Detection and Template Matching
A change in thinking

• So far we did feature extraction
  • “Unsupervised learning”

• Now we’ll do detection and classification
  • “Supervised learning”
Today’s lecture

• Detection of elements of interest

• Template matching / matched filters
  • Obtaining invariance/robustness
  • Efficient matching
  • Usage cases

• Probabilistic interpretation
Detection

- Find the presence of a predefined signal
  - Face/pedestrian/car detection
  - Speech, heartbeat, gunshot detection
  - Earthquakes, quasars, aliens, ...

- Many ways to go about it
  - We’ll start from the easy part
A simple example

- Can we detect a known face in a collection?
Vectorize the data (and scale to unit norm)

\[ \mathbf{x}^T = \text{vec}(\begin{bmatrix} \text{image} \end{bmatrix})^T = \]

\[ \mathbf{Y} = \text{vec}(\begin{bmatrix} \text{images} \end{bmatrix}) = \]
And get their dot product

\[ x^\top \cdot Y = \]

![Graph showing the dot product of x and Y]
Dot product for matching

• For two unit-length vectors $x, y$
  \[ x^\top \cdot y = 1 - \varepsilon \]

• When $x, y$ are increasingly similar, $\varepsilon$ tends to zero

• If the dot product is maximized we have a close match
Interpreting the dot product
And it works ok for noisy inputs too
Template matching

- Our query is a template
  - Use dot products to compare it with inputs

- Simple and very fast detector/classifier
  - But it won’t get you too far!

- Let’s signal-ize it
A tougher case

• “Heartbeat” data

• How do we detect the beats?
  • They can be all over the place
  • How do we do the dot product?
Brute force approach

- Do all possible dot products

\[ d(t) = \left[ x(1) \ldots x(N) \right] \cdot \left[ \begin{array}{c} y(t+1) \\ \vdots \\ y(t+N) \end{array} \right] \]
“Sliding” dot product

\[ d(t) = \begin{bmatrix} x(1) & \cdots & x(N) \end{bmatrix} \cdot \begin{bmatrix} y(t+1) \\ \vdots \\ y(t+N) \end{bmatrix} \]
A closer look at this ...

- Writing as a summation it becomes a convolution
- But with the template time-reversed

\[ d(t) = \left[ \begin{array}{c} x(1) \\ \vdots \\ x(N) \end{array} \right] \cdot \left[ \begin{array}{c} y(t + 1) \\ \vdots \\ y(t + N) \end{array} \right] = \sum_{i} x(i)y(t + i) = \hat{x} \ast y \]

\[ \hat{x}(t) = x(-t) \]
Matched filter

- This is known as a matched filter

\[ d(t) = \sum_{k} x(k) y(t - k) \]

- Why filter? Because we do a convolution
- Also known as cross-correlation
- Very common tool in communications, sonar, radar, etc ...
Matched filters in sonar/radar

- Radar/sonar
- Transmitted signal
- Received signal
- Object
- Reflection
- Template
- Input
- Matched filter output
Some post-processing

- The matched filter output is a little messy
- We mostly care about rough peaks, and its energy
- So we rectify and lowpass filter
And thankfully it works well with noise.
Simple communications example

• You send 0/1’s over the air
  • But you pick up some noise
• E.g.:

  How do you recover the original data?
Detecting the 1’s

- Make a template for them:
  - \( \mathbf{x} = [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1] \)
  - This will detect a series of “1” symbols

- Convolve template with the input
  - And measure when the result is above threshold
    - E.g. greater than 0.5
  - Where it is greater than 0.5 we have a series of “1”'s
Result

![Diagram showing sent, received, and recovered data with Convolution and Thresholding methods.](image-url)
An interesting connection

- What is this filter?
  - $x = [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1]$
  - Let’s look at its magnitude response:
An interesting connection

• What is this filter?
  • \( x = [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1] \)

• It’s a lowpass filter
  • We effectively remove the high frequencies (i.e. the noise)

• Two ways to achieve the same result
  • Template to match our signal
  • Or a denoising filter
Computational considerations

• The sliding dot product is slow
  • $N$ multiplications/additions per input sample
    • No fun with large templates

• Fortunately we can use the FFT!
  • Remember, convolutions can be performed efficiently with the FFT
FFT-based matched filtering

• Convolution in the time domain maps to element-wise multiplication in the frequency domain

\[ d = x \ast y \Rightarrow F \cdot d = (F \cdot x) \circ (F \cdot y) \]

• But we have three sizes here: template, input, output
  • template is length \( N \), input is \( M \), output is \( M+N-1 \)
  • What do we use?
Approach 1

- Zero pad everything to the largest size \((M+N-1)\)
  - Ensures that we have enough space to store the output
  - Doesn’t change the result since we convolve with more zeros
    - Slightly redundant though

- Example: \(N = 1,000, \ M = 10,000,000\)
  - Time domain takes about 10 GFLOPs
  - FFT convolution takes about 200 MFLOPs
Approach 2

- Take small snippets of the input, convolve them with the template and add them together (convolution is linear!)

- How small?
  - Same size as template \( N \) to avoid excessive zero padding
  - But, the output will be \( 2N-1 \), so we need to zero pad as much

- Example speedup, \( N = 1,000, M = 10,000,000 \):
  - Time: 10 GFLOPS
  - FFT: 200 MFLOPs
  - "Fast convolution": 60 MFLOPs
Matched filtering for event detection

- Let’s try to detect gunshot sounds in a video
Matched filter result

- Strong peaks where gunshots took place
- Processing: Time domain: 1.8 sec, Fast Conv: 0.06 sec
Going even faster

- The template and each time frame is a vector

\[
\begin{align*}
t_{\tau} * s_{\tau} &= F^H \cdot \left( F \cdot t_{\tau} \odot F \cdot s_{\tau} \right) = F^H \cdot (f_t \circ f_s) \\
\left\| t_{\tau} * s_{\tau} \right\|^2 &= (F^H \cdot (f_t \circ f_s))^H \cdot F^H \cdot (f_t \circ f_s) = (f_t \circ f_s)^H \cdot (f_t \circ f_s) = \left\| f_t \circ f_s \right\|^2
\end{align*}
\]
Generalizing to two dimensions

- In 1D the template was time-reversed
  - In 2D we flip both dimensions
    \[ d(i, j) = \sum_k \sum_l x(k, l) y(i + k, j + l) \]
- We now slide across both dimensions
  - E.g. left-to-right for all vertical offsets
  - FFT speedup still holds
    - But we now use 2D FFTs
    - Same caveats as before apply here as well (zero padding, etc.)
Digit recognition example

- Given the following input

\[
\begin{array}{|c|}
\hline
\text{Input} \\
\hline
3.141592654 \\
\hline
\end{array}
\]

- Find the digits by analyzing the image
Collect templates

• Create a set of templates for all digits

\[ x_1 = \begin{array}{c} 1 \end{array} \quad x_2 = \begin{array}{c} 2 \end{array} \quad \ldots \]

• Filter flipped versions with input image
  • Each template will peak at digit’s position

• Live demo!
A new problem

- What if we have a change in size?

```
Input
```

```
3.141592654
```
Broadening the search

- Resize the templates to more sizes and run template matching again
  - How much resizing?
- This is $N$ times more computation
  - When considering $N$ scales
- Live demo

\[
\begin{align*}
  x^L_1 &= 1 & x^L_2 &= 2 & \cdots \\
  x^M_1 &= 1 & x^M_2 &= 2 & \cdots \\
  x^S_1 &= 1 & x^S_2 &= 2 & \cdots
\end{align*}
\]
Image pyramids

- Scaling the templates is not too efficient
- Instead resample the input at multiple scales, not the template
Rotation?

• Same as before
  • Brute force search over all potential rotations

• One more multiplicative factor in computation

• Live demo!
Some relief

- Rotation and scale and ..., that’s a lot of searching that needs to be done!
- There are some tricks we can pull
  - e.g. radial filters for rotation
- Better features
  - Choose, e.g., a rotation invariant feature
- Lots and lots of papers ...
  - But some searching will be there
For example

- Treat significant pixels as points
  - Get their cross-distances
    - And normalize them

- The distances histogram is rotation- and scale- invariant
  - Do matching on that instead
One more problem ...

- What happened to the normalization?!  
  - Remember that vectors were unit-length?  
  - We have not enforced that!

- Here’s a potential problem case

![Image of the number 15]
Problems with scaling

- Regular cross-correlation is:

\[ d(t) = \sum_{k} x(k) y(t - k) \]

- Output is highly dependent on the local magnitude of the input
In fact, I’ve been cheating all this time

- I used an inverted colormap (white:0, black:1)
  - I made sure that larger values were parts of interest
  - Do you see a new problem?
Solution 1: Normalize

- We really want:
  \[ d(t) = \sum_k \frac{x(k) y(t - k)}{\sigma_x \sigma_y(t)} \]
  - Where the denominator ensures a unit-length dot product with the template
- Likewise we can generalize to 2-D, etc.
- But FFT speedup is a little trickier now ...
Solution 2: Gradient filtering

- Detect what matters
  - Remember what your retina likes? (hint: edges)

- Perform detection on the gradient image
  - This abstracts the color and local intensity

- Much more robust detector (for images at least)
Example

I Matched filter output I, peak = 1.2886

I Matched filter output I, peak = 1.2886
Movie example

- Track the ball movement
- Convolve in 4D space

Input movie

Ball template

\[ \text{Color channel average power} = 120 \times 160 \times 138 \]

120 × 160 × 3 × 138

\[ 28 \times 29 \times 3 \]

\[ \ast \]
Each channel separately
Template matching applications

• Popular tool in computer vision
  • face/car/pedestrian/footballer detection
    • Templates are example images of the above

• Good for event detection in audio
  • gunshot detection, room measurements

• Bio/geological/space/underwater signals etc ...
Matching without a template

- Sometime we don’t know the template
  - But we know it appears a lot

- We can use auto-correlation to find structure
  - Use the full input as a template

- Peaks will denote repeating elements
Back to the heartbeat example

- We don’t know the exact template, but there is a pattern that is repeating in time
- We can still find structure regardless
Autocorrelation

- Use the entire input as a template
- Match the input on itself

Point where the two sequences align will yield the maximum output

Correlation between oscillations within beat

Correlation between successive beats

Correlation between beats spaced by two
A popular case

- **Pitch tracking**
  - Musical instruments tend to repeat a similar waveform, the rate of its repetition being the pitch (i.e. note played)
  - But it can change over time!!
So we can’t really autocorrelate

- Many peaks corresponding to various elements
- No temporal information
Use localized autocorrelations instead

- For each analysis window perform an autocorrelation
- Which you can also efficiently compute with the FFT
Probing matching criteria

• Revisiting the starting point
  • Suppose we instead tried to find the minimal distance between the template and the input

\[ D(m, n) = \sum_{i} \sum_{j} \left| x(i - m, j - n) - y(i, j) \right|^2 \]

• This is a simple Euclidean distance
From Euclidean to dot product

- Assume a locally constant input $y$
  - Minimizing $D$, maximizes the last term
    - Which happens to be the dot product
- With the dot product we are ultimately minimizing a Euclidean distance

$$D(m,n) = \sum_i \sum_j |y(i,j)|^2 + \sum_i \sum_j |x(i,j)|^2$$
$$- 2 \sum_i \sum_j y(i,j)x(i-m, j-n)$$
Getting to a likelihood

- The Euclidean distance implies a Gaussian

\[ P(x; y) \propto e^{-\frac{1}{2}(x-y)^\top(x-y)} \]

- We can now get a likelihood of match
  - Is Gaussian the right model though?

- We can use various other metrics to define distance, each implying a different distribution
  - More on that later
And don’t forget features!

- We focused on detection on raw data
  - We can instead convolve on the feature weights
    - We sort of did that with the DFT and the gradients

- This can buy us noise robustness, invariance, and other desirable properties
  - You will rarely operate on raw data

- All of the previous theory applies here as well
Recap

• Detection of interesting elements

• Matched filtering
  • 1D to 2D
  • Scale/rotation invariance
  • Normalized cross-correlation
  • Detection on gradients
  • Autocorrelation

• From filters to likelihoods
Next lecture

- Linear classifiers
- Discriminant models
- Multi-class recognition
Reading

- Fast normalized correlation

- Transform invariant templates
  - http://www.springerlink.com/content/h345462721557808/
It’s time again

- Problem set 2 is out
  - It is due in two weeks
    - Talk to your TA early!

- Problem set 1 is due today
  - You’ll receive an email with feedback soon