CS 598RM: Algorithmic Game Theory, Fall 2018
Practice Exam

Instructions:

1. We advise you to read all the instructions and problems carefully before start writing the solutions.
2. There are four problems in total, each of 25 points. That is 100 points in total.
3. First problem is compulsory, while you are asked to do any 3 out of the next four.
4. Even if you are not able to solve a problem completely, do submit whatever you have. Partial proofs, high-level ideas, examples, and so on.
5. Be precise and succinct in your argument.

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1. Answer the following (each is of 5 points)
   - Agents 1 and 2 are bargaining over how to split a dollar. Each agent simultaneously demands share he would like to have, s_1 and s_2, where $0 \leq s_1, s_2 \leq 1$. If $s_1 + s_2 \leq 1$, then the agents receive the shares they named; if $s_1 + s_2 > 1$, then both agents receive zero. What is the set of pure strategy equilibria of this game?
   - A two player game represented by matrices (A, B) is called constant sum if $A(i,j) + B(i,j) = c$, $\forall i, j$ where $c \in R$. Is the following statement *True* or *False*: The set of Nash equilibria of this game is a convex set.
   - c In a stable roommate problem there is a set of m dorm rooms each of which can accommodate exactly two students, and a set n of students where $m > n/2$. Each student has a total ordering on all the other students (non-bipartite), and prefers to have a roommate then to live alone. Given an assignment of students to rooms, a pair of students (s_1, s_2) is unstable if they both prefer each other more than their current assignment. An assignment is called stable if there is no unstable pair. Construct an example that has no stable assignment.

     [Hint: Think of a three node graph.]
   - Consider a single item auction where highest bidder wins but pays third highest bid. Show that this auction is not truthful.
   - Compute the virtual valuation function for the uniform distribution on $[0, a]$ with $a > 0$.

Do any three out of the following four problems.

2. Consider a load balancing game with n jobs and m machines. Each job is a player who chooses a machine to run on, and trying to minimize its *completion time*. Job j has size p_j,

and any jobs can choose any of the m machines. Let $r_i(x)$ be the time needed by machine i to process the total load (sum of sizes of assigned jobs) is x. Assume $r_i(x) = x$ for all machines. A machine releases a job only after finishing all of its jobs, i.e., if the set of jobs that choose machine j is $S \subseteq \{1, \dots, n\}$, then completion time of job $j \in S$ is $\sum_{j \in S} p_j$.

- Is this a potential game?
- Suppose the social welfare is given by the maximum completion time. Show that the Price of Anarchy is upper-bounded by 2.

3. Recall the knapsack auction where each bidder i has a publicly known size w_i and a private valuation v_i. Consider a variant of a knapsack auction in which we have two knapsacks, with known capacities W_1 and W_2. Feasible sets of this single-parameter setting now correspond to subset S of bidders that can be partitioned into sets S_1 and S_2 satisfying $\sum_{i \in S_j} \leq W_j$ for $j = 1, 2$. We assume that $w_i \leq \min\{W_1, W_2\}, \ \forall i$.

   Consider the allocation rule that first uses the single-knapsack greedy allocation rule (sort jobs in decreasing order of $\frac{b_i}{w_i}$ and allocate until the knapsack is full, where b_i is the bid of agent i) to pack the first knapsack, and then uses it again on the remaining bidders to pack the second knapsack. Does this algorithm define a monotone allocation rule? Give either a proof of this fact or an explicit counter example.

4. Reverse (procurement) auction: Consider an undirected two-edge connected network, where each link (edge) is owned by a different agent. Auctioneer wants to buy a path from node s to node t. The cost of link e to its agent is c_e. Show that VCG mechanism for this problem can be computed in polynomial time, i.e., deciding which links to buy and what to *pay* to the agents of the bought links.

   Note that the social-welfare maximization is equivalent to social-cost minimization.

5. Consider the variant of stable matching problem, where the preference list can be incomplete, i.e., a woman (or a man) can exclude some men (or women) whom they does not want to be matched with.

   - Extend the definition of stable matching for this case.
   - Show that all stable matching are of the same size.
   - Extend the deferred acceptance algorithm (*proposal* algorithm) for this case.