Combinatorial Auctions (Spectrum Auction)

Tuesday, November 17, 2020 1:59 PM

☆ Spectrum (Frequencies to broadcast)
Every item here is (freq range, geographic location) } Heterogeneous items.

☆ Landing Slots on airports

↓

☆ Combinatorial Auction.

n: agents (N) every agent i has value $V_i(S)$ for any subset $S \subseteq M$.

m: different items (M)

Issues:

→ $\boxed{\text{Representation of each } V_i \text{ req. } 2^m \text{ many numbers.}}$

→ Substitutes vs Complements.

$A, B \subseteq M$

$V_i(A \cup B) \leq V_i(A) + V_i(B)$ $\qquad$ $V_i(A \cup B) \geq V_i(A) + V_i(B)$

↑ Easy case to handle $\qquad$ ↑ Hard.

☆ Sell every item separately through "Single Item Auction (SIA)".

m auctions.

$1 \; . \; . \; . \; . \; m$

$V_{i1} \; . \; . \; . \; . \; V_{im}$

Mistake 1 : Sequential SIA.

2-items similar

3-bidder

$\underset{\text{win 1}}{1000} > \underset{P_1 = 800}{\overset{\text{winner 2}}{800}} > 300$ $\qquad$ $P_2 = 300$

if the first bidder skips the first auction, (or under bids)

then she can win the second at \$300.

then she can win the second at $300.

★ March 2000 Switzerland.
3- seq. SIA.

Blocks : 28 MHz, 28 MHz, 56 MHz

Rev : 134 M. 121 M, 55M !

★ Mistake 2 : Sealed bid simultaneous SIA. (Vickery auction)

Bidder's options are : - participate in only one of them
(if she wants only one item) - participate in more than one but conservatively.

1990 Newzealand

Block : several similar blocks worth 250M.

Auction Rev : $\boxed{36M\ !}$

Highest bid $ 100,000 $ 7M

second highest bid $ 6 $ 5.

↳ Moved to sealed bid First price auction.

(FCC)
★ Simultaneous Ascending Auction (SAA)
Multi-Round Auction (SMR)

- Each item sold through an English Auction
- Happens in Rounds, starting with very low (reserve) price
- Each round has a fixed time when agents can place bid. They can see the winning bid from the previous round.

previous round.

$v_1=10$ $v_2=8$ $v_3=5$

2-item

$P_1 = P_2 = P_3 = 0$

Rounds	b_1	b_2	b_3
1.	(0.1, 0.1)	(1, 1)	(2, 2)
2.	(3, 0.1)	(1, 3)	(2, 2)
3.	(3, 0.1)	(1, 3)	(4, 2)
4.	(3, 4)	(1, 3)	(4, 2)
5.	(3, 4)	(1, 5)	(4, 2)
6.	($\boxed{5}$, 4)	(1, $\boxed{5}$)	(4, 2)

- s.w. maximizing
- VCG payment.
- similar price for similar item.

★ Pros :
→ Removes coordination issues for the bidders.
→ Need not know the valuations upfront.
→ Work well :
- similar items go for similar price
- No reselling, or reselling at similar price.
- Price discovery : winner + price at mid-auction are correlated
  w/ " " at the end.
- Exceeds the projected Revenue.

★ Cons :
→ Demand Reduction. (with both substitutes & complements)

$V_1(A) = V_1(B) = 10$ $V_1(AB) = 20$ ←

$V_2(A) = V_2(B) = V_2(A,B) = 8$

VCG : Give both A, B to 1 s.w. = 20

payment$_1$ = 8 Rev = 8

will get A, B

VCG : payment$_1$ = 0

SAA (truthful bidding) : 1 will get A, B pay 16   Rev = 16

" (non-truthful) : 1 bids only on A → 1 gets A pays little
2 bids " " B   2 gets B pays little

Rev = ε (very small).

→ Exposure Problem (complements)

$V_1(A) = V_1(B) = 0 \qquad V_1(A, B) = \underline{100}$

$V_2(A) = V_2(B) = V_2(AB) = 75$

SAA (truthful bidding) : Either 1 will win {A, B} pay 150! $(75+75)$
or 1 drops out at prices (50, 50)
then 2 wins both {A, B} & pays 100!
net utility < 0 in both cases

★ Improvement in SAA to eliminate exposure problem.
Allow package (combinatorial) bidding.

① First run SAA for single items.
Then a round of package bidding.

② Allow package bidding for fixed packages.

ABCD   EFGH

AB EF

③ Allowed fixed # packages with upper bound on sizes at most 12 items.

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Since 2014 :

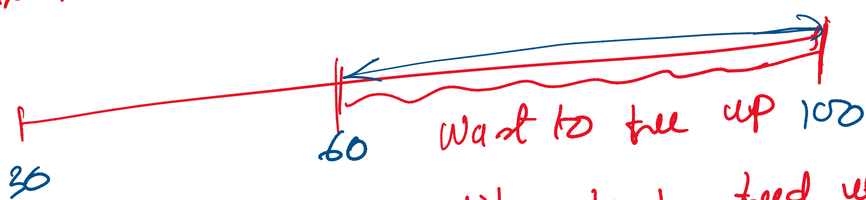
step-1 (to buy spectrum) : Reverse auction. to free up spectrum

step-2 : Repacking the remaining channels in a fixed range

Algorithmic question.

step-3 : Forward auction to sell the freed up spectrum.

Upfront Fix the frequency window to be freed.



Fix quantity to be freed up.

↓

Whatever remains should "fit" in range

say 30MHz - 60MHz .

Step-1 :

N : set of bidders willing to sell

v_i : private value of agent i for it's channel

b_i : bid at which it is willing to sell.

r_i : range it owns.

Direct (revealation) Mechanism :

Free = N , ToPack = ϕ

Repacking que.

while $\exists i \in$ Free s.t. ToPack $\cup \{i\}$ can be packed in 30-60 MHz range

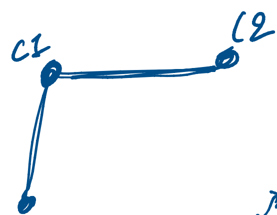
- move one such i from Free to ToPack.

... one to choose?

Which one to choose?
The least bid agent or
least bid / unit spectrum or
Most desirable spectrum range

Repacking: packing problem + coloring (NP-hard problems)

Two TV channels in the same geographic location should not "overlap".



colors: frequency bands.