Functionally Rich Signatures

Lecture 24

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- Definitions sometimes have subtleties (not all of them have ideal functionality specifications)

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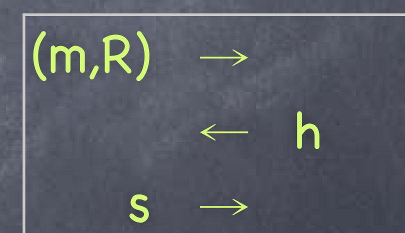

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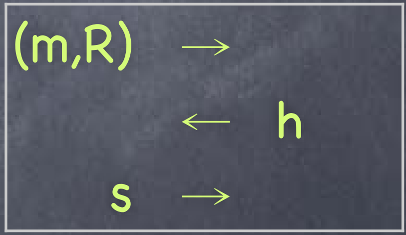

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  - So that final signature $s_n = r + h_1 x_1 + \ldots + h_n x_n$ where $R= g^r$

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- General aggregation: signatures can be created independently and then aggregated in arbitrary order without knowing the secret keys

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    - Aggregate signatures saves on bandwidth and verification time, but needs coordination among signers and does not allow un-aggregating the signatures

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- Similarly for pairing equations, but with further optimizations
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    - Can save on number of pairing operations using $\Pi_i e(S_i,g)^{w_i} = \Pi_i e(S_i^{w_i},g) = e(\Pi_i S_i^{w_i},g)$

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  - Signature shows that message was signed by some member of a group
  - But a group manager can "trace" the signer
  - However, the group manager or other group members "cannot frame" a member

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- **Full-Traceability**: If a set of group members collude and create a valid signature, the tracing algorithm will trace at least one member of the set. This holds even if the group manager is passively corrupt.

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  - Even if adversary knows secret keys of all group-members, and has oracle access to the "tracing algorithm" (but not allowed to query it on the challenge)
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- **Full-Traceability**: If a set of group members collude and create a valid signature, the tracing algorithm will trace at least one member of the set. This holds even if the group manager is passively corrupt.
  - Implies unforgeability (i.e., with no group members colluding with it, adversary cannot produce a valid signature) and framing-resistance (even colluding with the group manager)

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- Tracing algorithm decrypts C to find SK^*_i and hence ID_i

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- Unwitting collaborators: F_i's could be the verification keys for a standard signature scheme

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- Security requirements: Unforgeability and Hiding

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- c.f. Mesh signatures: here, instead of multiple parties signing a message, a single party with multiple attributes

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- Zero-knowledge verification: A verifier cannot transfer a signature that it verified

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  - e.g. a ring signature with a ring of size 2, containing the signer and the designated verifier

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- Not discussed: Blind signatures, P-signatures, anonymous credentials...