

# Quantum Cryptography

Lecture 26

# Quantum Cryptography

- Quantum information: Using microscopic physical state of “quantum systems” (spin of atoms/sub-atomic particles, polarization of photons etc.) to encode information (and generate randomness)
- **Quantum Key-Distribution**: Can expand a short (one-time) shared secret key into a long one over public channels, **without computational restrictions on the adversary**, (with some physical idealization assumptions, and assuming quantum mechanics)
  - Need special “quantum channels” (optic fibers, free space...)
  - Commercially available today
- Beyond QKD: some (limited) multi-party computation results; also, security for “quantum information”

# Qubits

- Qubit refers to a quantum state that allows encoding (and decoding) one bit of information
  - State of a system (or of some aspect of it -- like polarization of a photon) is represented, according to quantum mechanics, by a **vector of complex numbers**
- But there are several possible ways to encode/decode the information in a qubit, leading to interesting properties
- A system of multiple qubits shows even more interesting properties, beyond just holding all the bits of information

# Measuring

- Measurement: reading the state of a qubit (presumably to decode the information encoded in it)
- **Basic principle: measuring alters the system**
- A metaphor: need to read the direction of a virtual needle using a "cross"
  - If either leg of the cross is aligned with the needle, we just learn its alignment (nothing happens to the needle)
  - Otherwise the needle will move to one of the legs, and we learn which one (but not whether it moved or not)
    - To which leg it moves is probabilistic, depending on its original position (which we do not learn)
  - In either case at the end the needle is aligned along a leg of the cross (as reported by the measurement)

# Measuring: Another metaphor

- Qubits as “cards” that can be read using “card readers”
- Cards come in two colors (red and blue), and have a value 0/1 on them. Cannot tell the color or the value of a card w/o “reading” it
  - If a red card is inserted into a red reader, it reports the value on the card correctly
  - If a red card is read by a blue reader, then the card gets transformed into a blue card with a random value!
    - And the reader will report that value
- Think of color as “axis-parallel” or “diagonal” needle/cross position; value indicates which of 2 legs in a cross the needle is aligned with
  - Note: not exploiting all possibilities, but already useful

# BB84

- A protocol for “key distribution” by Bennett and Brassard
- Alice and Bob want to generate a long one time pad (for information theoretically secure encryption)
- But only public channels to communicate over
  - Suppose in addition a “quantum channel” (controlled by the adversary) to send qubits
  - And the public channel is authenticated (for now), so that the adversary cannot inject messages into it
- BB84 allows them to generate a secret shared keys
- Will describe in terms of red/blue cards and card-readers

Alice

# BB84

Bob

Prepare several cards, with  
random colors and values  
Send the cards to Bob (via Eve)



Read all cards using red or blue  
readers randomly. Tell Alice which  
color reader was used for each card



Now tell Bob which color  
each card originally was



Discard all cards which were read  
using the wrong color

Among the undiscarded cards, Alice and Bob check for consistency:



Send values obtained for a  
random subset of the cards

If any value wrong, abort



- If consistency check OK, Alice and Bob "almost agree on" the values on the remaining cards and it is "mostly hidden" from Eve: Raw keys

# BB84

- If consistency check OK, Alice and Bob “almost agree on” the values on the remaining cards and it is “mostly hidden” from Eve: Raw keys
- **No-cloning**: Eve cannot save copies of the cards
  - And reading a card alters it
- If Eve reads a card (using red or blue reader) she doesn't know its original color
  - Suppose she sends it to Bob as a blue card. With prob  $1/4$ , originally the card was red and Bob reads it using red reader
  - If this card is chosen for consistency check, will discover the tampering if the random value obtained by Bob doesn't match original value on card
- Eve might get lucky and remain undetected if she alters only a few cards (so Alice and Bob may disagree on those cards)
  - But then Eve can read only a few cards (half get altered)

# Raw Keys to Good Keys

- Raw Keys:
  - A few positions where Alice's and Bob's keys may differ
  - Eve may have a small amount of information about the keys
- Distilling raw keys to good (i.e., almost uniformly random) keys is important in other contexts too
- Two step (classical) protocol, over authenticated public channel
  - **Reconciliation:** Alice and Bob calculate and compare several randomized "parity check bits" to isolate and discard errors
    - This gives further information to Eve, but now Alice and Bob agree on the same raw key (with overwhelming probability)
  - **Privacy amplification:** Use a randomness extractor to derive a suitably shorter key so that Eve has little information about the new key
    - Alice picks a seed at random and publicly sends it to Bob; shared key is defined as  $\text{Extract}(\text{RawKey}, \text{Seed})$

# Using QKD

- Alice and Bob need an authenticated public-channel
  - Can use one-time MAC with a short key (2-Universal Hash functions work)
- Originally several idealizations required for security: crucially depends on reliable quantum channels and devices
  - Many idealizations can be removed using quantum error-correction, quantum repeaters, self-testing devices
- Commercial products available



# Quantum Channel

- Transmitting an unknown qubit is delicate (even if uncertainty is a single bit of information): the entire state needs to be sent over a “quantum channel”
  - e.g.: optic fibers carrying photons
  - Recall that we can't measure the information in an unknown qubit accurately. (Else could have used a classical channel to send that information)
- **Quantum teleportation**: Pre-processing quantum communication
  - If some “entangled” qubits are shared a priori, then can use a classical channel to “teleport” an unknown qubit (without reading it)

# Entanglements

- A system with multiple qubits exhibits complex behavior
- Two qubits can be correlated in more ways than two classical cards/needles (with probabilistic values) can be
  - More complex correlation than between classical cards, even with hidden state variables (other than color and value)
  - Called entanglement
  - “EPR (Einstein–Podolsky–Rosen) paradox”: spooky action at a distance
    - Measuring two entangled qubits (cards) appears co-ordinated, as if the two card readers communicate with each other (but not in a way that can be used to communicate information we choose – non-signalling)
  - Bell inequality: limit of correlation that is possible classically. Experimentally violated by quantum systems (with caveats)

# QKD History

- Bennett and Brassard proposed BB84 in 1984
  - Similar ideas by Wiesner in early 1970s
- QKD scheme based on entanglement by Ekert in 1990
- Several other schemes by now
- Original proofs of security considered restricted Eve (e.g., in BB84 Eve measured/transformed each transmitted qubit separately)
  - Complete proof in 1996, followed by several refined proofs
- Security definitions originally based on information leaked to Eve
  - But key distribution needs composability (because key will be used for other tasks later, and attack may not be separately on QKD and subsequent use)
  - Universally Composable Security for QKD (2005)

# QKD History

- BB84 implemented at IBM Research in 1989: 32cm free air quantum channel
  - 144km in 2006
- Geneva, 2002: 23 km optical fiber cable quantum channel
  - 307 km in 2015
- DARPA network, Boston (since 2003): Between Boston University, Harvard and BBN Technologies
  - With wireless links too
- Towards longer links, larger networks
  - Possibly using "quantum repeaters"

# Beyond QKD

- Information-theoretically secure coin-tossing?
  - Impossible classically: an adversary can completely bias
  - With quantum channels, known to exist when some limited adversarial bias is allowed
  - Zero bias coin-tossing is still impossible
- Information-theoretically secure commitment?
  - Impossible even with quantum channels
- Secret-sharing: requiring quantum communication for reconstruction

# Beyond QKD

- **Quantum computation:** a major field (still not practical), using quantum gates to manipulate qubits
  - “Efficient” algorithm for factorization
- **Cryptography for qubits**
  - Authenticating qubits
  - Encrypting qubits
  - Multi-party computation when inputs and outputs are qubits
    - Known when 5/6th-majority is honest
- **Post-Quantum Cryptography:** Classical/Quantum cryptography secure against computationally bounded quantum adversaries?
  - Several OWF candidates are not quantum-OWF

# Quantum Cryptography

- Goal: Don't depend on computational restrictions on the adversary
- Quantum Key Distribution: information theoretic security, if reliable quantum channels/devices available
  - Still needs a small (one-time) shared key to authenticate the classical channel (MAC)
  - Needs quantum channels: today limited to short distances
  - Also need to counter "quantum hacking"
- No magic bullet: QKD doesn't have all functionalities of PKE. Other primitives (e.g. commitment) still impossible without computational assumptions.
- Evolving theory and practice

# A Quick Summary

## Standardized

- Encryption
- Authentication

- Multi-party computation

- Voting

- Fancy Signatures

- Obfuscation

- Quantum crypto

- Lattices, Pairings, Multi-linear maps, Generic groups, Random Oracle

- SKE/PKE. (Also, Homomorphic Encryption, IBE, ...)
- Security definitions: CPA/CCA, SIM & IND
- Abstractions: OWF/Hardcore bits, Trapdoor-OWP, ...
- Constructions: DDH, RSA, bilinear pairings, lattices, ...

- Hash functions, MACs, Digital Signatures
- Security definitions: Collision resistance (various), existential forgery, ..
- Constructions: Based on hash functions (also Random Oracles + Trapdoor-OWP)

- MPC: Oblivious Transfer, Commitment, ZK Proofs, Yao's garbled circuit, GMW, BGW
- Didn't cover: more protocols for general tasks, more efficient protocols for specific tasks, other security definitions...

- Didn't cover: Cryptanalysis, Leakage-resilience, Oblivious RAM, e-Cash, Functional encryption, Steganography, Game-theoretic crypto, ...

# That's All Folks!

