

# CS 598RM: Algorithmic Game Theory, Fall 2019

## HW 0

### Instructions:

1. The purpose of this homework is to reacquaint you with topics, ideas and tools that will be needed in this course.
  2. This homework is **NOT** for submission and will **NOT** be graded.
- 

1. Let  $a_1, a_2, \dots, a_n$  be fixed real numbers and  $X$  be a random variable that takes value  $a_i$  with some probability  $p_i$ . Define the set of probability distributions that maximize  $E[X]$ .
2. Consider throwing  $n$  balls into  $n$  bins where each ball is thrown independently and uniformly at random into a bin.
  - (a) What is the probability that a given bin (say the first bin) is empty?
  - (b) What is the probability that it contains exactly  $k$  balls?
  - (c) What is the expected number of bins that are empty?
3. Consider the following linear program:

$$\begin{aligned} \max \quad & c^T x \\ \text{s.t.} \quad & Ax \leq b \\ & x \geq 0 \end{aligned}$$

- (a) Write the dual linear program of the above LP.
  - (b) Write the corresponding complementary slackness conditions.
  - (c) Using the complementary slackness conditions, derive the strong duality theorem.  
(If  $x^*$  is an optimal solution to the primal LP and  $y^*$  is an optimal solution to the dual LP, then  $c^T x^* = b^T y^*$ .)
4. A *wheel* of size  $k$  consists of a cycle on  $k$  vertices along with an additional vertex connected to every vertex in the cycle. As an example, you can see a wheel of size 8 in the figure below. The WHEEL problem is the following: Given an undirected graph  $G = (V, E)$  and an integer  $k$ , does  $G$  contain a wheel of size  $k$  as a subgraph? Prove that WHEEL is NP-Complete.

