Functionally Rich Signatures

Lecture 24
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Functionally Rich Signatures

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- Simple and efficient ones in the Random Oracle Model
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  - Simple and efficient ones in the Random Oracle Model
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  - Using minimal/general assumptions, often simple, but not very efficient (e.g., involving NIZK for general NP statements)
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  - Simple and efficient ones in the Random Oracle Model
  - Relatively efficient ones under specific assumptions (often relatively strong/new assumptions)
  - Using minimal/general assumptions, often simple, but not very efficient (e.g., involving NIZK for general NP statements)
- Definitions sometimes have subtleties (not all of them have ideal functionality specifications)
Multi-Signatures
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Multiple signers signing the same message
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- Multiple signers signing the same message
- Each signer has an (SK,VK) pair
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Security requirement: Unforgeability (chosen message security)
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- Each signer has an \((SK, VK)\) pair
- Resulting signature must be "compact": size independent of the number of signers
- Security requirement: Unforgeability (chosen message security)
- Adversary can collude with all but one signer
Schnorr Signature
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A regular (i.e., non-multi) digital signature scheme secure in the Random Oracle model under the discrete log assumption
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**KeyGen:** Signing key is $x$ and Verification key is $X = g^x$
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KeyGen: Signing key is $x$ and Verification key is $X = g^x$

Sign($m; x$): Compute $R = g^r$, $h = H(m, R)$, $s = r + hx$. Output $(R, s)$
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Security: $\approx$ a ("simulation-sound") HVZK PoK of $x$
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Extended to a multi-signature scheme [BN’06] →
A Multi-Signature Scheme
A Multi-Signature Scheme

Schnorr: \( \text{Sign}(m; x) = (R, s) \) where \( R = g^r, \ s = r + hx \) for \( h = H(m, R) \).
Verify\((m, (R, s); X)\) checks if \( g^s = RX^h \) for \( h = H(m, R) \)
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For multiple signers with keys X_1,...,X_n can create an “aggregated” signature (R,s) such that g^s = R.X_1^{h_1}...X_n^{h_n}, where:
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For multiple signers with keys $X_1,...,X_n$ can create an “aggregated” signature $(R,s)$ such that $g^s = R.X_1^{h_1}...X_n^{h_n}$, where:

Pick $R$: each party picks $r_i$ and publishes $g^{r_i}$. Set $R = g^{r_1+...+r_n}$
A Multi-Signature Scheme

**Schnorr:** \( \text{Sign}(m;x) = (R,s) \) where \( R=g^r, \ s = r + hx \) for \( h=H(m,R) \). \( \text{Verify}(m,(R,s);X) \) checks if \( g^s = RX^h \) for \( h = H(m,R) \)

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- **Pick \( R \):** each party picks \( r_i \) and publishes \( g^{ri} \). Set \( R = g^{r_1+...+r_n} \)
  - Ensure **simultaneous** announcement of \( g^{ri} \). (Commit & reveal.)
A Multi-Signature Scheme

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- \( h_i = H(m,R,X_i,L) \), where \( L = <X_1,...,X_n> \)
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- Then, sequentially \( s_i = s_{i-1} + r_i + h_ix_i \) (starting with \( s_0 = 0 \))
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- Then, sequentially \( s_i = s_{i-1} + r_i + h_ix_i \) (starting with \( s_0 = 0 \))
- So that final signature \( s_n = r + h_1x_1 + ... + h_nx_n \) where \( R = g^r \)
Waters Signature

A regular (non-multi) signature scheme that is secure if the Computational Diffie-Hellman assumption holds in a group with bilinear pairings (no RO)
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Keys: Signing key is $x$ and verification key is $X := e(g,g)^x$, and generators $u_0,u_1,\ldots,u_k$ (for k bit long messages)
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\[ \text{Sign}(m;x) = (R,S) \text{ where } R = g^r \text{ and } S = g^x H^r, \text{ where } \]
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Verify$(m,(R,S);X,u,u_1,\ldots,u_k)$: check $e(S,g) = e(R,H).X$
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**Verify($m,(R,S);X,u,u_1,....,u_k$):** check $e(S,g) = e(R,H).X$

Extended to a multi-signature scheme [LOSSW’06] →
LOSSW Scheme

**Keys:** For user $i$ verification key is $X_i := e(g,g)^{x_i}$, and $u_{i0}, u_{i1}, \ldots, u_{ik}$. Signing key is $x_i$ and $y_{i0}, y_{i1}, \ldots, y_{ik}$ where $u_{ij} = g^{y_{ij}}$
LOSSW Scheme

**Keys:** For user i verification key is $X_i := e(g,g)^{x_i}$, and $u_{i0}, u_{i1}, ..., u_{ik}$. Signing key is $x_i$ and $y_{i0}, y_{i1}, ..., y_{ik}$ where $u_{ij} = g^{y_{ij}}$

**Signature = (R,S),** where $R = g^{r_1 + ... + r_n}$, $S = g^{x_1 + ... + x_n} (H_1 \ldots H_n)^{r_1 + ... + r_n}$ where $H_i = u_{i0}.(u_{i1})^{m_1}...(u_{ik})^{m_k}$
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Verification of signature $(R, S)$: check $e(S, g) = e(R, H_1)^{X_1} \ldots e(R, H_n)^{X_n}$
LOSSW Scheme

**Keys:** For user $i$ verification key is $X_i := e(g,g)^{x_i}$, and $u_i^0, u_i^1, ..., u_i^k$. Signing key is $x_i$ and $y_i^0, y_i^1, ..., y_i^k$ where $u_i^j = g^{y_{ij}}$

**Signature** = $(R,S)$, where $R = g^{r_1 + ... + r_n}$, $S = g^{x_1 + ... + x_n} (H_1 ... H_n)^{r_1 + ... + r_n}$ where $H_i = u_i^0(u_i^1)^{m_1}...(u_i^k)^{m_k}$

**Verification of signature** $(R,S)$: check $e(S,g) = e(R,H_1)X_1 ... e(R,H_n)X_n$

Signing done sequentially by individual signers. Initially set $R = 1$ and $S = 1$ (identity in the group). Then:
LOSSW Scheme

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**Signature** = $(R,S)$, where $R=g^{r_1+\ldots+r_n}$, $S = g^{x_1+\ldots+x_n} (H_1 \ldots H_n)^{r_1+\ldots+r_n}$ where $H_i = u_{i0}^i(u_{i1}^i)^{m_1}\ldots(u_{ik}^i)^{m_k}$

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Signing done sequentially by individual signers. Initially set $R=1$ and $S = 1$ (identity in the group). Then:

**AddSign**$(m,(R',S'); x_i, y_{i0}^i,y_{i1}^i,\ldots,y_{ik}^i) = \text{ReRand}(R'',S'')$, where $R''=R'$ and $S'' = S'.g^{x_i}(R')^{h_i}$ where $h_i \text{ s.t. } g^{h_i} = H_i$
**LOSSW Scheme**

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**Signature** = $(R,S)$, where $R = g^{r_1 + ... + r_n}$, $S = g^{x_1 + ... + x_n} (H_1 ... H_n)^{r_1 + ... + r_n}$ where $H_i = u_{i0}.(u_{i1})^{m_1}...(u_{ik})^{m_k}$

**Verification of signature** $(R,S)$: check $e(S,g) = e(R,H_1)X_1 ... e(R,H_n)X_n$

Signing done sequentially by individual signers. Initially set $R=1$ and $S = 1$ (identity in the group). Then:

- $\text{AddSign}(m, (R',S'); x_i, y_{i0}, y_{i1}, ..., y_{ik}) = \text{ReRand}(R'', S'')$, where $R'' = R'$ and $S'' = S'.g^{x_i}.(R')^{h_i}$ where $h_i \text{ s.t. } g^{h_i} = H_i$
- $\text{ReRand}(R'', S'') = (R, S)$, where $R = R''g^t$ and $S = S'' (H_1 ... H_i)^t$
Aggregate Signatures
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Generalization of multi-signatures where multiple signers may have different messages
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- Sequential aggregation: each signer gets the aggregated signature so far and adds her signature into it (using own signing key)
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  e.g., LOSSW’06: If \((m_1, ..., m_n)\) where \(m_i=(m_{i1}, ..., m_{ik})\), then let

  \[ H_i = u_{i0}.(u_{i1})^{m_{i1}}...(u_{ik})^{m_{ik}} \]
Aggregate Signatures

Generalization of multi-signatures where multiple signers may have different messages

Sequential aggregation: each signer gets the aggregated signature so far and adds her signature into it (using own signing key)

\[ H_i = u_{i_0} (u_{i_1})^{m_{i_1}} ...(u_{i_k})^{m_{i_k}} \]

e.g., LOSSW’06: If \((m_1,...,m_n)\) where \(m^i=(m_{i_1},...,m_{i_k})\), then let

General aggregation: signatures can be created independently and then aggregated in arbitrary order without knowing the secret keys
Batch Verification
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To speed up verification of a collection of signatures
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To speed up verification of a collection of signatures

Batching done by the verifier
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Batch verifiable signature scheme reduces verification time, but does not reduce the total size of signatures that verifier gets. No co-ordination among signers.
Batch Verification

- To speed up verification of a collection of signatures
- Batching done by the verifier
- Incomparable to aggregate signatures

- Batch verifiable signature scheme reduces verification time, but does not reduce the total size of signatures that verifier gets. No co-ordination among signers.

- Aggregate signatures saves on bandwidth and verification time, but needs coordination among signers and does not allow un-aggregating the signatures
Batch Verification
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Idea: to verify several equations of the form $Z_i = g^{z_i}$, pick random weights $w_i$ and check $\prod_i Z_i^{w_i} = g^{\sum z_i w_i}$
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If one (or more) equation is wrong, probability of verifying is at most $1/q$, where $q$ is the size of the domain of $w_i$
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Efficiency by using a small domain for $w_i$. e.g., use $w_i \in \{0,1\}$, and repeat $k$ times (independent of number of signatures)
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Similarly for pairing equations, but with further optimizations
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e.g. Waters’ signature: \( e(S,g) = e(R,H).X \) (\( g \) same for all signers)
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Efficiency by using a small domain for $w_i$. e.g., use $w_i \in \{0,1\}$, and repeat $k$ times (independent of number of signatures).

Similarly for pairing equations, but with further optimizations.

- e.g. Waters' signature: $e(S,g) = e(R,H).X$ (g same for all signers)

- Can save on number of pairing operations using $\prod_i e(S_i,g)^{w_i} = \prod_i e(S_i^{w_i},g) = e(\prod_i S_i^{w_i},g)$
Group Signatures
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To sign a message “anonymously” [Chaum-vanHeyst’91]
Group Signatures

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Signature shows that message was signed by some member of a group
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But a group manager can “trace” the signer
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To sign a message “anonymously” [Chaum-vanHeyst’91]

Signature shows that message was signed by some member of a group

But a group manager can “trace” the signer

However, the group manager or other group members “cannot frame” a member
Group Signatures
**Group Signatures**

**Full-Anonymity:** Adversary gives \((m, ID_0, ID_1)\) and gets back \(\text{Sign}(m; ID_b)\) for a random bit \(b\). Advantage of the adversary in finding \(b\) should be negligible.
Group Signatures

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Even if adversary knows secret keys of all group-members, and has oracle access to the “tracing algorithm” (but not allowed to query it on the challenge)
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Implies unlinkability (can’t link signatures from same user)
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*Implies unlinkability* (can’t link signatures from same user)

**Full-Traceability**: If a set of group members collude and create a valid signature, the tracing algorithm will trace at least one member of the set. This holds even if the group manager is passively corrupt.
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**Implies unlinkability** (can’t link signatures from same user)

**Full-Traceability**: If a set of group members collude and create a valid signature, the tracing algorithm will trace at least one member of the set. This holds even if the group manager is passively corrupt.

**Implies unforgeability** (i.e., with no group members colluding with it, adversary cannot produce a valid signature) and **framing-resistance** (even colluding with the group manager)
Group Signatures
Group Signatures

A general construction: using a digital signature scheme, a CCA secure encryption scheme, and a “simulation-sound” NIZK [BMW’03]
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Each member’s signing key $SK^*_i = (SK_i, VK_i, ID_i, \sigma)$ where $(SK_i, VK_i)$ are signing/verification keys, and $\sigma$ is a signature (w.r.t. $VK_{\text{group}}$) from the group-manager on $(VK_i, ID_i)$
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\[
\sigma = \text{Sign}(\text{message}; SK_i)
\]
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- $\pi = \text{a proof (w.r.t CRS}_{\text{group}})$ that $C$ is correct

Tracing algorithm decrypts $C$ to find $SK^*_i$ and hence $ID_i$
Ring Signatures
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For "leaking secrets"
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Similar to group signatures, but with unwitting collaborators
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i.e. the “ring” is not a priori fixed
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Similar to group signatures, but with unwitting collaborators

i.e. the “ring” is not a priori fixed

And no manager who can trace the signer
Ring Signatures
Ring Signatures

Recall T-OWP/RO based signature
Ring Signatures

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$(SK, VK) = (F^{-1}, F)$
Ring Signatures

Recall T-OWP/RO based signature

\[(SK, VK) = (F^{-1}, F)\]

\[\text{Sign}(m; F^{-1}) = F^{-1}(H(m))\]
Ring Signatures

Recall T-OWP/RO based signature

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\(\text{Verify}(S; F): \text{ check if } H(m) = F(S)\)
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Extended to a ring signature [RST’01]
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Extended to a ring signature [RST’01]

\[\text{Verify}(m, (S_1, ..., S_n); (F_1, ..., F_n)) : \text{check } H(m) = F_1(S_1) + \cdots + F_n(S_n)\]
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\(\text{Verify}(m, (S_1, \ldots, S_n); (F_1, \ldots, F_n))\) : check \(H(m) = F_1(S_1) + \ldots + F_n(S_n)\)

\(\text{Sign} (m; F_1^{-1}, F_2, \ldots, F_n) = (S_1, \ldots, S_n)\) where \(S_2, \ldots, S_n\) are random and \(S_1 = F_1^{-1}\left( H(m) - F_2(S_2) - \ldots - F_n(S_n) \right)\)
Ring Signatures

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$$(SK, VK) = (F^{-1}, F)$$

$$\text{Sign}(m; F^{-1}) = F^{-1}(H(m))$$

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$$\text{Sign } (m; F_1^{-1}, F_2, ..., F_n) = (S_1, ..., S_n) \text{ where } S_2, ..., S_n \text{ are random and } S_1 = F_1^{-1} ( H(m) - F_2(S_2) - ... - F_n(S_n) )$$

Unwitting collaborators: $F_i$’s could be the verification keys for a standard signature scheme
Mesh Signatures
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Ring signature allows statements of the form
\((P_1 \text{ signed } m) \text{ or } (P_2 \text{ signed } m) \text{ or } .... \text{ or } (P_n \text{ signed } m)\)
Mesh Signatures

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Mesh signatures extend this to more complex statements
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\(\text{e.g., } (P_1 \text{ signed } m_1) \text{ or } ((P_2 \text{ signed } m_2) \text{ and } (P_3 \text{ signed } m_3))\)

\(\text{e.g., some two out of the three statements } (P_1 \text{ signed } m_1), \quad (P_2 \text{ signed } m_2), \quad (P_3 \text{ signed } m_3) \text{ hold}\)
Mesh Signatures

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  e.g., some two out of the three statements \((P_1 \text{ signed } m_1), \ (P_2 \text{ signed } m_2), \ (P_3 \text{ signed } m_3)\) hold 

- Signature is produced by the relevant parties collaborating
Mesh Signatures

Ring signature allows statements of the form 
(P₁ signed m) or (P₂ signed m) or .... or (Pₙ signed m)

Mesh signatures extend this to more complex statements

 e.g., (P₁ signed m₁) or ( (P₂ signed m₂) and (P₃ signed m₃) )

 e.g., some two out of the three statements (P₁ signed m₁), 
(P₂ signed m₂), (P₃ signed m₃) hold

Signature is produced by the relevant parties collaborating

Security requirements: Unforgeability and Hiding
Attribute-Based Signatures
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“Claim-and-endorse”: Claim to have attributes satisfying a certain policy, and sign a message
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Also unlinkable: cannot link multiple signatures as originating from the same signer
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Also unlinkable: cannot link multiple signatures as originating from the same signer

c.f. Mesh signatures: here, instead of multiple parties signing a message, a single party with multiple attributes
Undeniable Signatures
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Suppose Signer wants to control when/how often the signature can be verified, but signature is a commitment to a message
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- Signer refusing to deny can be taken as accepting.
Undeniable Signatures

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Verification is via an interactive protocol.

It lets the signer verifiably accept or deny endorsing the message.

Signer refusing to deny can be taken as accepting.

Zero-knowledge verification: A verifier cannot transfer a signature that it verified.
Designated Verifier
Signatures
Designated Verifier Signatures

Signature addressed to a single designated verifier
Designated Verifier Signatures

- Signature addressed to a single designated verifier
- Verifier cannot convince others of the validity of the signature
Designated Verifier Signatures

- Signature addressed to a single designated verifier
- Verifier cannot convince others of the validity of the signature
- e.g. a ring signature with a ring of size 2, containing the signer and the designated verifier
Today
Today

Signatures
Today

- Signatures
- Multi-signatures
Today

- Signatures
- Multi-signatures
- Aggregate Signatures
Today

- Signatures
- Multi-signatures
- Aggregate Signatures
- Signatures with Batch verification
Today

- Signatures
  - Multi-signatures
  - Aggregate Signatures
  - Signatures with Batch verification
  - Group signatures
Today

- Signatures
  - Multi-signatures
  - Aggregate Signatures
  - Signatures with Batch verification
  - Group signatures
  - Ring and Mesh signatures
Today

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- Not discussed: Blind signatures, P-signatures, anonymous credentials...