A}(t)]_{ij} = w_{ij}(t). \tag{4}$$

- If the network has symmetric weights, $\mathcal{A}(t)$ is symmetric.
- If the weights satisfy $w_{ij}(t) \in {0, 1}$, the powers of $\mathcal{A}(t)$ are closely related to network connectivity.

Theorem 1 (Graph Connectivity)

The entry $[\mathcal{A}^k(t)]_{ij}$ of the matrix $\mathcal{A}^k(t)$ is the number of paths of length k from node i to node j in $\mathbb{G}$. Therefore, $\mathbb{G}$ is connected if and only if there exists an integer K such that all the entries of the matrix $\mathcal{C}_K(t) = \sum_{k=0}^{K} \mathcal{A}^k(t)$ are non-zero.

Another way to study the graph connectivity is using Laplacian matrix of the network $\mathbb{G}$.

Definition (Laplacian matrix)

$$\mathcal{L}(t) \triangleq \mathcal{D}(t) - \mathcal{A}(t) \tag{5}$$

where $\mathcal{D}(t) = \text{diag}(\sum_{j=1}^{n} w_{ij}(t))$ denotes the diagonal matrix of degrees of the network.

The network connectivity is closely related to the spectral properties of $\mathcal{L}(t)$, shown by the following theorem.

Theorem 2 (Graph Connectivity)

Let

$$0 = \lambda_1(\mathcal{L}(t)) \leq \lambda_2(\mathcal{L}(t)) \leq \cdots \leq \lambda_n(\mathcal{L}(t))$$

be the ordered eigenvalues of the Laplacian matrix $\mathcal{L}(t)$. Then $\lambda_2(\mathcal{L}(t)) > 0$ if and only if $\mathbb{G}$ is connected.

$\lambda_2(\mathcal{L}(t))$ is also a measure of the robustness of the network to link failures.

Definition (*k*-connectivity)

Let $\eta(\mathbb{G})$ be the minimum number of edges that if removed from $\mathbb{G}$ increase its number of connected components. Then, for any $k \leq \eta(\mathbb{G})$ the undirected graph $\mathbb{G}$ is called *k*-connected.

The edge connectivity $\eta(\mathbb{G})$ and algebraic connectivity $\lambda_2(\mathcal{L}(t))$ are related by the inequality

$$\lambda_2(\mathcal{L}(t)) \leq \eta(\mathbb{G})$$

If $\lambda_2(\mathcal{L}(t)) > k - 1$, then the network $\mathbb{G}$ is *k*-connected.
When $k = 1$, the *k*-connectivity reduces to the usual definition of connectivity.

Network problem foundation

Given an initially connected state-dependent network $\mathbb{G}$, design distributed controllers $u_i(t)_{i=1}^{n}$ for the robots, so that the closed loop system guarantees that $\mathbb{G}$ is *k*-connected for all time.

Optimimization-based Connectivity Control

Since $\lambda_2(\mathcal{L}(t))$ is a concave function of $\mathcal{L}(t)$, i.e.

$$\lambda_2(\mathcal{L}(t))z^{\mathrm{T}}z \leq z^{\mathrm{T}}\mathcal{L}(t)z$$

the 2nd largest eigenvalue of $\mathcal{L}(t)$

$$\lambda_2(\mathcal{L}(t)) = \inf_{z \in \mathbf{1}^{\perp}} \frac{z^{\mathrm{T}}\mathcal{L}(t)z}{z^{\mathrm{T}}z}, \tag{6}$$

maximization of $\lambda_2(\mathcal{L}(t))$ gives rise to convex optimization approaches to the connectivity control problem, i.e.

$$\max_{x \in \mathrm{R}^{dn}} \lambda_2(\mathcal{L}(x)) \tag{7}$$

where $x = [x_1 \ x_2 \dots x_n]^T \in \mathrm{R}^{dn}$ denotes vectors of all robot positions.

Centralized Connectivity Maximization

Lemma 1

Let $\mathcal{P} = [p_1 \ \ldots \ p_{n-1}] \in \mathrm{R}^{n \times (n-1)}$, be such that $p_i^T \mathbf{1} = 0$ for all $i = 1, \ldots, n-1$ and $p_i^T p_i = 0$ for all $i \neq j$. Then, $\lambda_2(\mathcal{L}) > 0$ if and only if $\mathcal{P}^T \mathcal{L} \mathcal{P} \succ 0$.

Proof.

Since $\mathcal{L} \succeq 0$ and $\mathcal{L}\mathbf{1} = \mathbf{0}$, the smallest eigenvalue $\lambda_1(\mathcal{L}) = 0$ and $\mathrm{rank}(\mathcal{L}) \leq n-1$. This implies that $\lambda_2(\mathcal{L}) > 0$ if and only if $w^T \mathcal{L} w > 0$ for all $w \in \mathbf{1}^\perp$.

Let $z \in \mathrm{R}^{n-1}$ and consider $z^T \mathcal{P}^T \mathcal{L} \mathcal{P} z = (\mathcal{P}z)^T \mathcal{L} \mathcal{P} z$. Let $w = \mathcal{P}z$. Since $\mathcal{P}$ is full rank, $w = \mathcal{P}z$ defines an injective mapping between R^{n-1} and R^n. Therefore, $w^T \mathcal{L} w > 0$ for all $w \in \mathrm{R}^n$ if and only if $z^T \mathcal{P}^T \mathcal{L} \mathcal{P} z > 0$ for all $z \in \mathrm{R}^{n-1}$. $\qquad\square$

This results in an equivalent convex formulation for the problem by

$$\max_{\mathcal{L}(t) \in \mathbf{S}^n} \gamma \tag{8}$$

$$\text{s.t.} \qquad \mathcal{P}^T \mathcal{L} \mathcal{P} \succ \gamma \mathcal{I}_{n-1}, \tag{9}$$

This can be solved for the optimal Laplacian matrix $\mathcal{L}^*$ from semidefinite programming.

Centralized Connectivity Maximization (Continued)

Introduce state-dependent $\mathbb{G}$ via the set of edge weights, along with a set of minimum distance constraints $\|x_{ij}\|_2 \geq \rho_1$.

The solution for a trajectory $x(t) \in \mathrm{R}^{dn}$ is achieved by an iteration algorithm that maximizes the algebraic connectivity at every step. The distances $\|x_{ij}\|_2^2$ are differentiated and then discretized by Euler's first order method:

$$2([x_{s+1}]_i - [x_{s+1}]_j)^{\mathrm{T}}([x_s]_i - [x_s]_j) = [\chi_{s+1}]_{ij} - [\chi_s]_{ij}$$

where $\chi \in \mathrm{R}_+^{n \times n}$ is a Euclidean distance matrix, s.t. $[\chi]_{ij} = \|x_{ij}\|_2^2$.

Differentiate and discretize the weights w_{ij}, gives

$$[w_{s+1}]_{ij} = [w_s]_{ij} + \frac{\partial f([\chi]_{ij})}{\partial [\chi]_{ij}}|_s([\chi_{s+1}]_{ij} - [\chi_s]_{ij})$$

Substituting these, and results the optimization problem:

$$\max_{x_{s+1} \in \mathrm{R}^{dn}} \gamma$$

$$\text{s.t.} \quad \mathcal{P}^T \mathcal{L}(x_{s+1})\mathcal{P} \succ \gamma \mathcal{I}_{n-1}, [\chi_{s+1}]_{ij} \geq \rho_1^2$$

$$2([x_{s+1}]_i - [x_{s+1}]_j)^{\mathrm{T}}([x_s]_i - [x_s]_j) = [\chi_{s+1}]_{ij} - [\chi_s]_{ij} \quad (10)$$

Distributed Connectivity Maximization

Observe that

$$\lambda_2(\tilde{\mathcal{L}})z_2^{\mathrm{T}}z_2 \leq z_2^{\mathrm{T}}\tilde{\mathcal{L}}z_2 \tag{11}$$

where $\tilde{\mathcal{L}} \neq \mathcal{L}$, and $z_2 \in \mathbf{1}^{\perp}$ is the unit eigenvector of $\mathcal{L}$ corresponding to $\lambda_2(\mathcal{L})$.
Since

$$z_2^{\mathrm{T}}\tilde{\mathcal{L}}z_2 = z_2^{\mathrm{T}}\mathcal{L}z_2 + z_2^{\mathrm{T}}(\tilde{\mathcal{L}} - \mathcal{L})z_2$$
$$= \lambda_2(\mathcal{L}) + \langle z_2 z_2^{\mathrm{T}}, (\tilde{\mathcal{L}} - \mathcal{L}) \rangle$$

then,

$$\lambda_2(\tilde{\mathcal{L}})z_2^{\mathrm{T}}z_2 \leq \lambda_2(\mathcal{L}) + \langle z_2 z_2^{\mathrm{T}}, (\tilde{\mathcal{L}} - \mathcal{L}) \rangle$$

Thus, $\mathcal{G} = z_2 z_2^{\mathrm{T}}$ is a supergradient for $\lambda_2(\mathcal{L})$, and the update rule for Laplacian $\mathcal{L}$ is

$$\mathcal{L}_{s+1}^{*} = \mathcal{L}_s^{*} + \alpha_s \mathcal{G}_s \tag{12}$$

where α_s is the step size.

Distributed Connectivity Maximization (Continued)

Robot i is captured by the following optimization problem:

$$\min_{x_i \in \mathrm{R}^d} \|[\mathcal{L}(x)]_i - [\mathcal{L}_s^*]_i\|_2^2 \tag{13}$$

where $\mathcal{L}(x)$ is the i-th row of the Laplacian as a function of robots' position, and $[\mathcal{L}_s^*]_i$ is the i-th row of the optimal Laplacian computed by robot i at the s-th step of the supergradient.

The controller for robot i

$$u_i(t) = -\sum_{j \in \mathbb{N}_i} \nabla_{x_i} V_{ij}(t), \tag{14}$$

where

$$V_{ij}(t) = \begin{cases} (\|x_{ij}\|_2^2 - [\mathcal{L}_s^*]_{ij}^{-1})^2, & \text{if } \|x_{ij}\|_2 \le \rho_2 \\ (\rho_2 - [\mathcal{L}_s^*]_{ij}^{-1})^2, & \text{if } \|x_{ij}\|_2 > \rho_2 \end{cases} \tag{15}$$

Here, $[\mathcal{L}_s^*]_{ij}^{-1}$ is the desired distance between robots i and j.

Under certain boundedness conditions on the tracking error associated with the optimal Laplacian $\mathcal{L}_s^*$, the supergradient algorithm converges.

Continuous feedback connectivity control

Recall Proposition 1: Let $\mathcal{P} = [p_1 \ \ldots \ p_{n-1}] \in \mathrm{R}^{n \times (n-1)}$, be such that $p_i^T \mathbf{1} = 0$ for all $i = 1, \ldots, n-1$ and $p_i^T p_i = 0$ for all $i \neq j$. Then, $\lambda_2(\mathcal{L}(t)) > 0$ if and only if $\mathcal{P}^T \mathcal{L} \mathcal{P} \succ 0$.

Theorem 1

Define the potential function $\phi : \mathrm{R}^{dn} \to \mathrm{R}_+$ as

$$\phi(x) = \log \det(\mathcal{P}^T \mathcal{L}(x)\mathcal{P})^{-1}, \tag{16}$$

The closed loop system with $u = -\nabla_x \phi(x)$ guarantees that $\mathbb{G}$ is connected for all time.

Proof Scheme

The proof of this result relies on positive invariance of the level sets $\phi^{-1}([0, c]) = \{x \in \mathrm{R}^{dn} | \phi(x) \leq c\}$ of ϕ, which is due to the fact that $\dot{\phi}(x) = -\|\nabla_x \phi(x)\|_2^2 \leq 0$.

The potential ϕ is a convex function of the Laplacian, but the dependence of the Laplacian on the state via the edge weights makes ϕ a non-convex function of the $x \in \mathrm{R}^{dn}$. Thus, the proposed control scheme ensures only local maximization of $\lambda_2(\mathcal{L}(x))$.

Proposition 1

The controller $u = -\nabla_x \phi(x)$ is given by

$$u = \frac{1}{\det \mathcal{M}(x)} \begin{bmatrix} \text{tr}[\mathcal{M}^{-1}(x)\frac{\partial}{\partial x_1}\mathcal{M}(x)] \\ \vdots \\ \text{tr}[\mathcal{M}^{-1}(x)\frac{\partial}{\partial x_n}\mathcal{M}(x)] \end{bmatrix} \tag{17}$$

where $\mathcal{M}(x) = \mathcal{P}^T \mathcal{L}(x) \mathcal{P}$.

The proof process is skipped due to straightforward but heavy math.

Hybrid feedback connectivity control

The approach discussed is centralized since every robot requires knowledge of the whole network structure captured by $\mathcal{L}(x)$ to compute its controller. To regulate the structure of the proximity-based network $\mathbb{G}$ in a distributed fashion, introduce a binary control signal $\sigma \in \{0, 1\}^{n \times n}$, such that

$$[\sigma]_{ij} = \begin{cases} 1, & \text{to activate the link } (i,j) \in \vec{\mathbb{E}} \\ 0, & \text{to deactivate the link } (i,j) \in \vec{\mathbb{E}} \end{cases}$$

This gives rise to the weighted graph $\mathbb{G}_\sigma = (\mathbb{V}, \mathbb{W}_\sigma)$ where $\mathbb{W}_\sigma : \mathbb{V} \times \mathbb{V} \times \mathrm{R}_+ \to \mathrm{R}_+$ is the set of edge weights such that

$$w_{ij}^\sigma(t) = \mathbb{W}_\sigma(i, j, t)$$

with $w_{ij}^\sigma = w_{ij}[\sigma]_{ij}$.
The control signal σ is a discrete switch on the links of the network $\mathbb{G}$, but only affects existing links for which $w_{ij} > 0$.
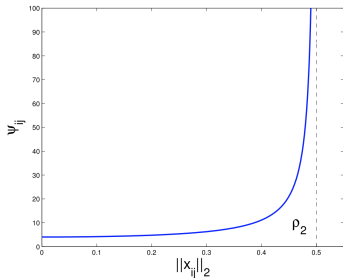
Figure 5: The artificial function $\psi_{ij}(\|x_{ij}\|_2)$ is symmetric w.r.t x_i and x_j, and when bounded, it guarantees edge preservation for $\|x_{ij}\|_2 \to \rho_2$.

The edge and neighbor sets associated with the graph $\mathbb{G}_\sigma$ are defined by $\vec{\mathbb{E}}_\sigma = \{(i,j)|w_{ij}^\sigma > 0\}$ and $\mathbb{N}_i^\sigma = \{j \in \mathbb{V}|(i,j) \in \vec{\mathbb{E}}_\sigma\}$, respectively. The hybrid model for the mobile network $\mathbb{G}$ consisting of single integrator robots and controllers given by ([5] [6])

$$u_i^\sigma = -\sum_{j \in \mathbb{N}_i^\sigma} \nabla_{x_i} \psi_{ij}. \qquad (18)$$

where $\psi_{ij} : \mathrm{R}_+ \to \mathrm{R}_+$ are artificial potential functions defined on the links of the network. In connectivity control, take the form:

$$\psi_{ij} = \frac{1}{\rho_2^2 - \|x_{ij}\|_2^2}, \qquad (19)$$

to ensure link preservation between adjacent robots.
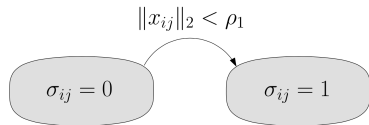
Maintaining Communication Links

$$\|x_{ij}\|_2 < \rho_1$$

$\sigma_{ij} = 0$ $\sigma_{ij} = 1$

Figure 6: Hysteresis protocol for adding internet energy functions to the total energy function only when agents get within a distance ρ_1 of each other, rather than when they first encounter each other at a distance ρ_2.

Introduce a hysteresis into the system through the signal σ given by the state machine in Fig.4. The signal $[\sigma]_{ij}$ is such that the total energy is affected by an edge (i,j) that was previously not contributing to the total energy only when $\|x_{ij}\|_2 < \rho_1$, where $0 < \rho_1 < \rho_2$ is the predefined switching threshold that regulates how fast inter-robot information is included in the control law.

Once the edge is allowed to contribute to the total energy, it keeps doing so for all subsequent times.

$$[\sigma]_{ij}(t^+) = \begin{cases} 0, & \text{if } [\sigma]_{ij}(t^-) = 0 \text{ and } \|x_{ij}\|_2 \geq \rho_1 \\ 1, & \text{otherwise} \end{cases}$$
(20)

where $[\sigma]_{ij}(t^+)$ and $[\sigma]_{ij}(t^-)$ denotes the value of $[\sigma]_{ij}$ before and after the state transition in Fig.4.

Proposition 4

Consider the closed loop system (1)-(18). Then, all links in $\mathbb{G}_\sigma$ are maintained.

Proof.

Let

$$\psi_\sigma = \frac{1}{2} \sum_{i=1}^n \psi_i^\sigma,$$

where $\psi_i^\sigma = \sum_{j \in \mathbb{N}_i^\sigma} \psi_{ij}$, denote the total energy of the system.

$$\frac{1}{2} \sum_{i=1}^n \dot{\psi}_i^\sigma = \frac{1}{2} \sum_{i=1}^n \sum_{j \in \mathbb{N}_i^\sigma} \dot{x}_{ij}^{\mathrm{T}} \nabla_{x_{ij}} \psi_{ij}$$

$$= \frac{1}{2} \sum_{i=1}^n \sum_{j \in \mathbb{N}_i^\sigma} (\dot{x}_i^{\mathrm{T}} \nabla_{x_{ij}} \psi_{ij} - \dot{x}_j^{\mathrm{T}} \nabla_{x_{ij}} \psi_{ij})$$

(21)

$$= \frac{1}{2} \sum_{i=1}^n \sum_{j \in \mathbb{N}_i^\sigma} (\dot{x}_i^{\mathrm{T}} \nabla_{x_i} \psi_{ij} + \dot{x}_j^{\mathrm{T}} \nabla_{x_j} \psi_{ij})$$

$$= \sum_{i=1}^n \sum_{j \in \mathbb{N}_i^\sigma} \dot{x}_i^{\mathrm{T}} \nabla_{x_i} \psi_{ij} = \sum_{i=1}^n \sum_{j \in \mathbb{N}_i^\sigma} \dot{x}_i^{\mathrm{T}} \nabla_{x_i} \psi_i^\sigma$$

by the symmetry of ψ_{ij}. Therefore,

$$\dot{\psi}_\sigma = - \sum_{i=1}^n \|\nabla_{x_i} \psi_i^\sigma\|_2^2 \le 0.$$

This implies that the level sets $\psi_\sigma^{-1}([0, c])$ of ψ_σ are positively invariant, and hence, no line are lost. $\qquad\square$
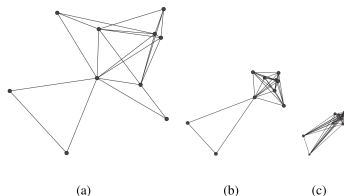
Connectivity Preserving Rendezvous

Figure 7: Execution of the rendezvous control strategy until the graph is a complete graph

Robots are required to meet at a common, not a priori specified location without relying on global positioning, i.e. robot i, at position x_i, has access to $x_j - x_i$ if i and j are neighbors. One control strategy is given in [6],

$$\dot{x}_i = \sum_{j \in \mathbb{N}_i^\sigma} \frac{2\rho_2^2}{(\rho_2^2 - \|x_{ij}\|_2^2)^2}(x_j - x_i), \quad (22)$$

As long as $\mathbb{G}$ is connected for all times. This ensures that no edges are lost, and all agents asymptotically approach the same location.

- Rendezvous control law serves a cohesion purpose.
- Some collision-avoidance controller may need to be added.

Connectivity Preserving Formation Control

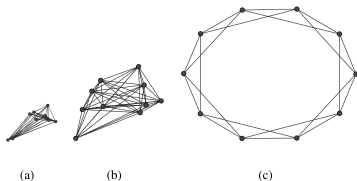(a)   (b)   (c)

Figure 8: Illustration of how the complete
graph is changed to the desired formation
using only local information.

Drive the robots to a desired target
configuration.

Assume the target configuration can
be encoded through $\zeta_1, \ldots, \zeta_n \in \mathrm{R}^d$,
where ζ_i is the location that agent i
should go. The formation control
objective is to achieve

$$x_i = \zeta_i + \tau, \ \forall i = 1, \ldots, n \qquad (23)$$

for some constant $\tau \in \mathrm{R}^d$, where τ is
the constant offset from the target
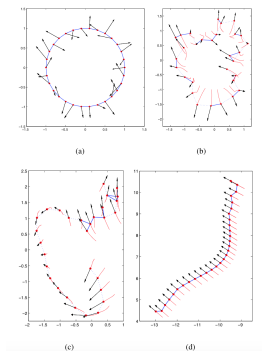configuration.

Connectivity Preserving Flocking

Figure 9: Connectivity preserving flocking for a sparse initial configuration where connectivity can not be trivially maintained. The network remains connected while all robot velocities are asymptotically aligned. Dotted lines indicate communication links that are candidates for deletion.

Reynolds model of flocking:

- Alignment: Steer towards the average heading of local flock mates.

- Seperation: Steer to avoid crowding of local flock mates.

- Cohesion: Steer towards the average position of local flock mates.

Flocking requires information from nearest neighbor flock mates only. Superposition of the three rules results in all robots moving as a flock while avoiding collision.

Connectivity Preserving Flocking (continued)

Double integrator dynamics of the robots team

$$\dot{x}_i = v_i \tag{24}$$

$$\dot{v}_i = -\sum_{j \in \mathbb{N}_i} (v_i - v_j) - \sum_{j \in \mathbb{N}_i} \bar{\psi}_{ij} \tag{25}$$

The connectivity control framework and the artificial potentials

$$\bar{\psi}_{ij} = \begin{cases} \frac{1}{\|x_{ij}\|_2^2 + P_1(\|x_{ij}\|_2)}, & \|x_{ij}\|_2 \in (0, \rho_0] \\ 0, & \|x_{ij}\|_2 \in (\rho_0, \rho_1) \\ \frac{1}{\rho_2^2 - \|x_{ij}\|_2^2 + P_2(\|x_{ij}\|_2)}, & \|x_{ij}\|_2 \in [\rho_1, \rho_2) \end{cases} \tag{26}$$

with $0 < \rho_0 < \rho_1 < \rho_2$ and $P_k(\|x_{ij}\|_2) \triangleq a_k\|x_{ij}\|_2^2 + b_k\|x_{ij}\|_2 + c_k$ for $k = 1, 2$ such that $\psi_{ij} \in \mathrm{C}^2$ in $(0, \rho_2)$.

This control architecture guarantee the flocking behavior of the team, while preserving connectivity of the network.

Conclusion

- Basic knowledge of network connectivity associated with spectral graph theory
- Control theoretic methods for connectivity preservation (optimization-based connectivity control, continuous feedback connectivity control, and hybrid feedback connectivity control)
- Applications of connectivity control (connectivity preserving rendezvous, connectivity preserving formation control, and connectivity preserving flocking)

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