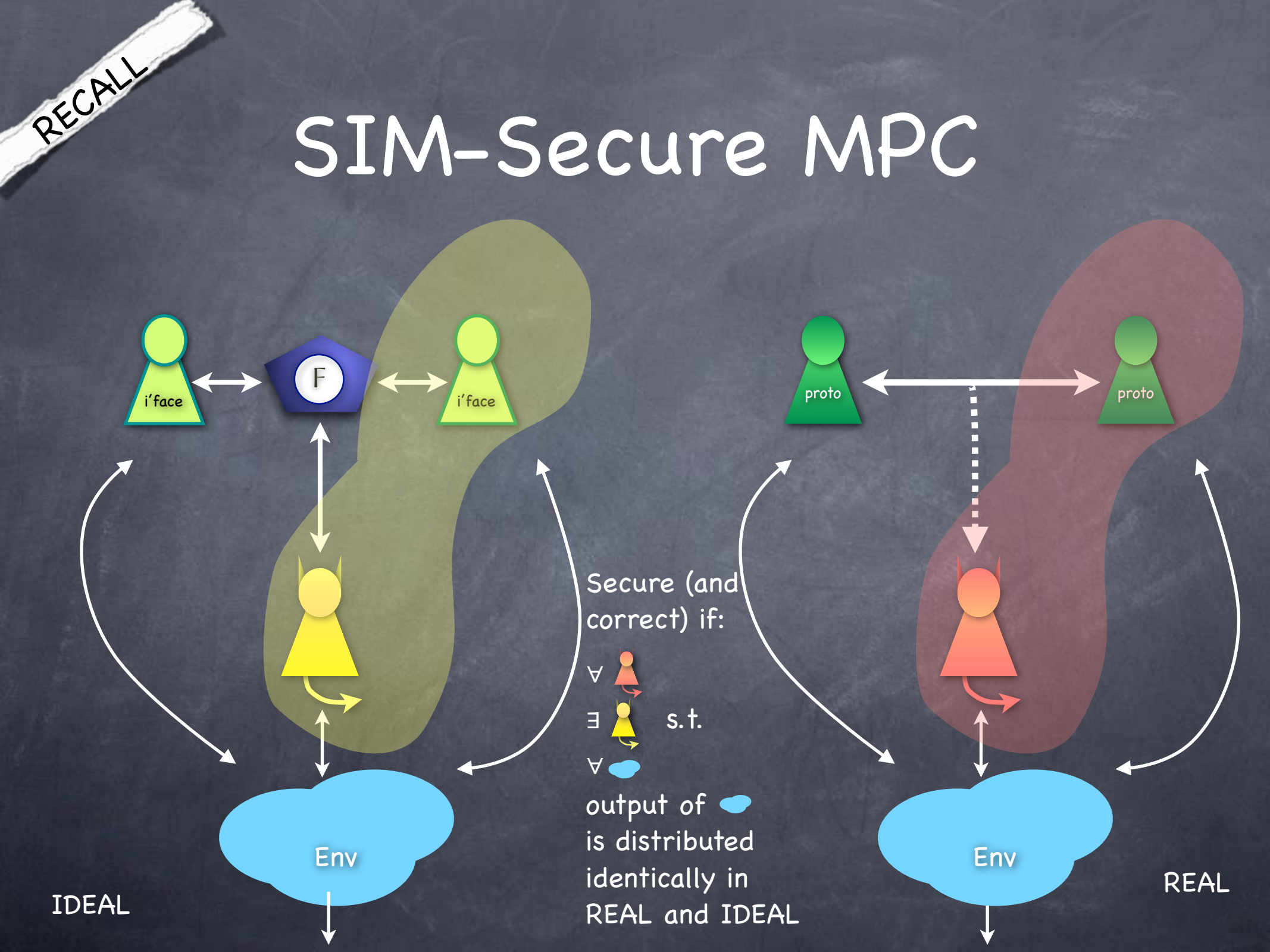
Secure 2-Party Computation

Lecture 14

Yao's Garbled Circuit

RECALL

SIM-Secure MPC



RECALL

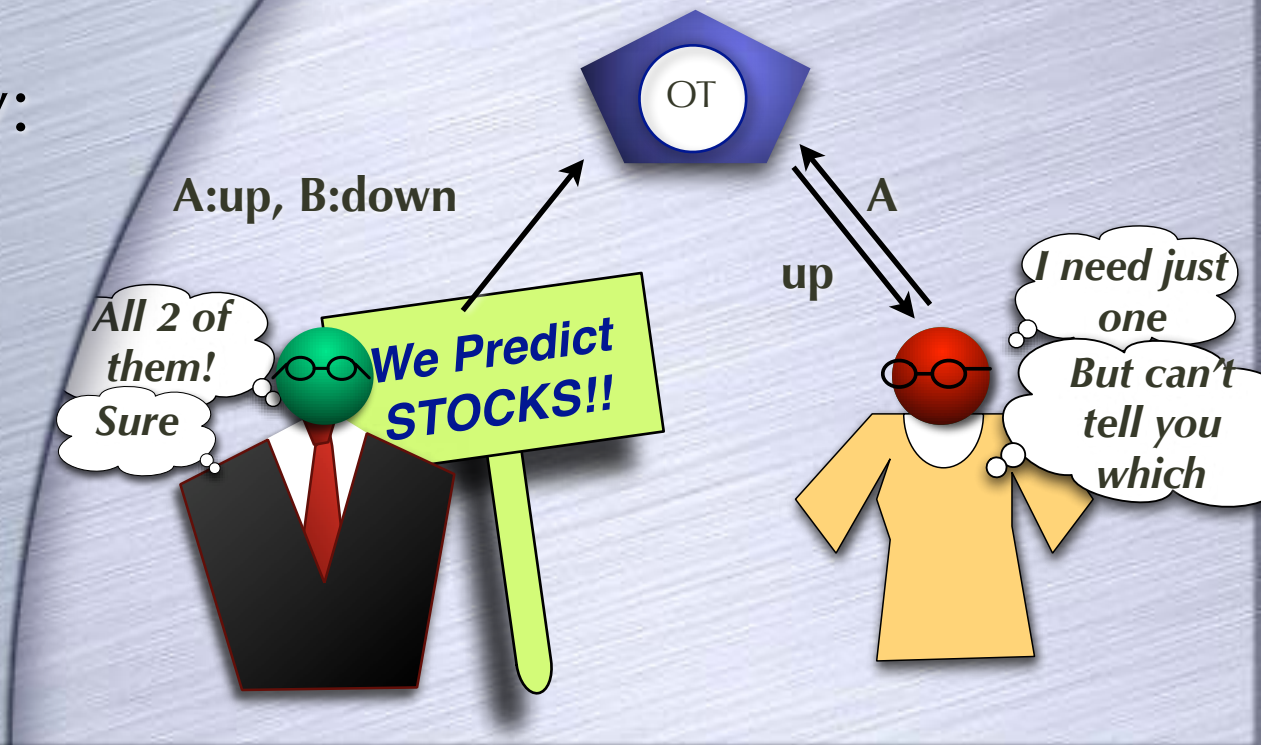
Passive Adversary

- Gets only read access to the internal state of the corrupted players (and can use that information in talking to environment)
  - Also called "Honest-But-Curious" adversary
  - Will require that simulator also corrupts passively
- Simplifies several cases
  - e.g. coin-tossing [why?], commitment [coming up]
- Oddly, sometimes security against a passive adversary is more demanding than against an active adversary
  - Active adversary: too pessimistic about what guarantee is available even in the IDEAL world
  - e.g. 2-party SFE for OR, with output going to only one party (trivial against active adversary; impossible without computational assumptions against passive adversary)

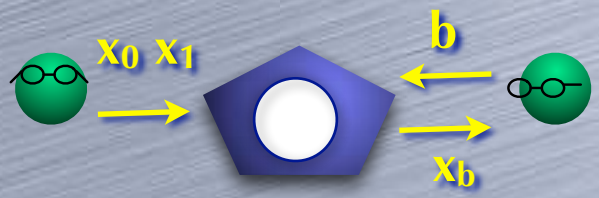
RECALL

Oblivious Transfer

- Pick one out of two, without revealing which
  - Intuitive property: transfer partial information "obliviously"



An OT Protocol
(passive corruption)



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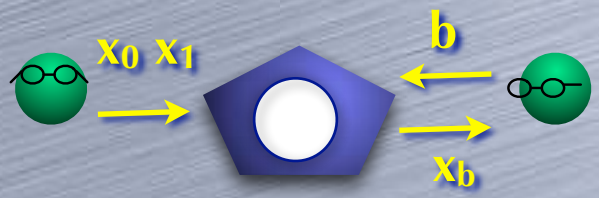

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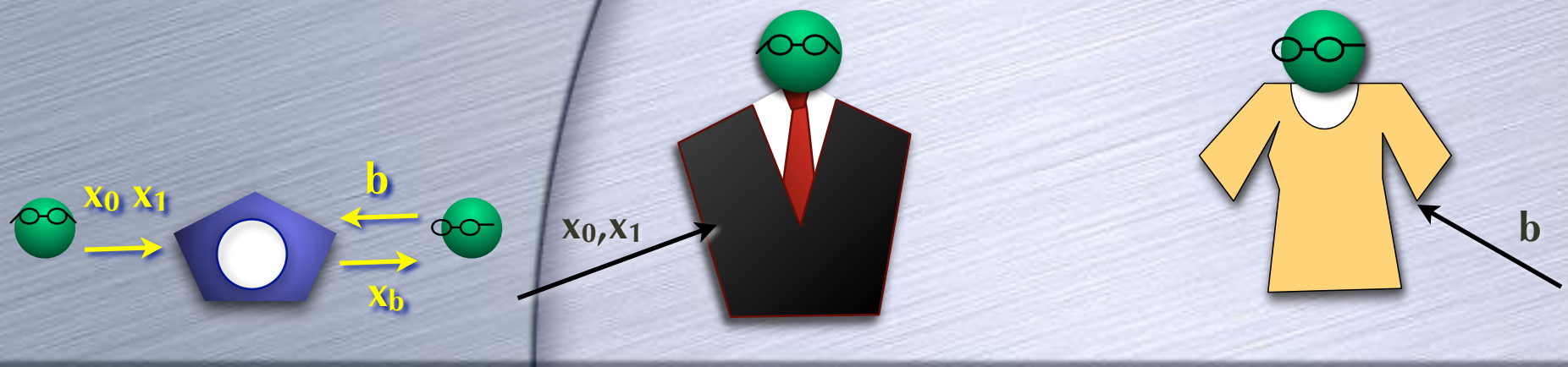

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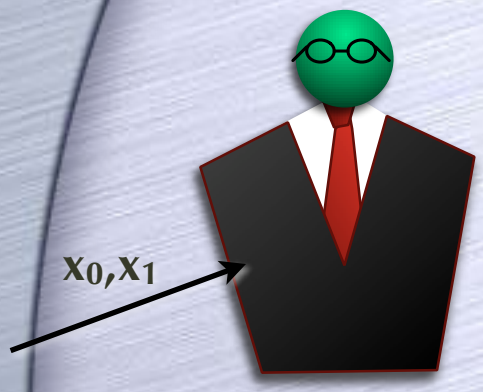

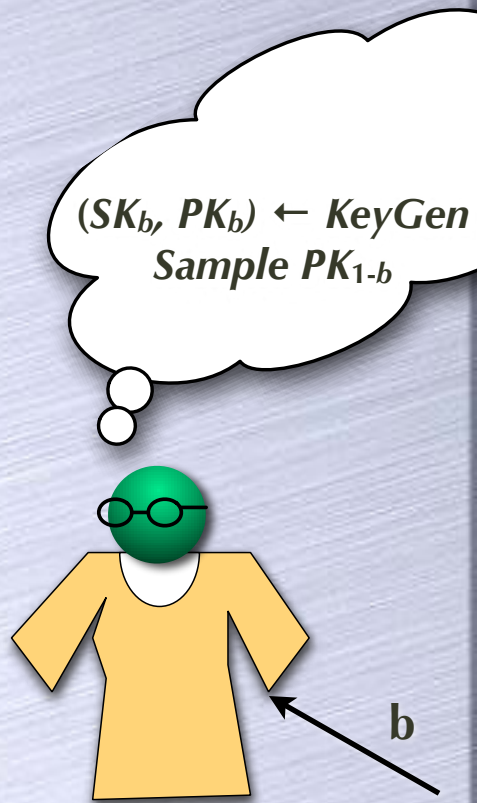

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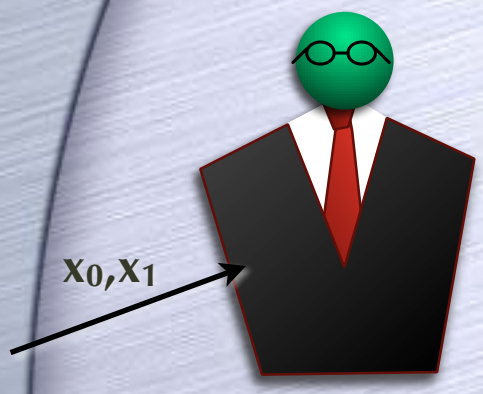

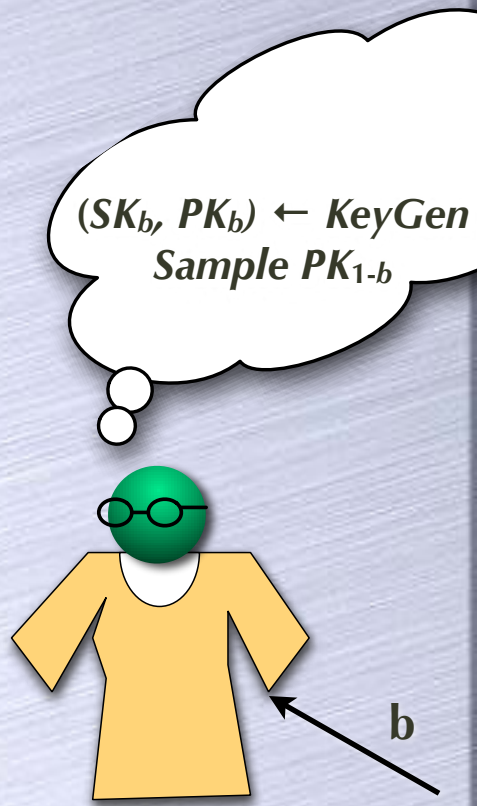

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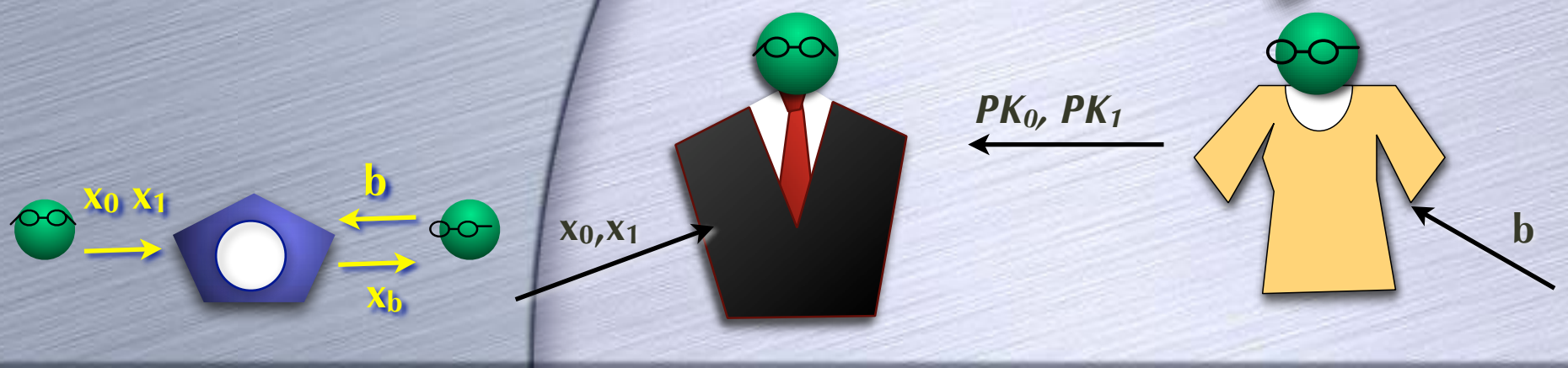

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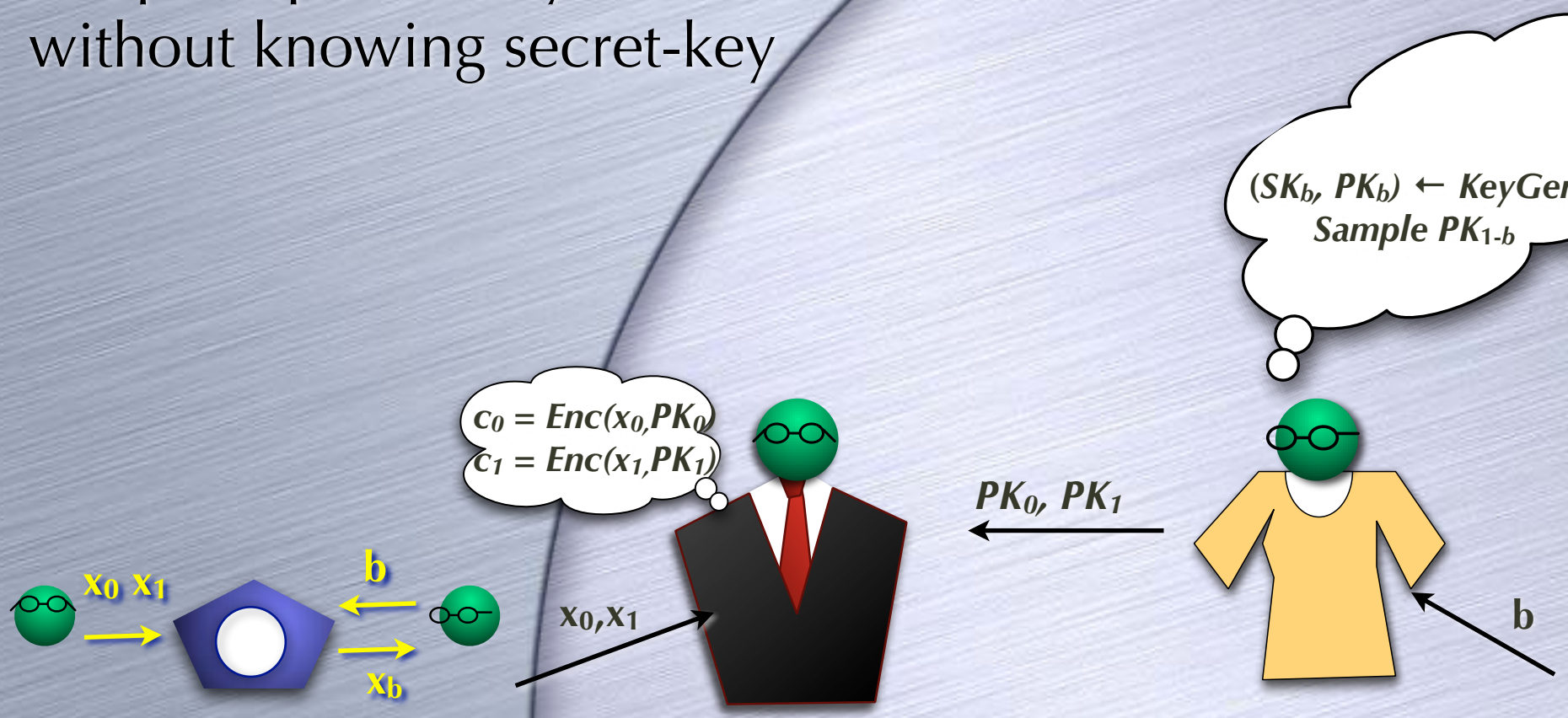

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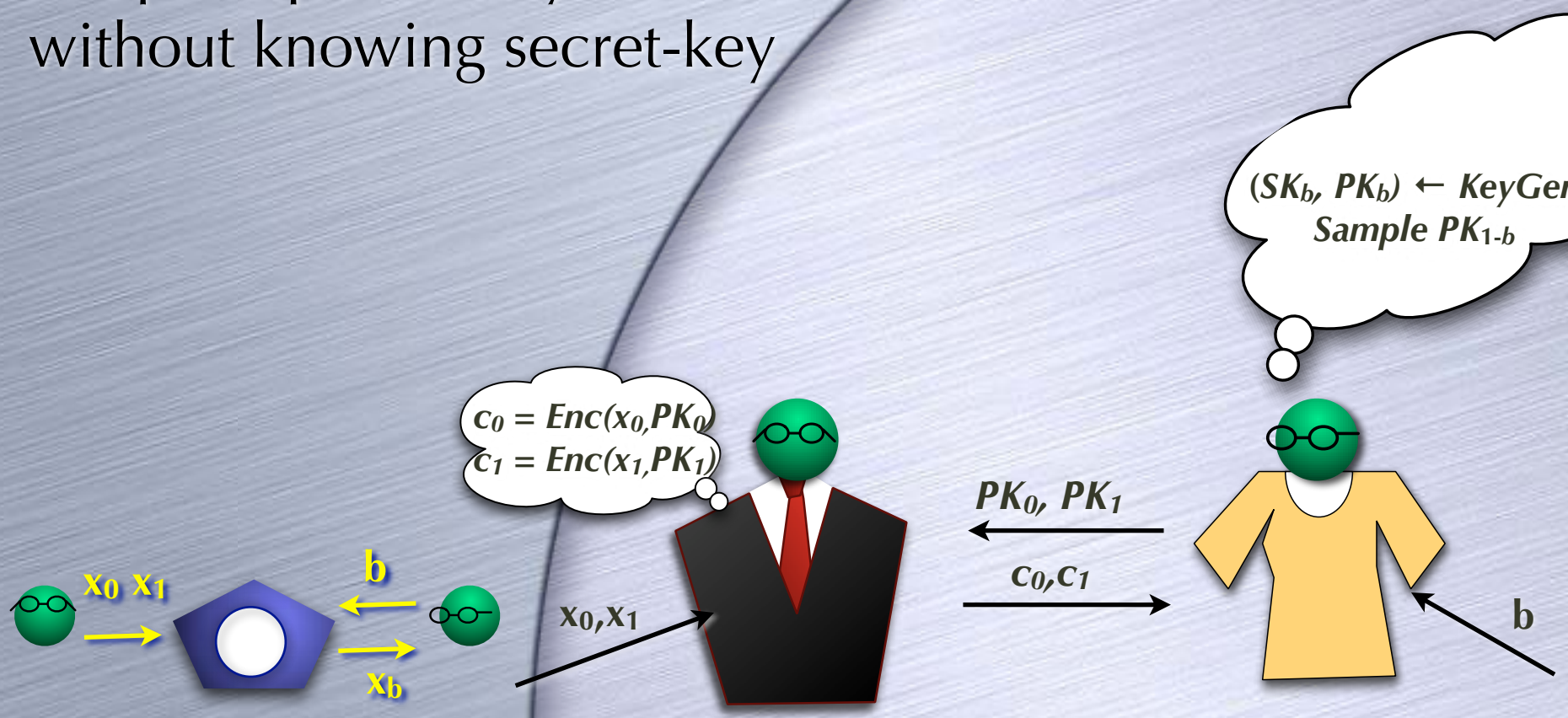

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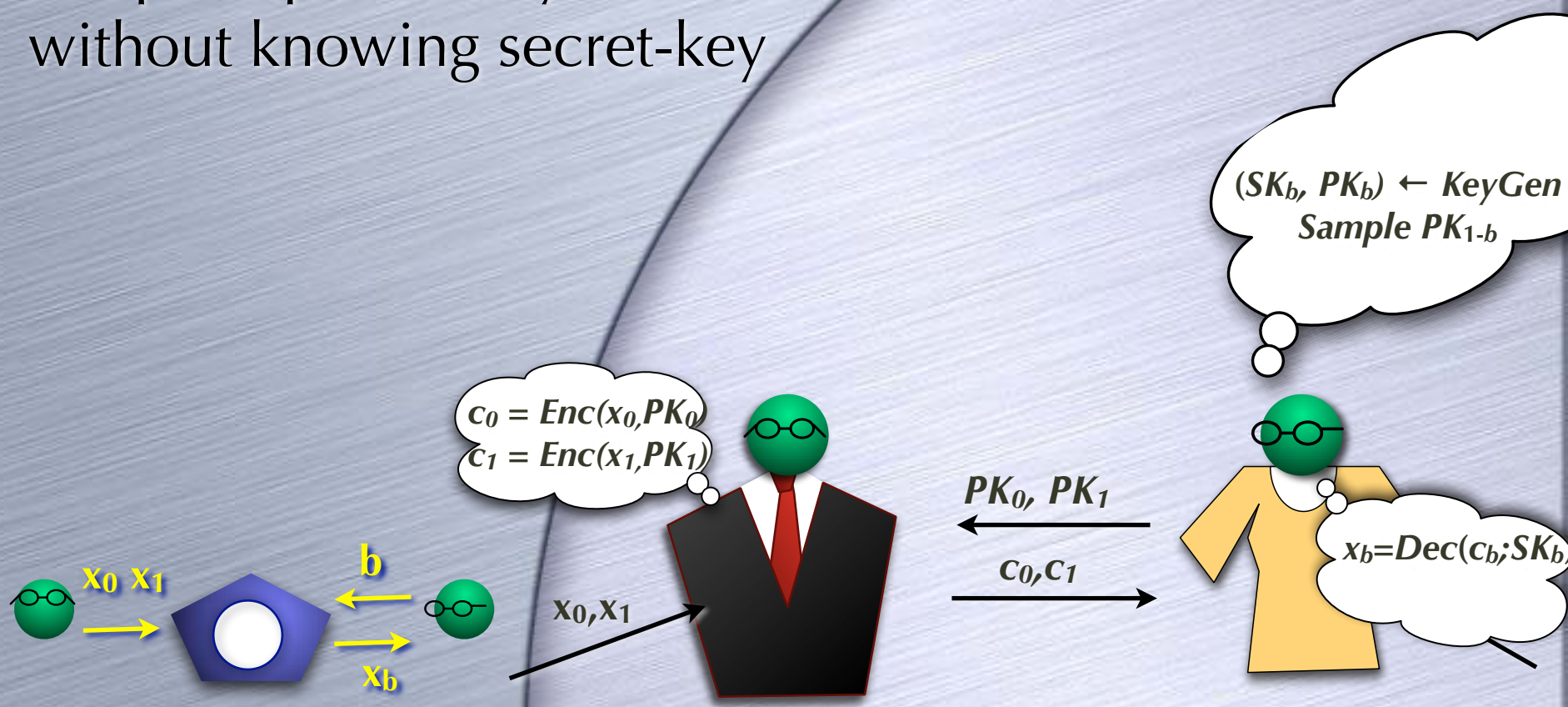

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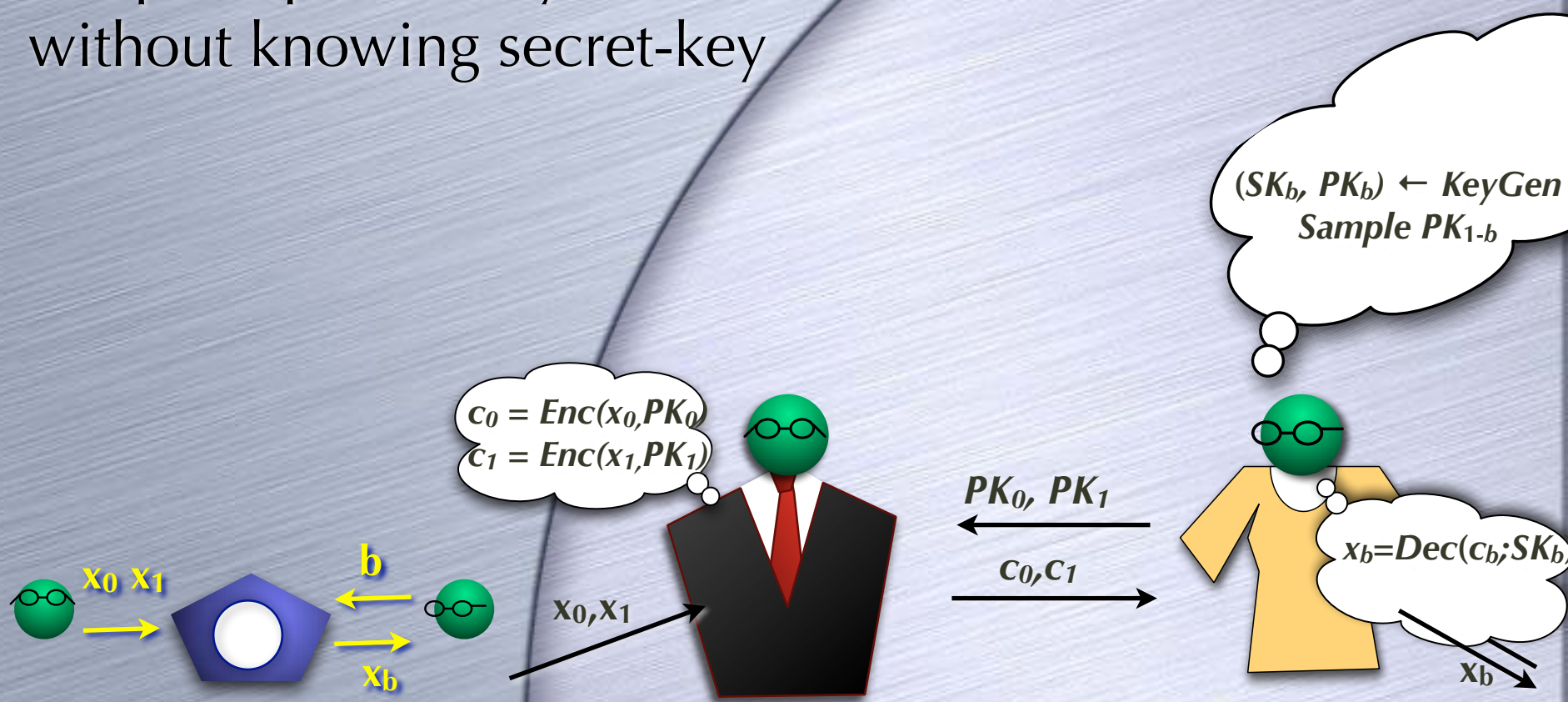

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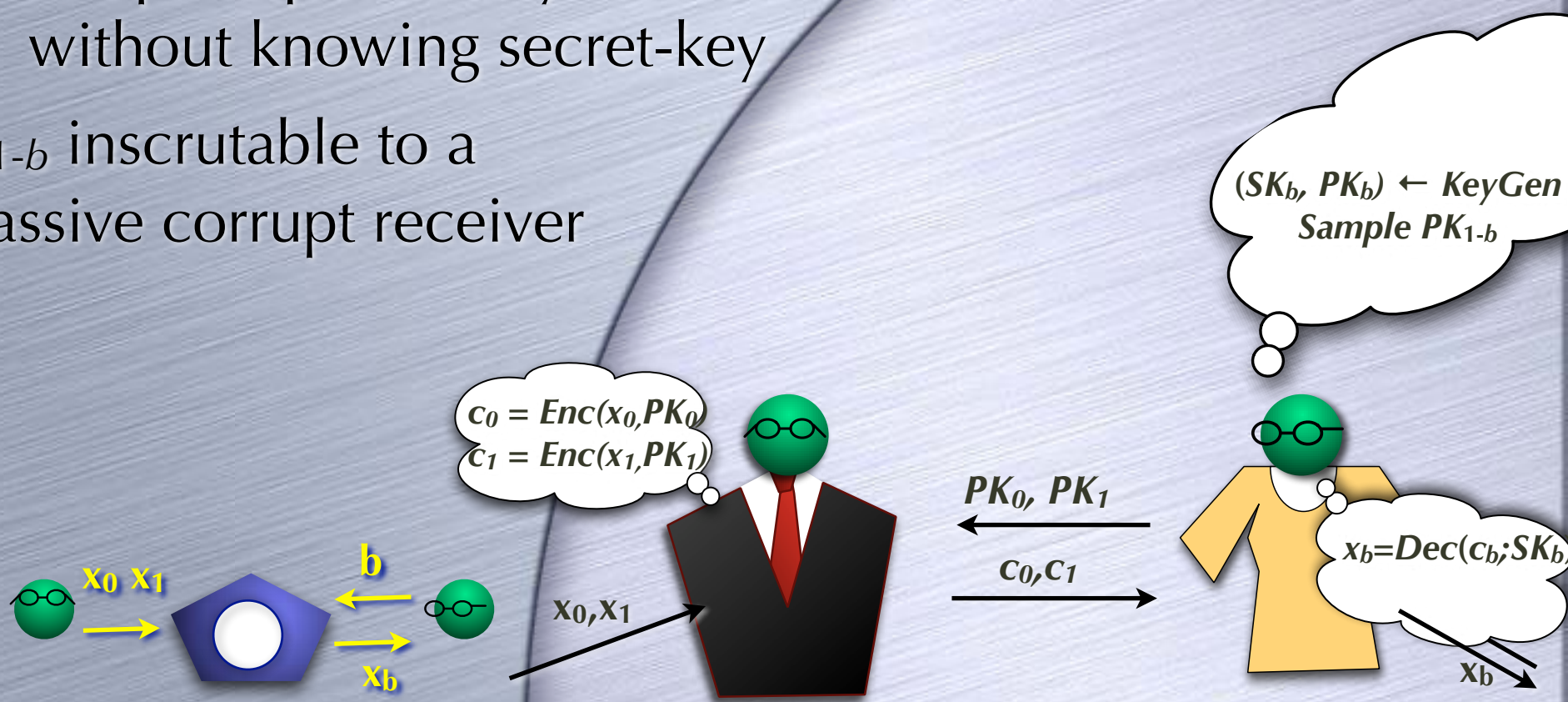

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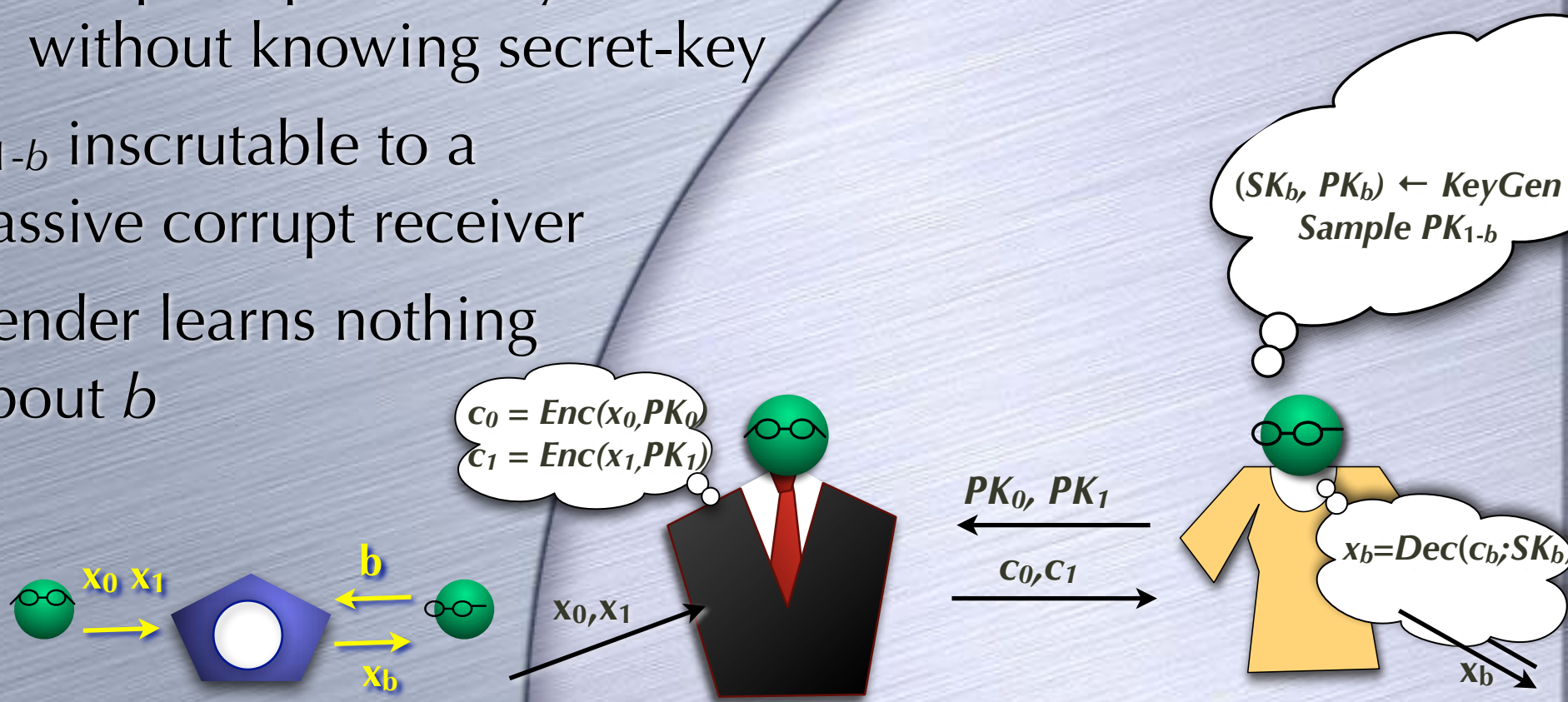

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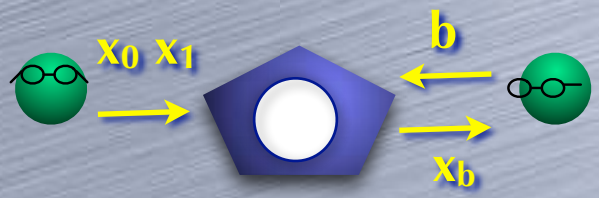

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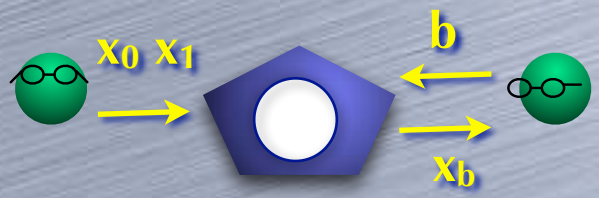

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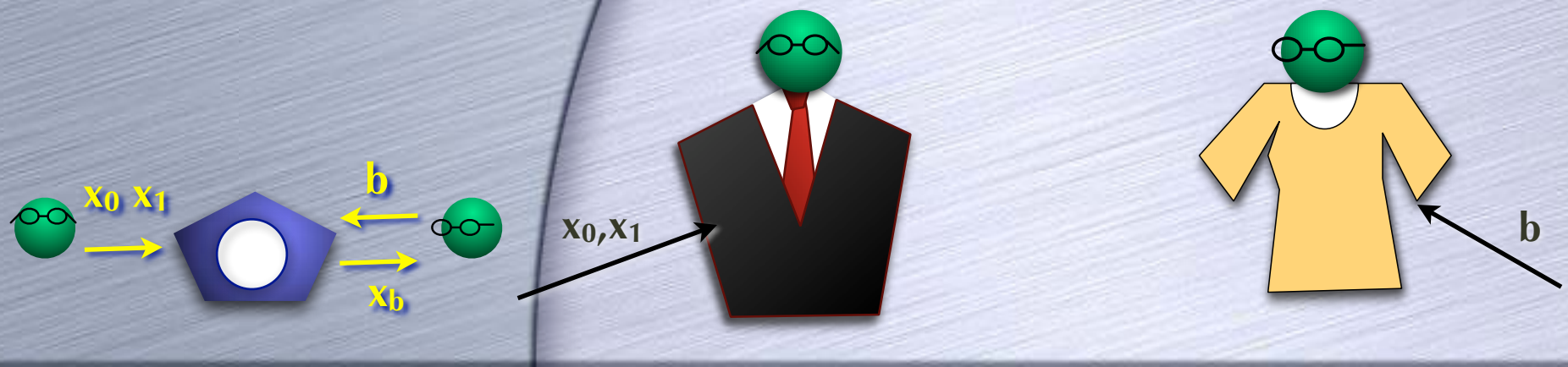

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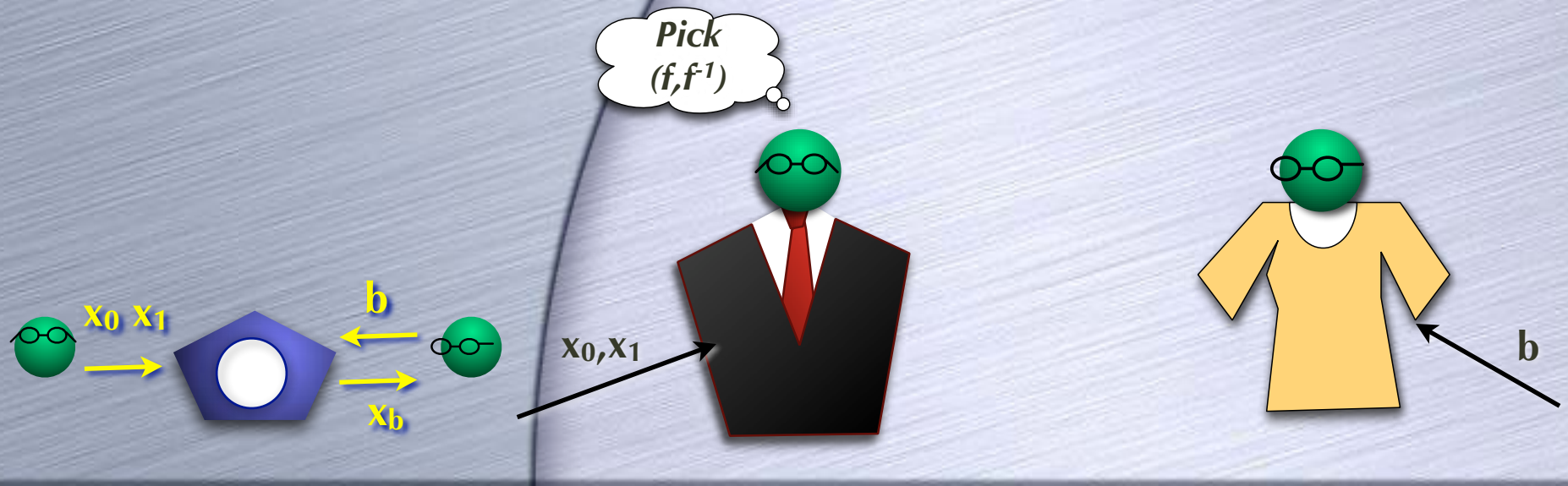

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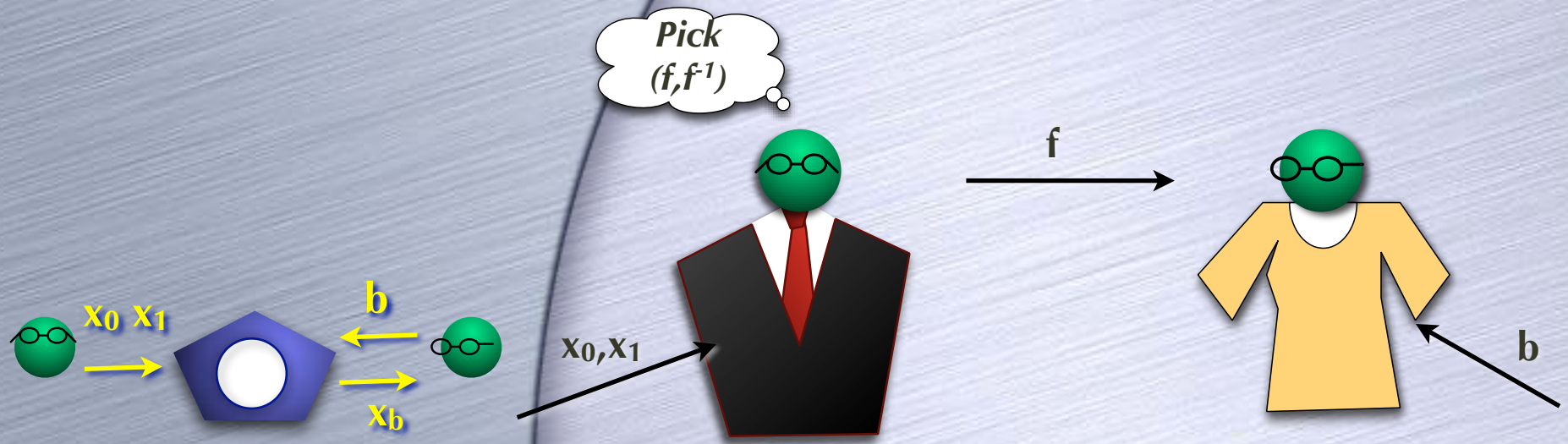

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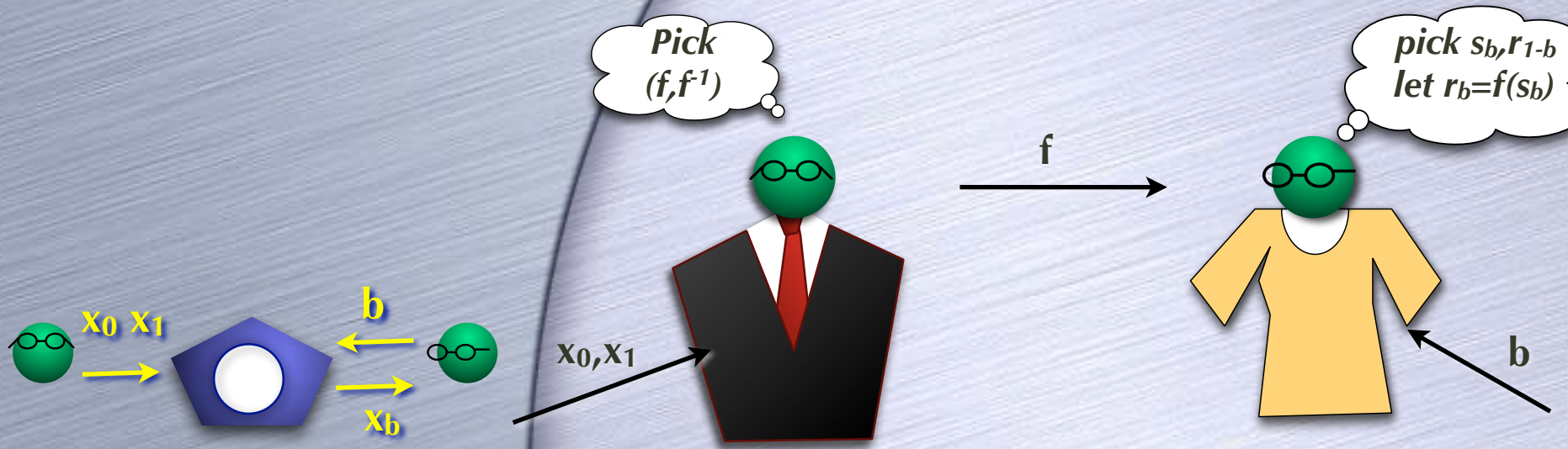

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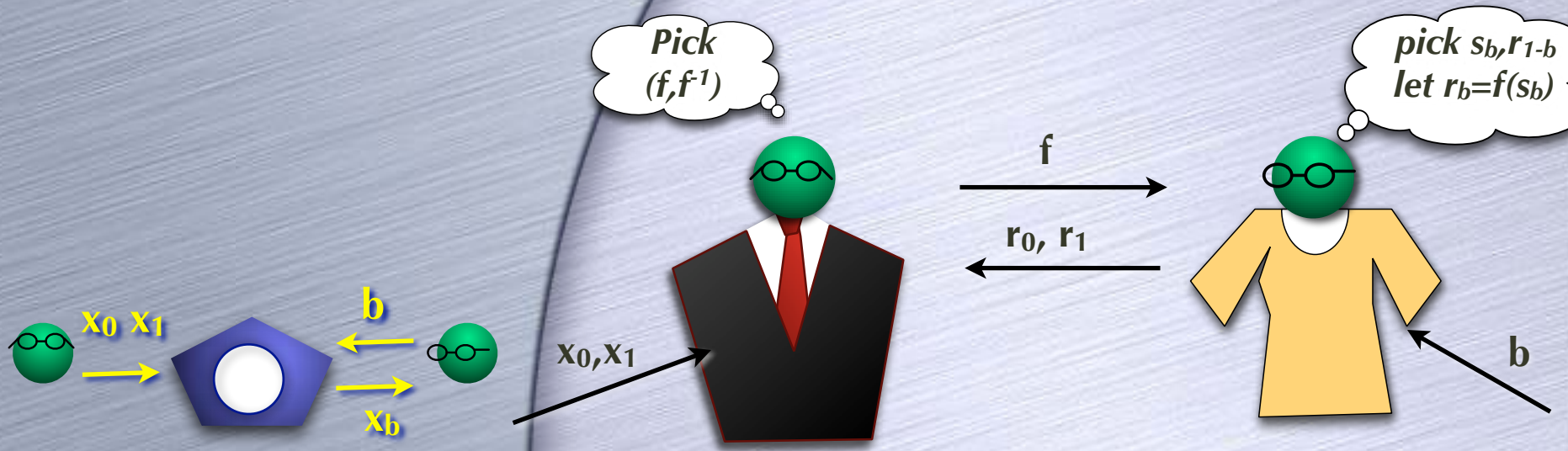

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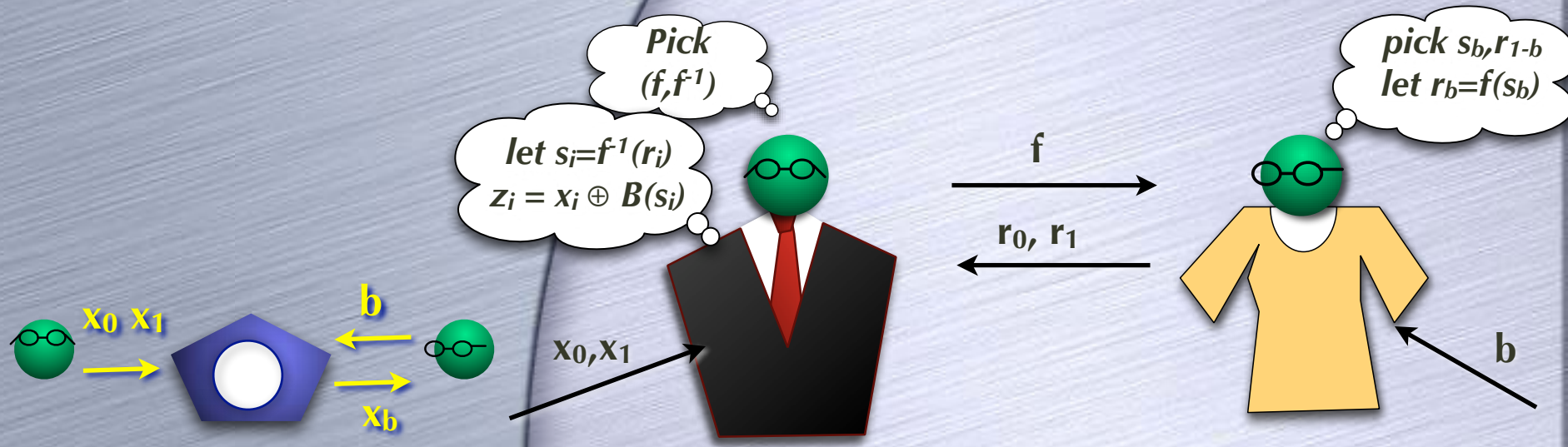

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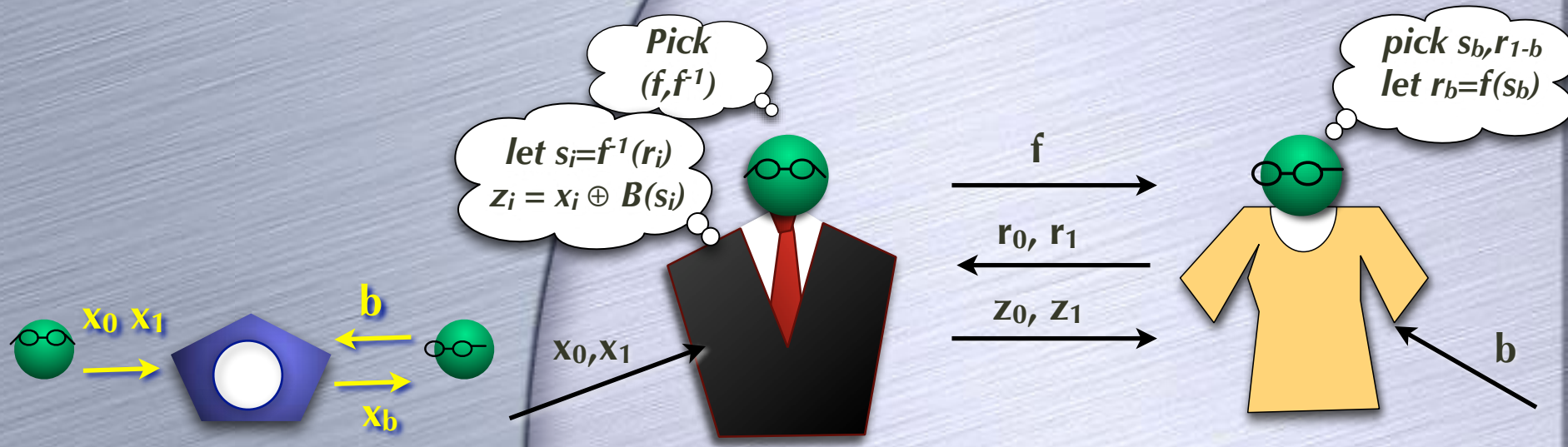

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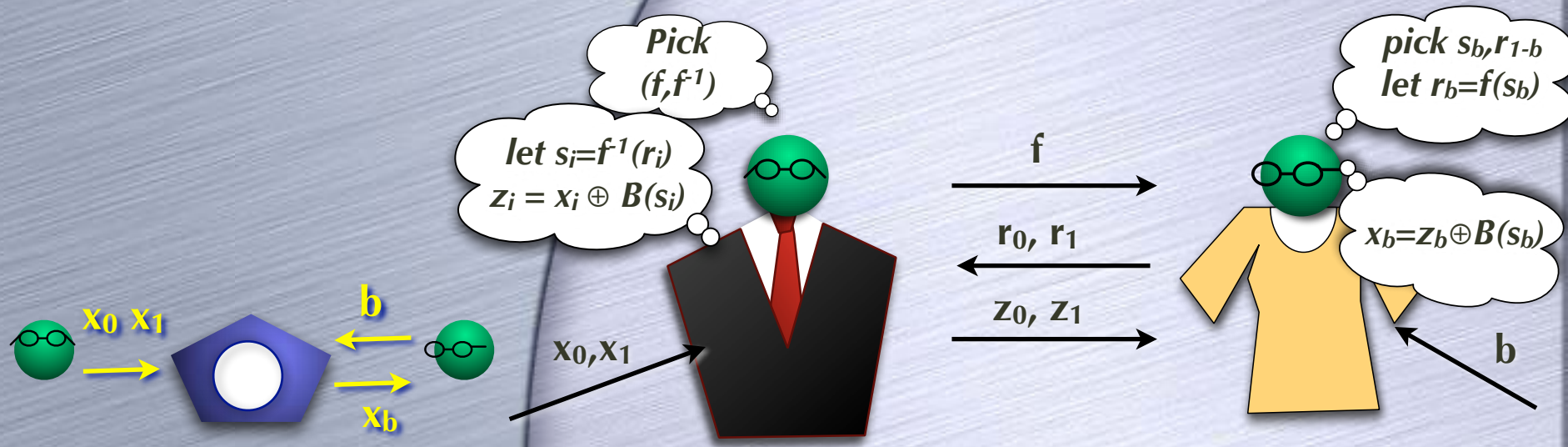

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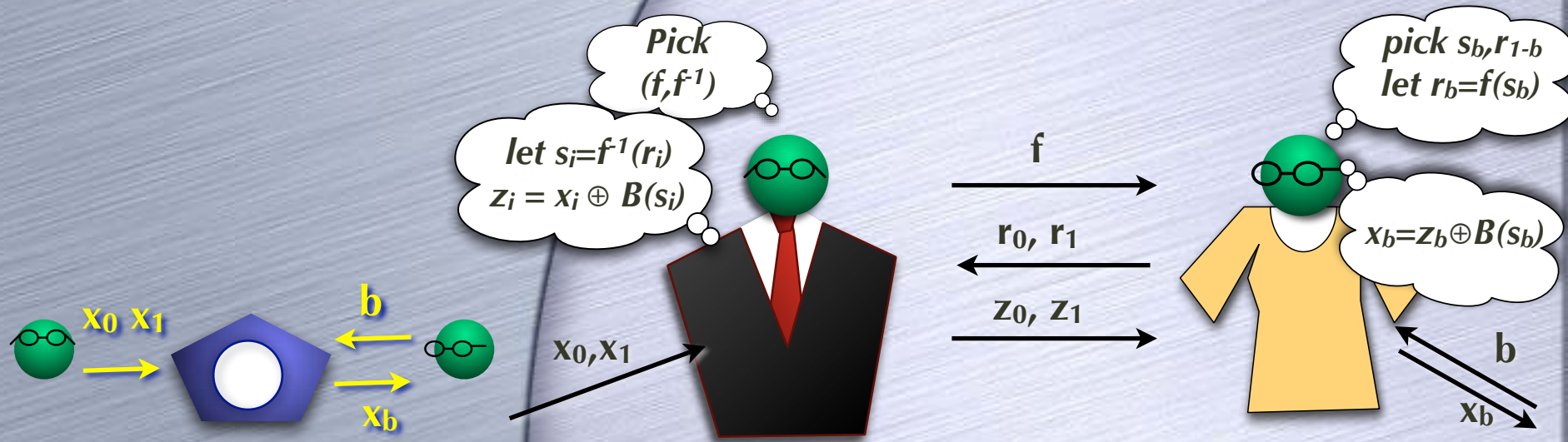

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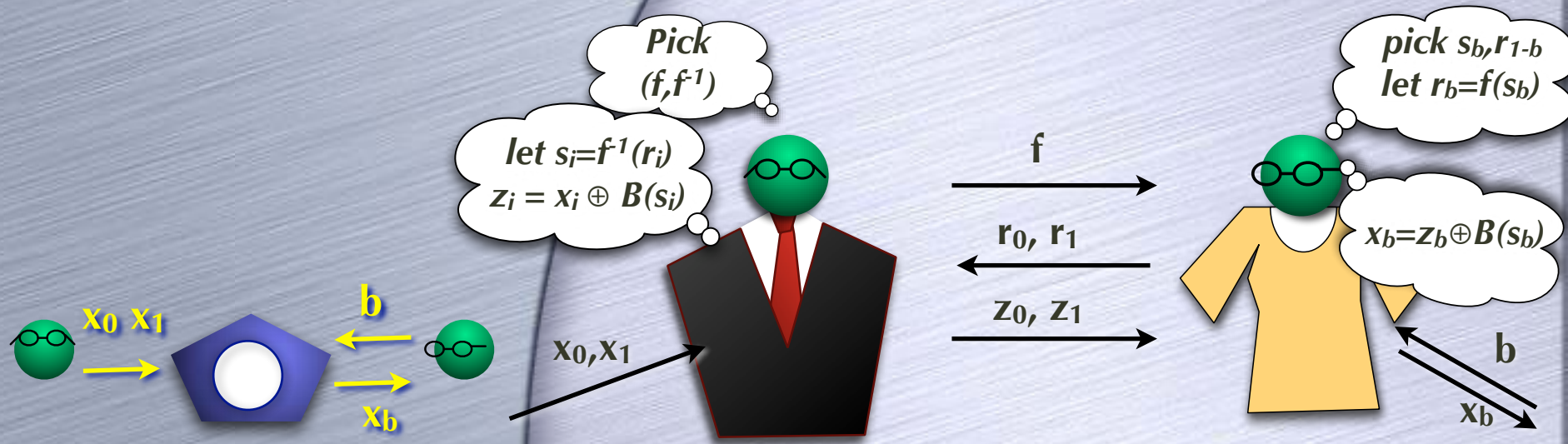

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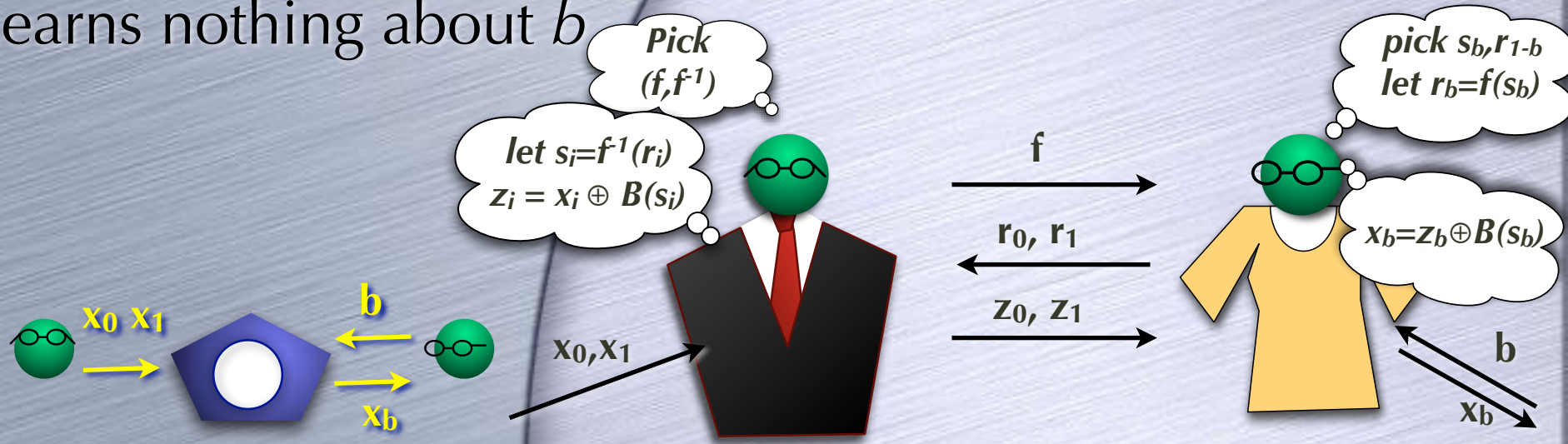

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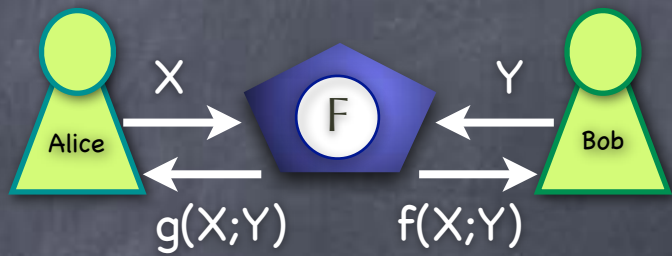
  - Trusted party takes (X;Y). Outputs g(X;Y) to Alice, f(X;Y) to Bob



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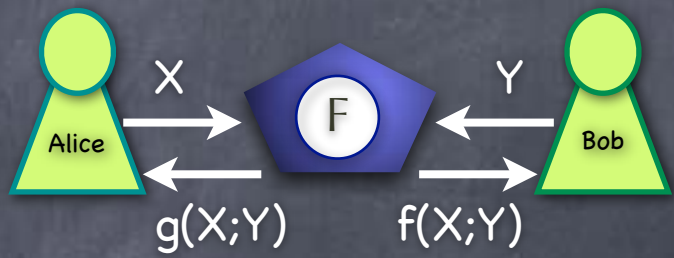

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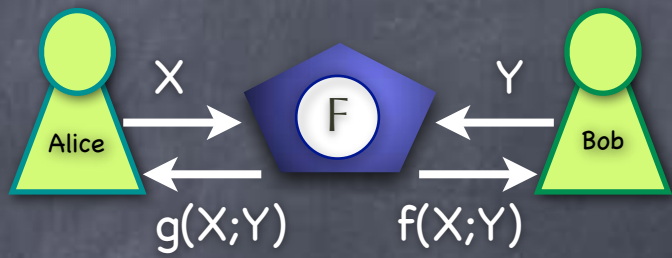

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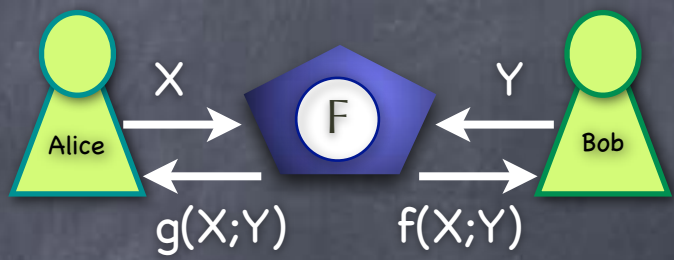

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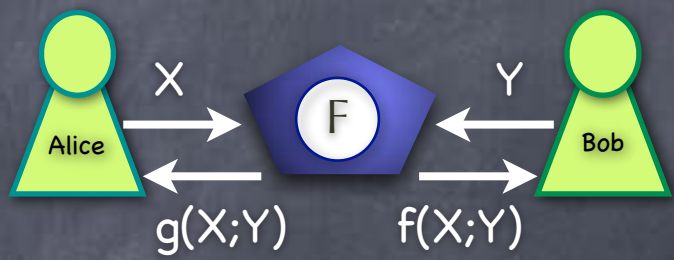

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  - Single-Output SFE: only one party gets any output

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- Fact: OT is complete even for active security

"Completeness" of OT: Proof of Concept

- Single-output 2-party function f

- Alice (who knows x, but not y) prepares a table for $f(x,\cdot)$ with $N = 2^{|y|}$ entries (one for each y)

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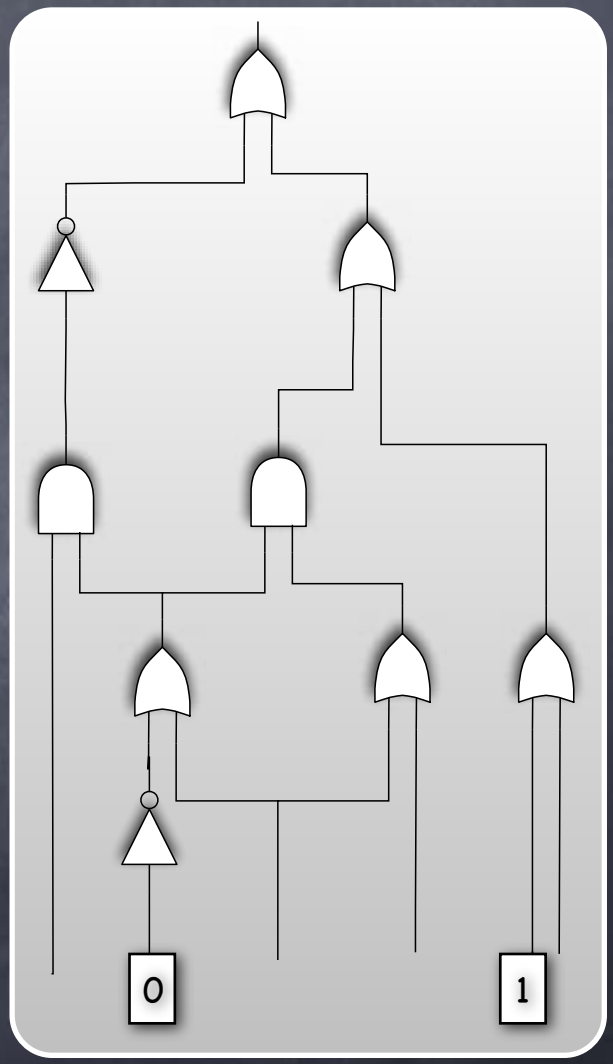
- Problem: N is exponentially large in |y|

Functions as Circuits

- Directed acyclic graph
  - Nodes: AND, OR, NOT, CONST gates, inputs, output(s)
  - Edges: Boolean valued wires
  - Each wire comes out of a unique gate, but a wire might fan-out



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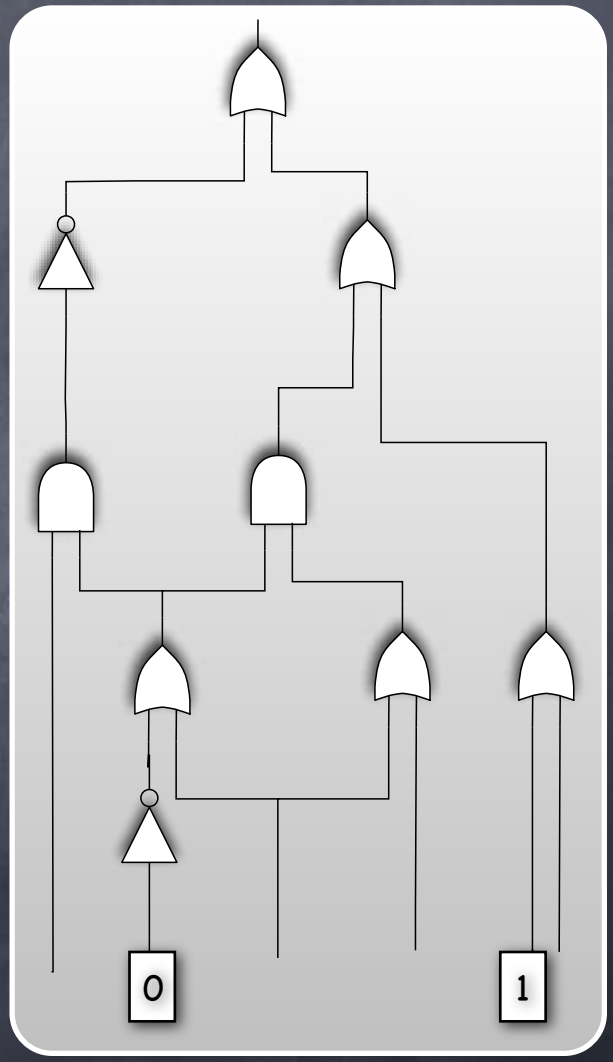

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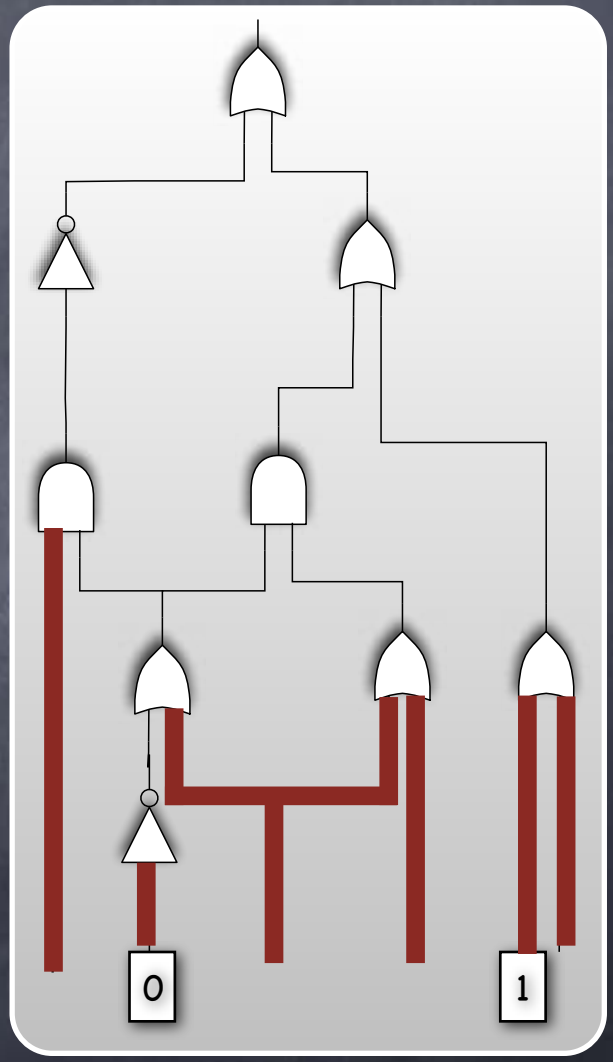

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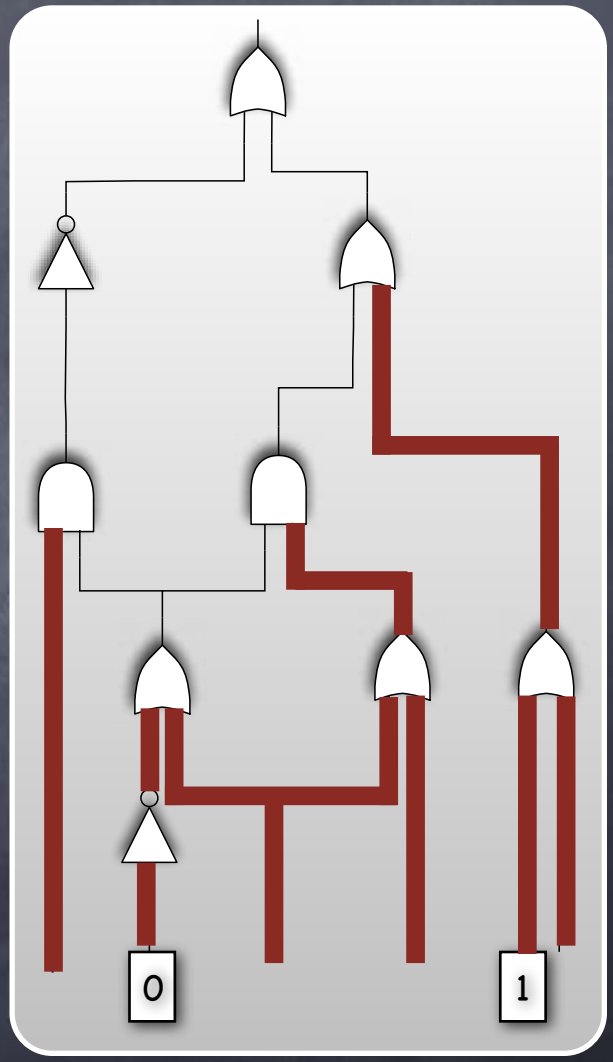

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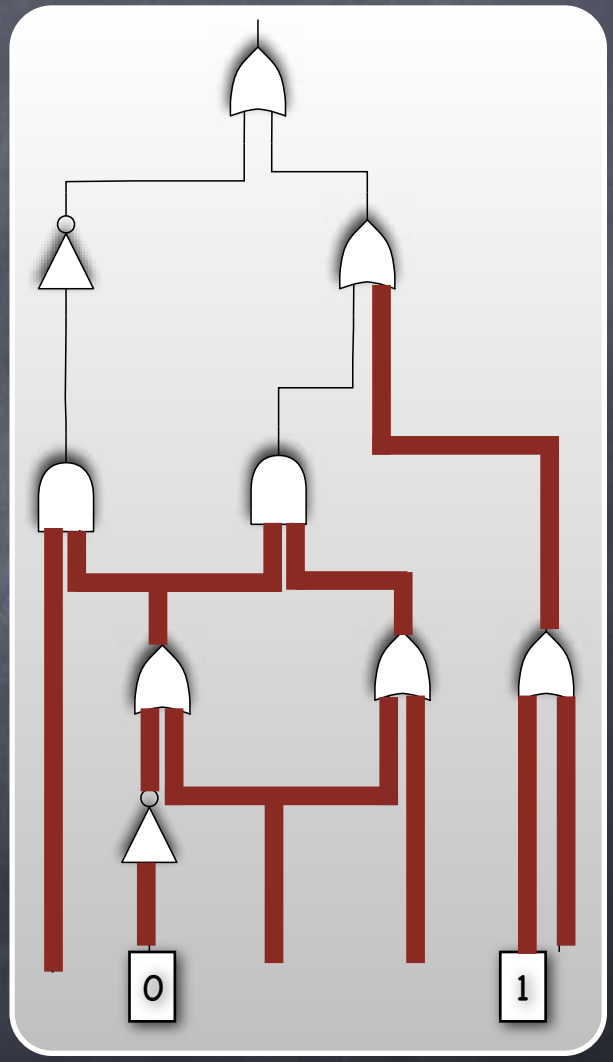

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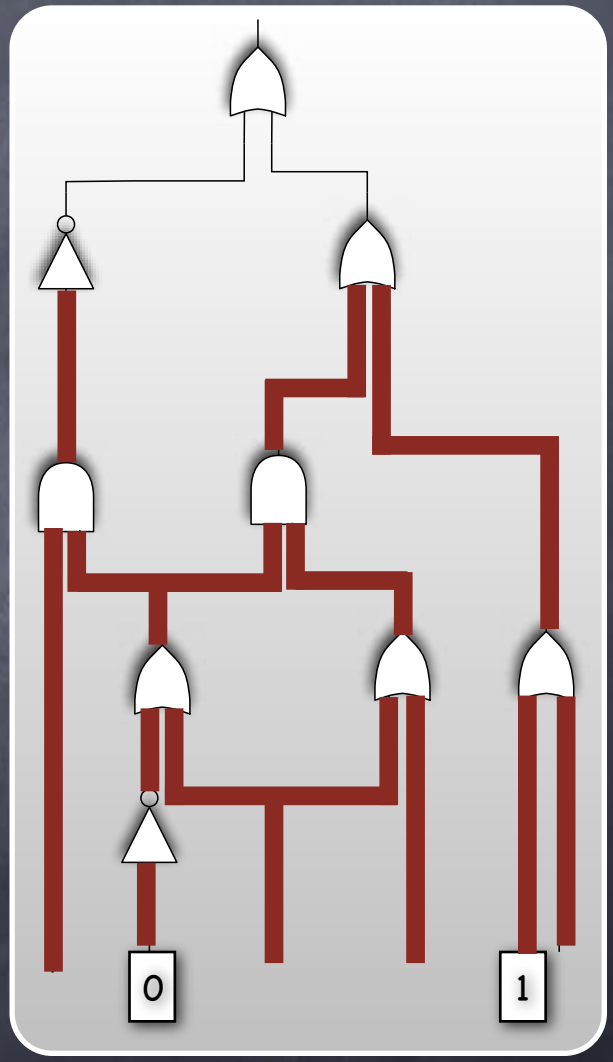

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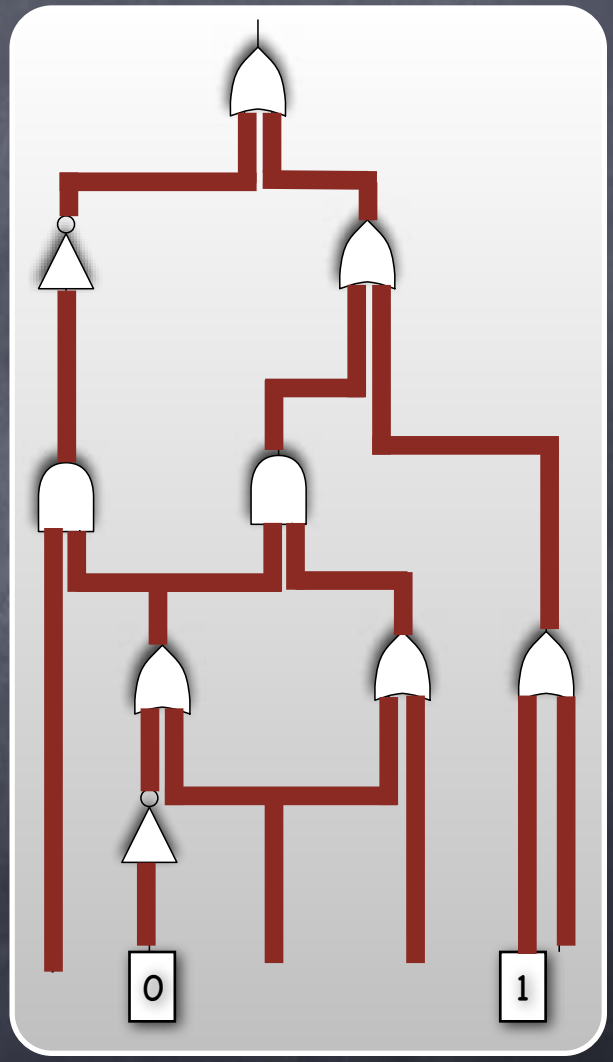

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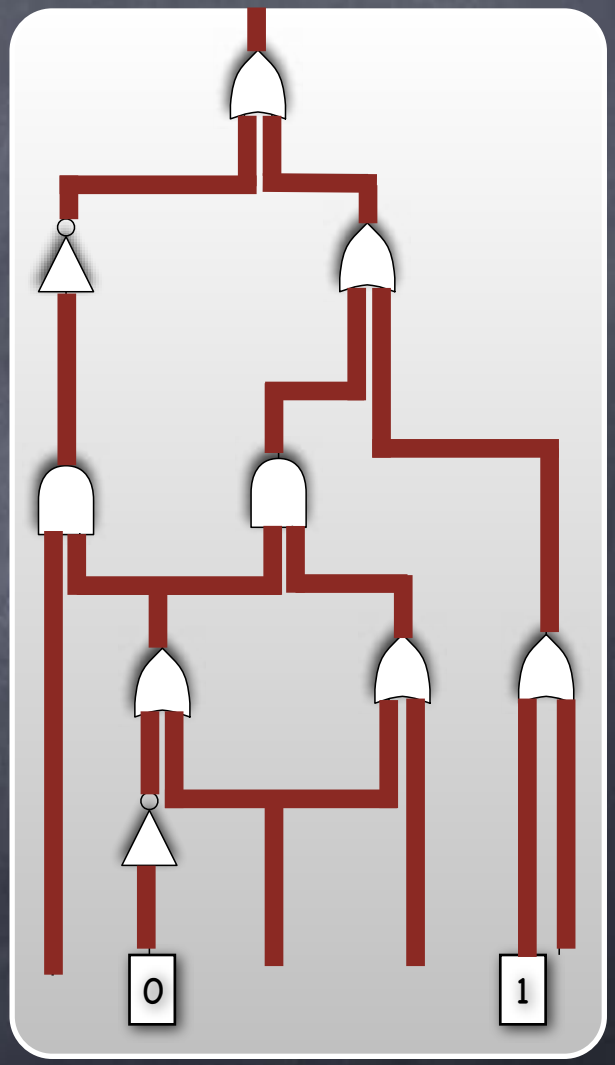

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- Interesting problems already given as succinct programs/circuits

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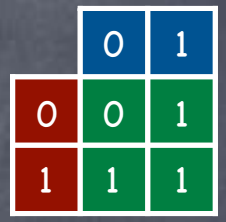

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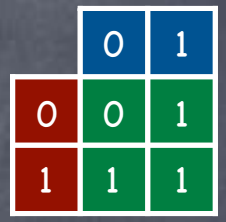

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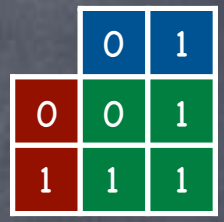
  - Alice holds x=a, Bob has y=b; Bob should get OR(x,y)

A Physical Protocol



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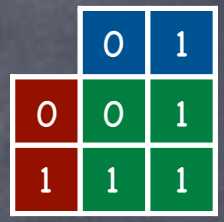

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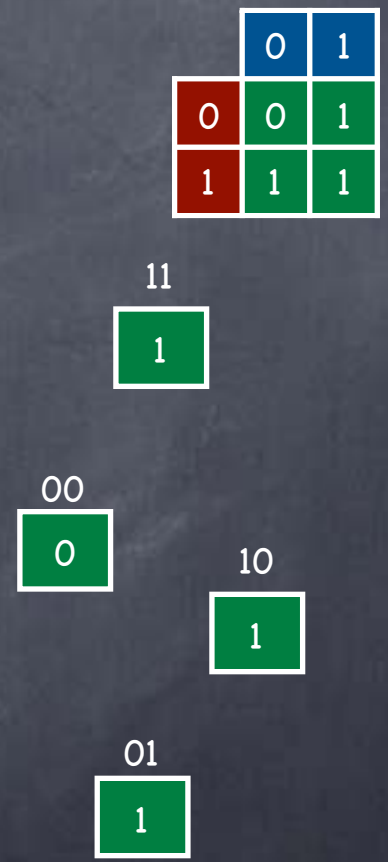

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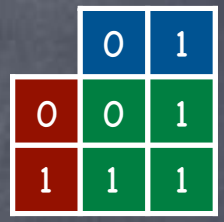

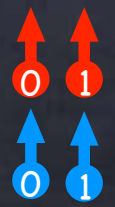

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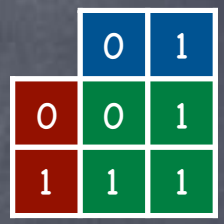

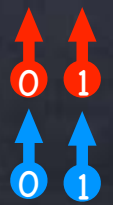

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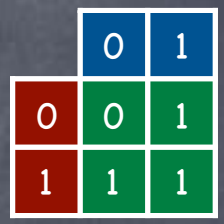

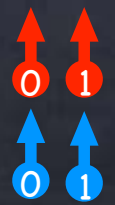

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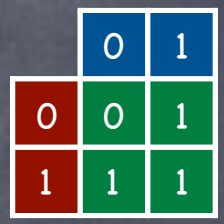

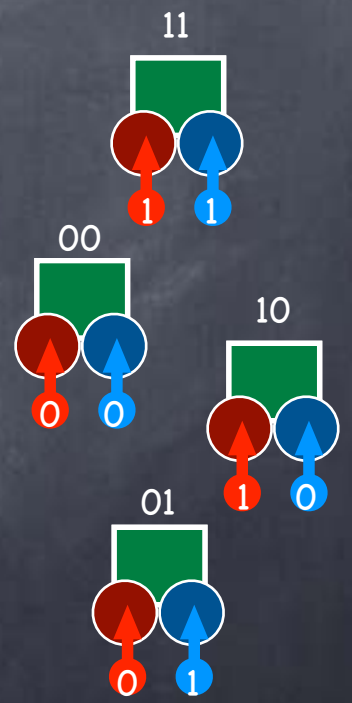

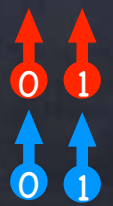

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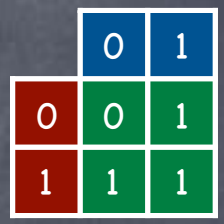

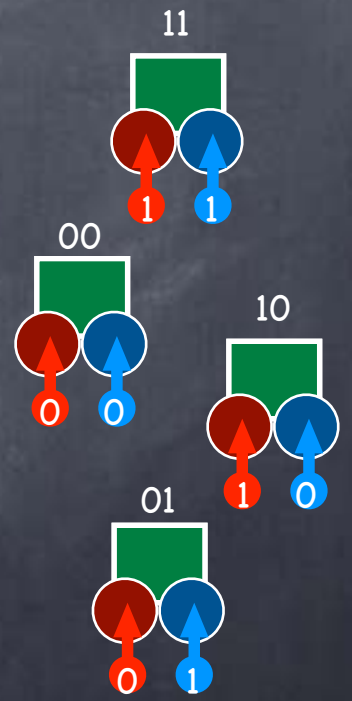

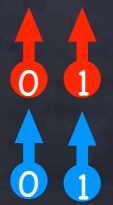

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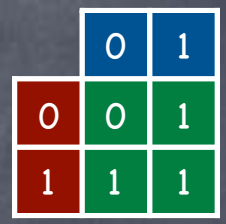

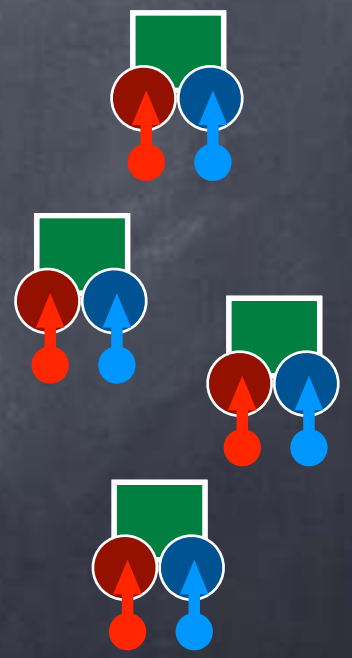

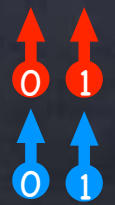

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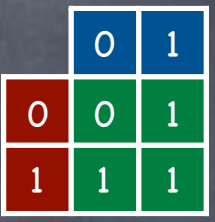

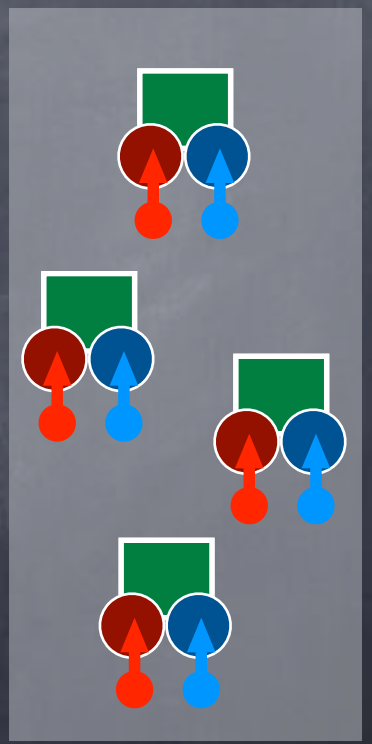

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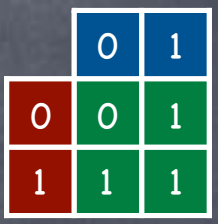

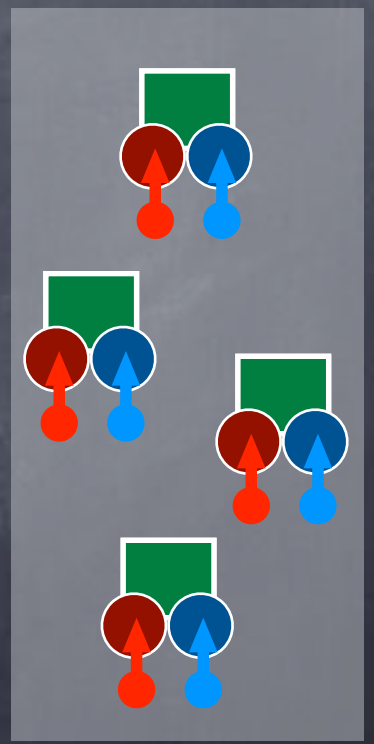

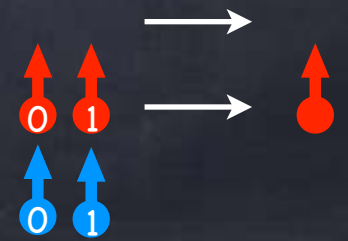

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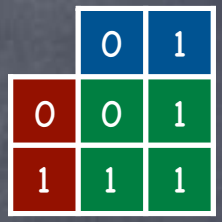

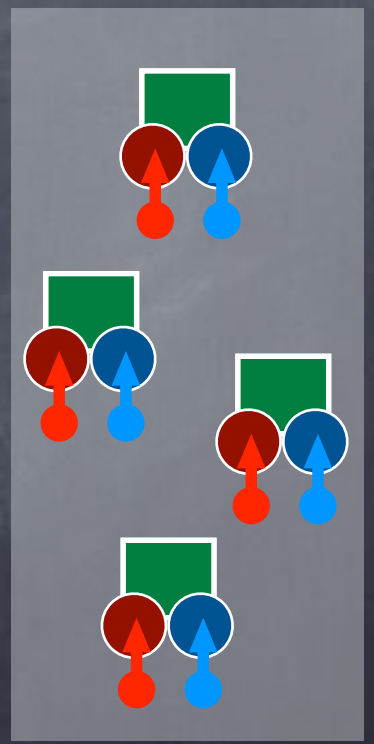

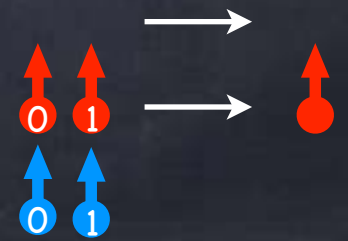

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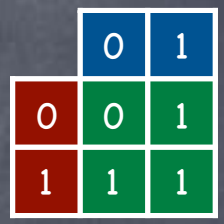

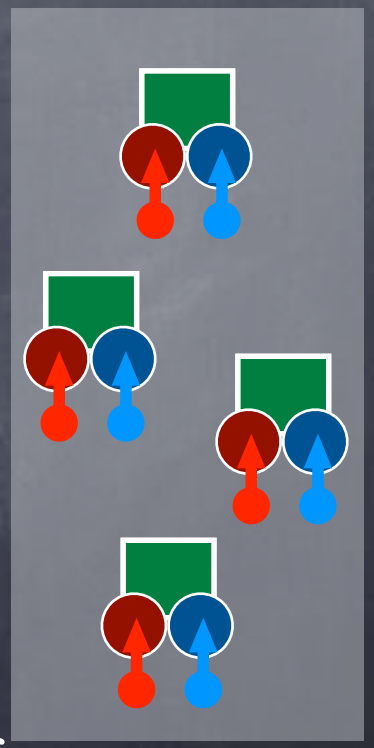

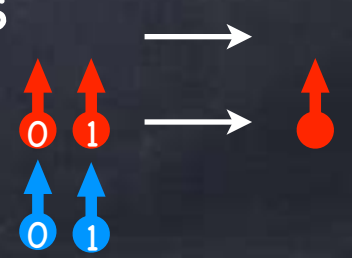

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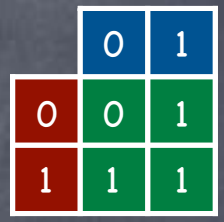

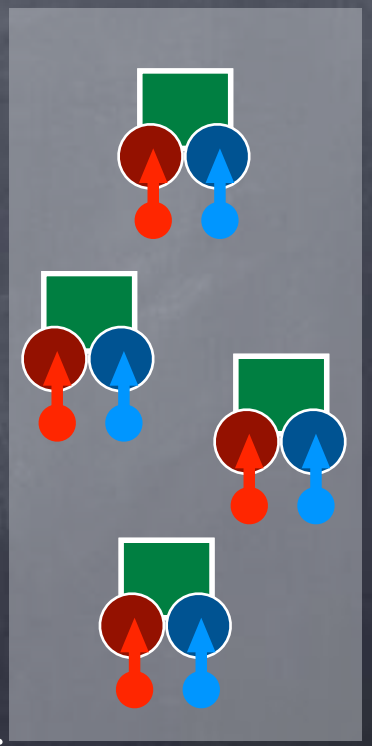

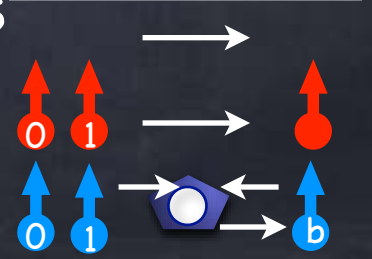

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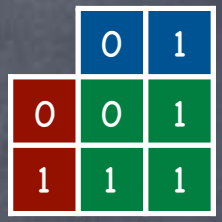

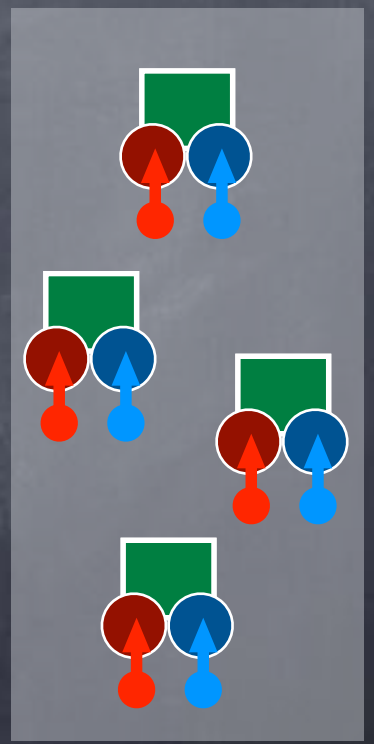

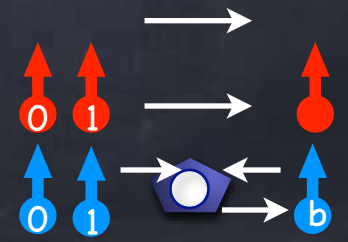

A Physical Protocol

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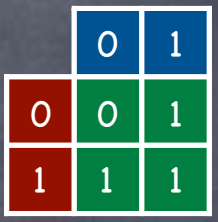

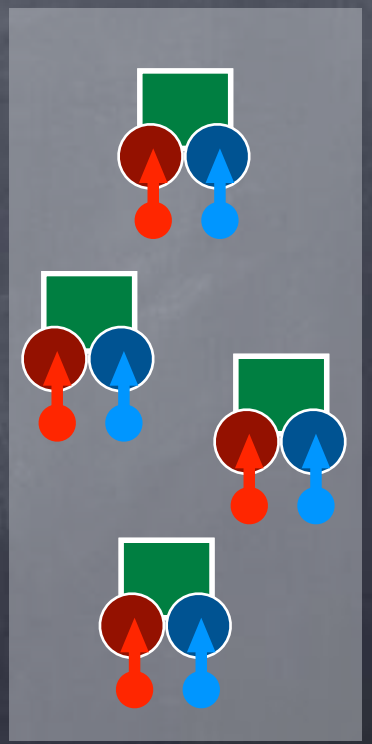

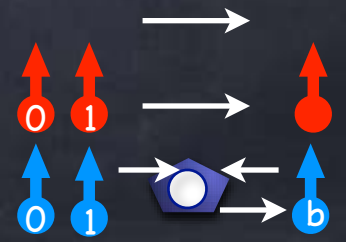

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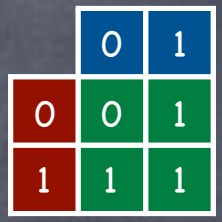

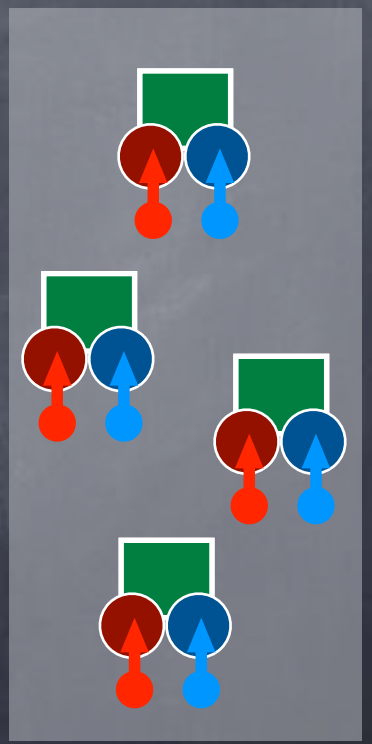

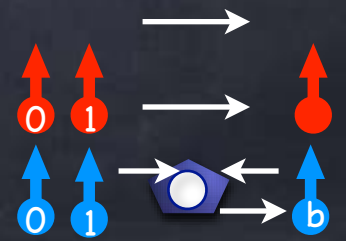

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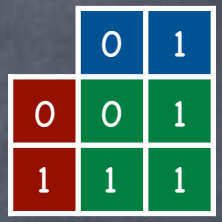

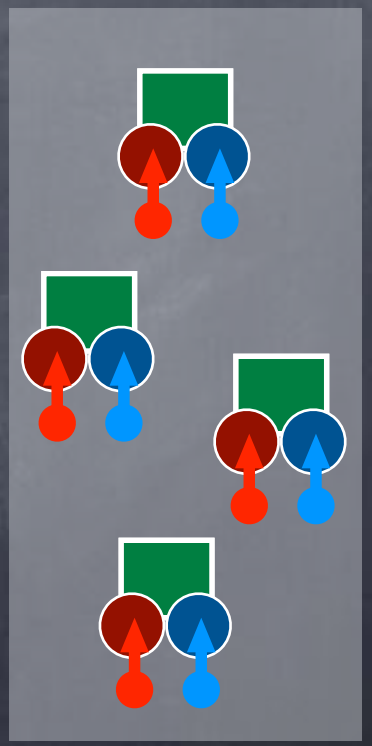

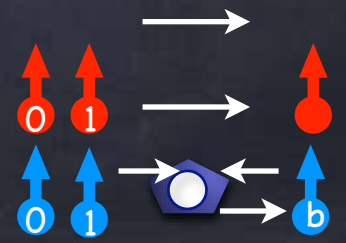

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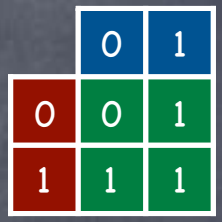

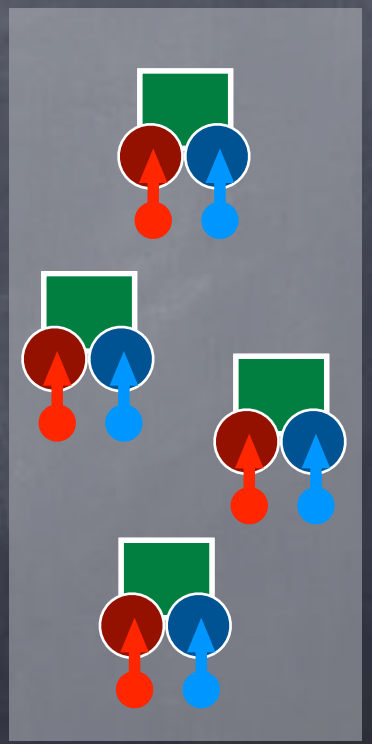

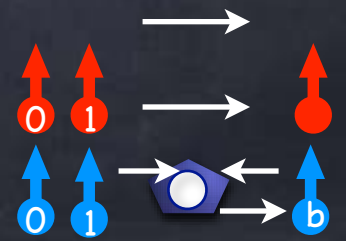

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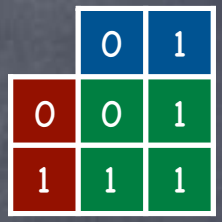

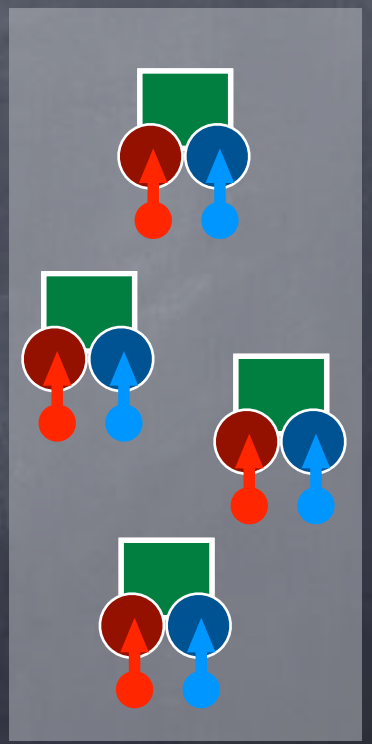

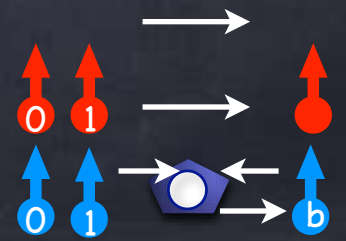

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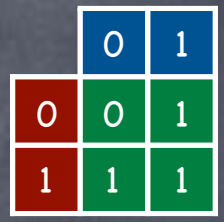
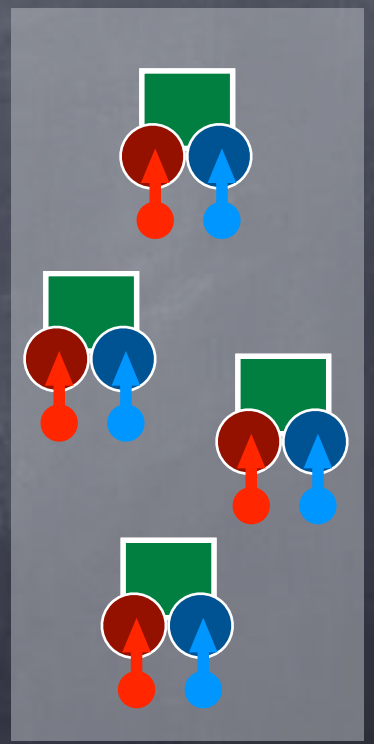
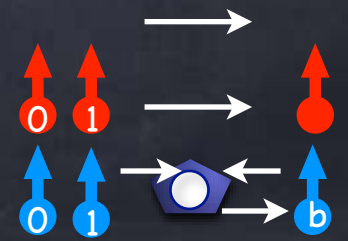
  - Note when y=1, cases x=0 and x=1 appear same







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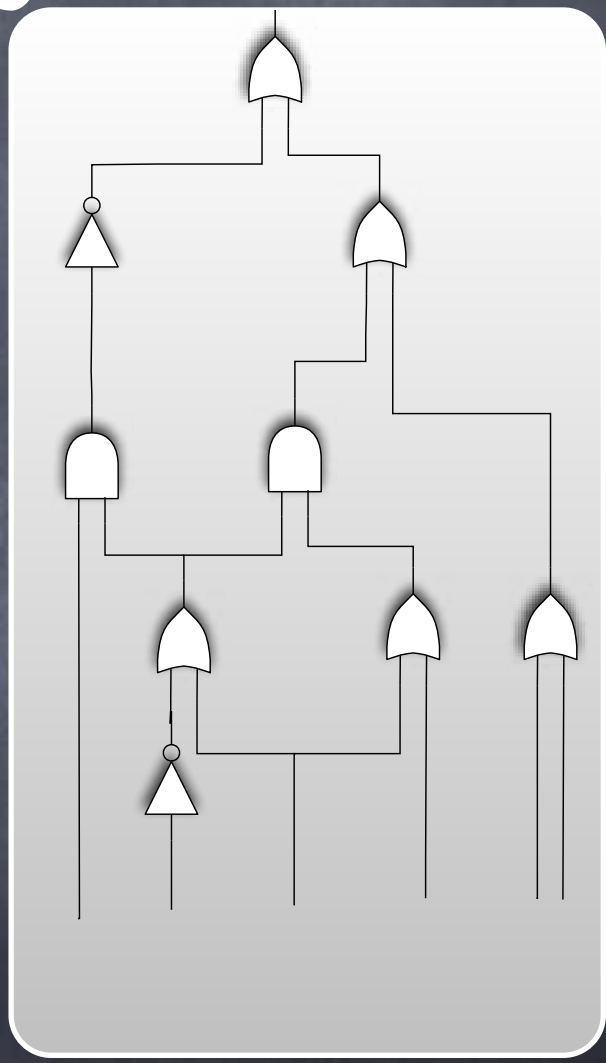

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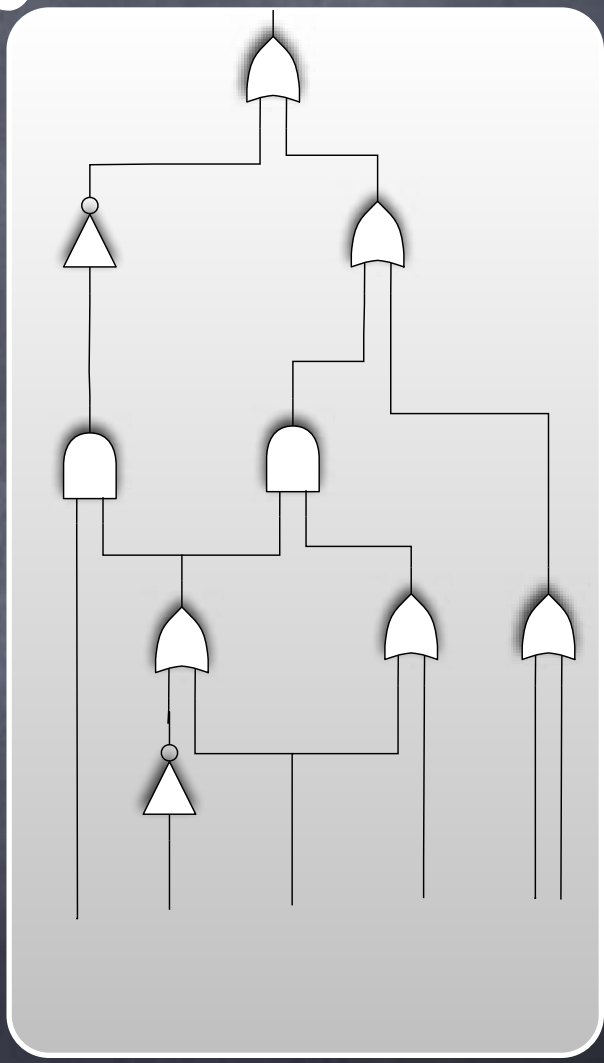

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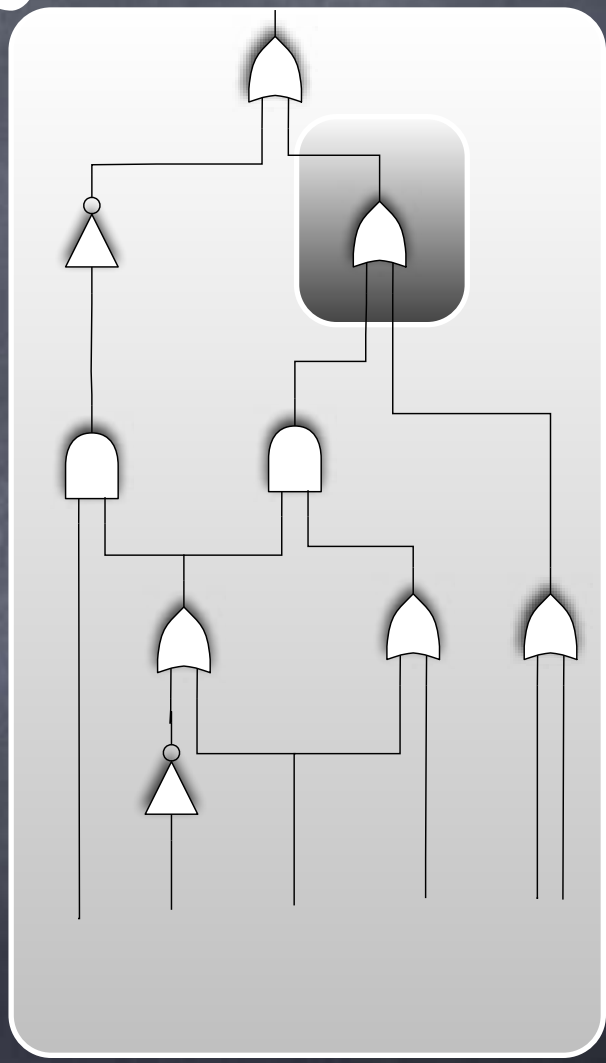

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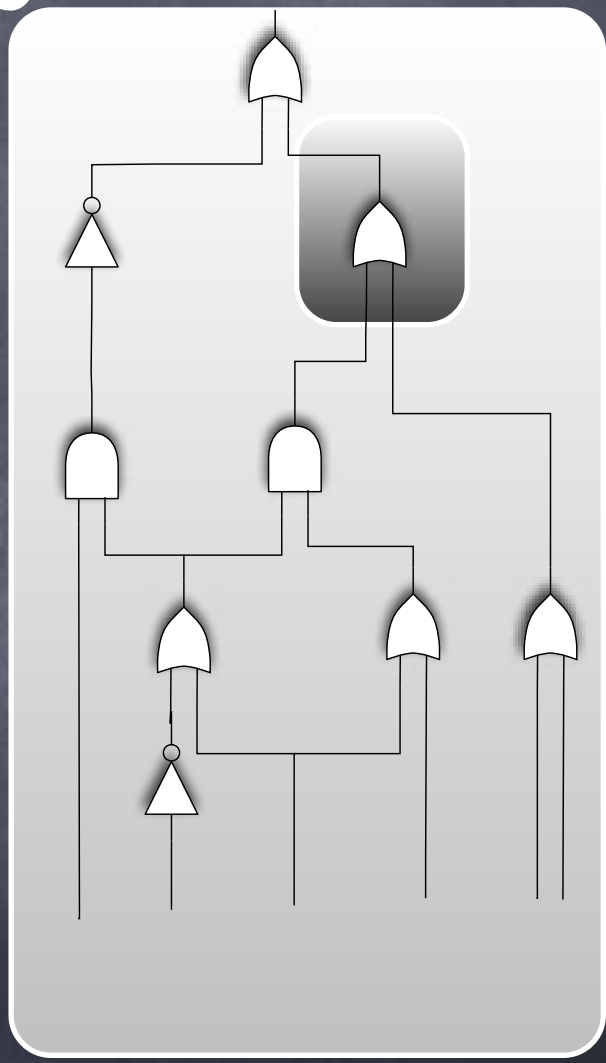

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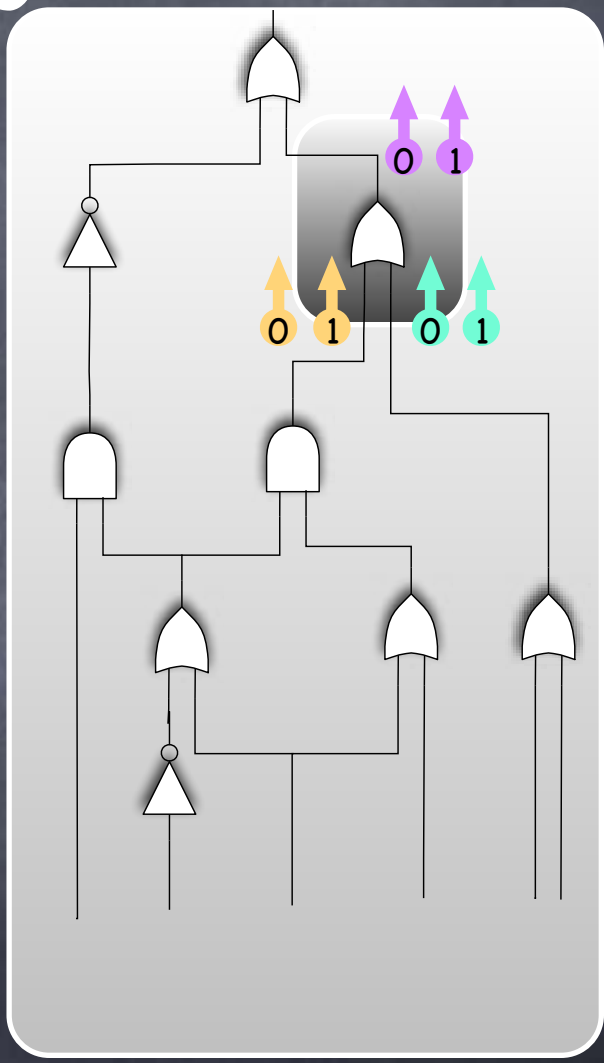

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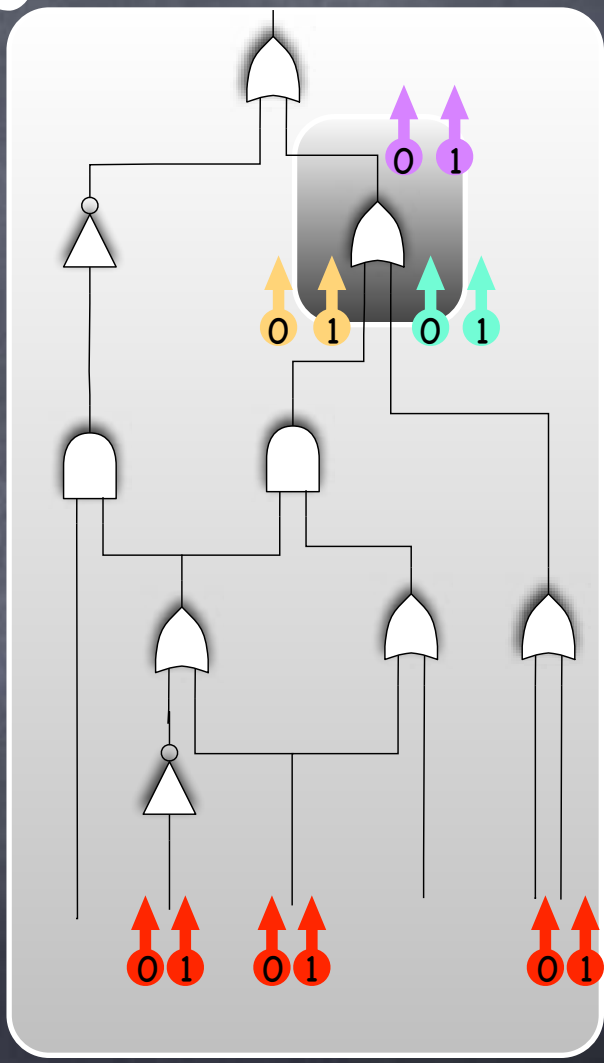

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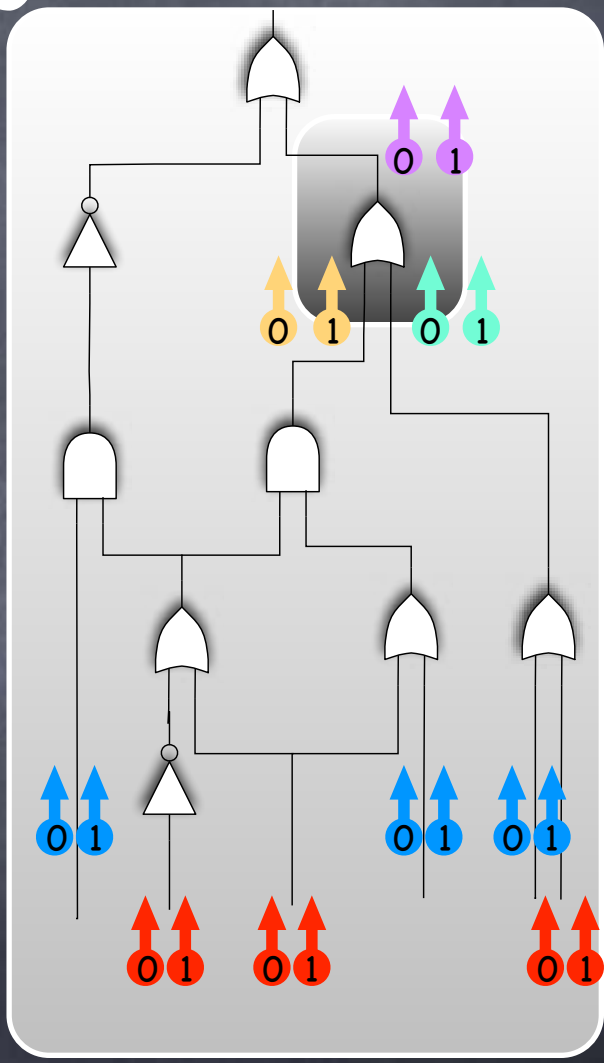

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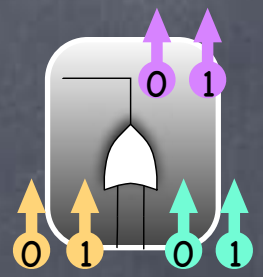

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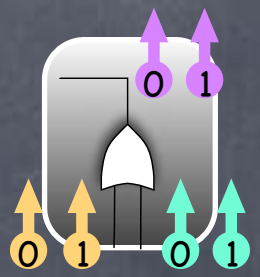

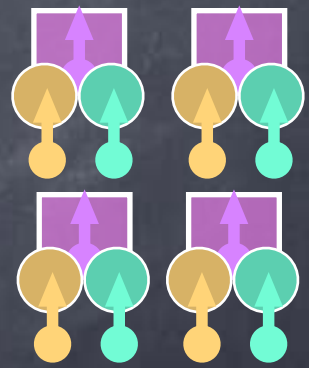

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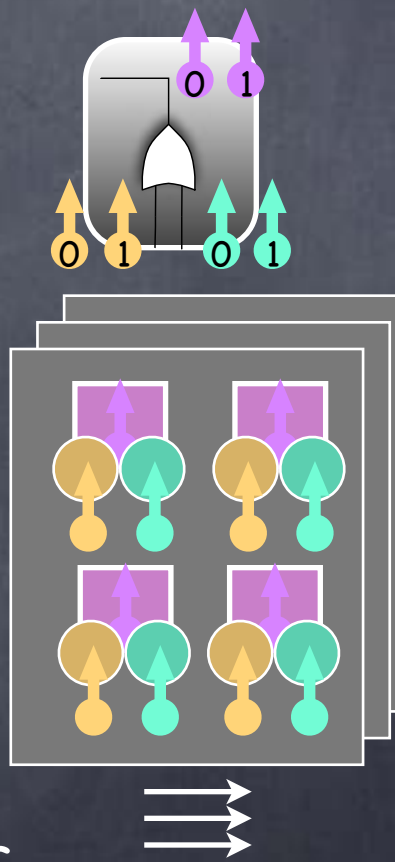

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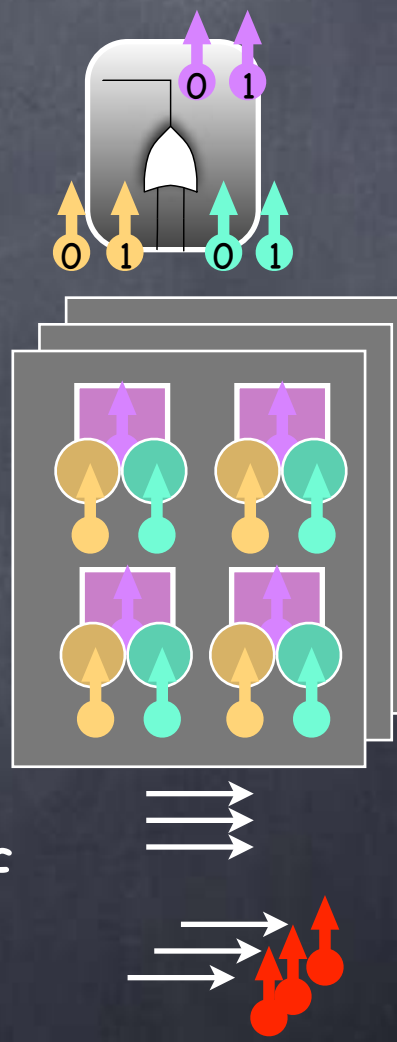

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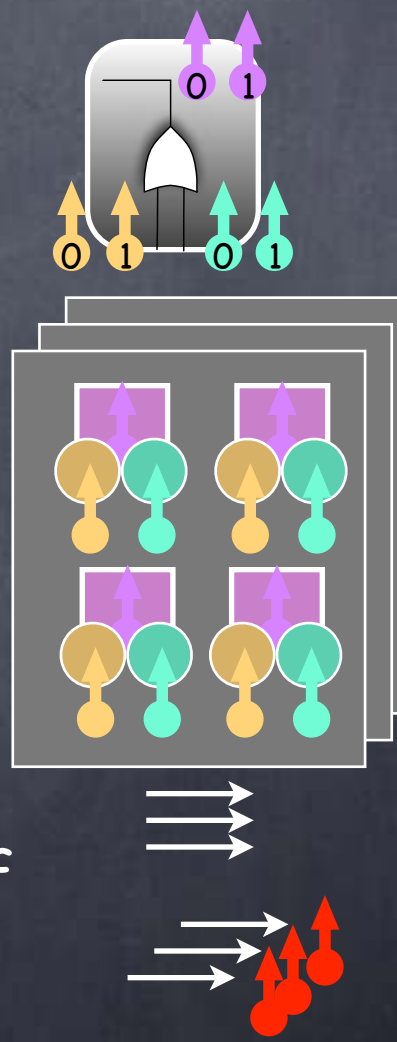

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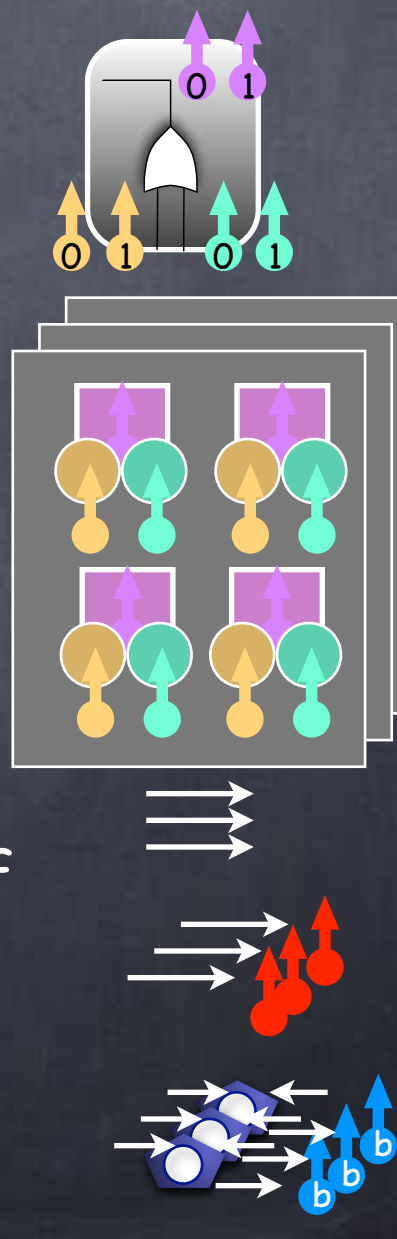

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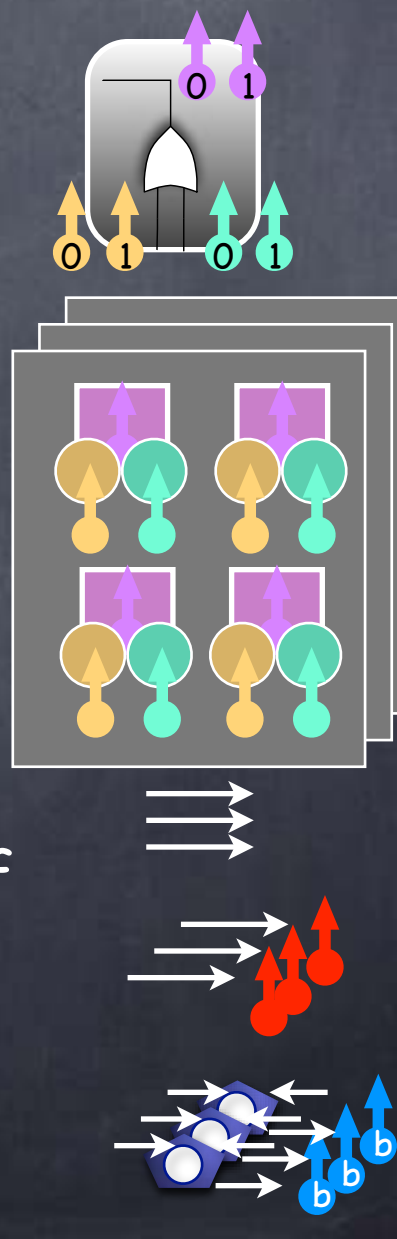

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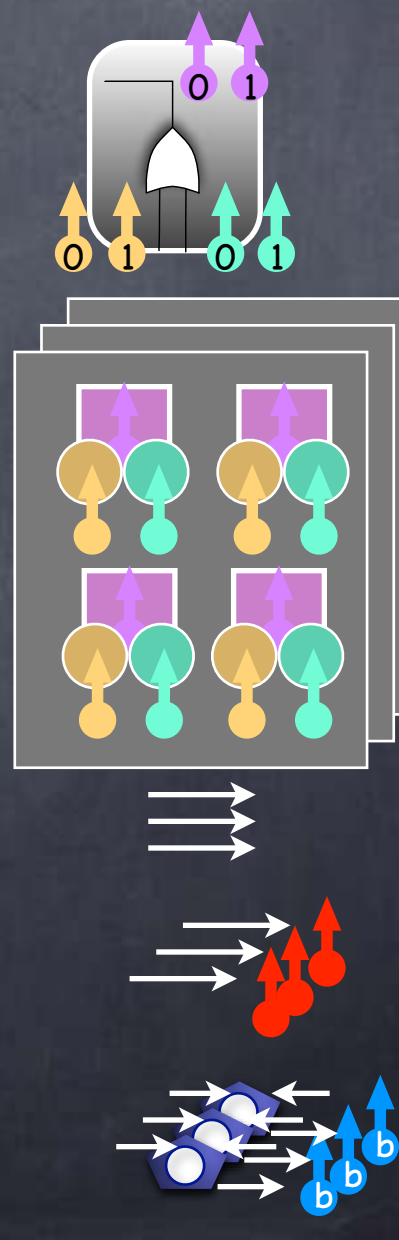

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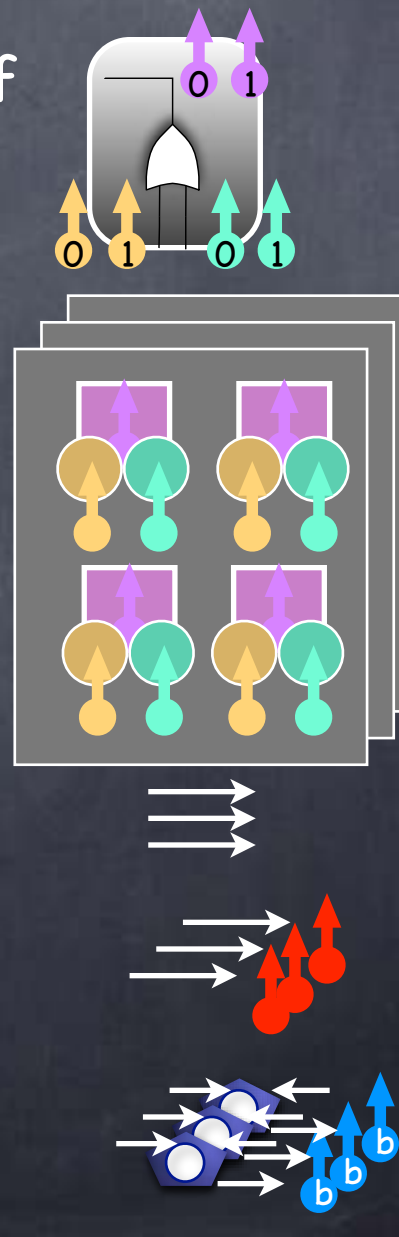

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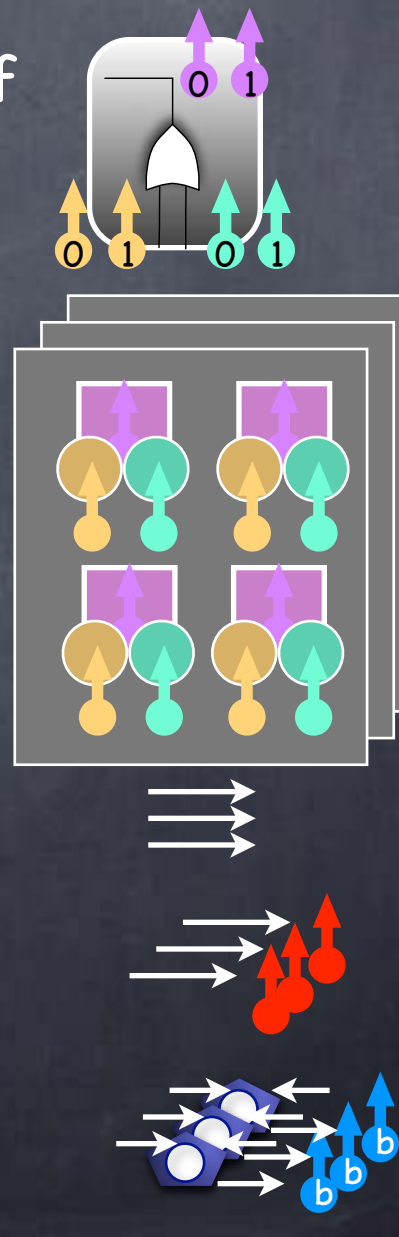

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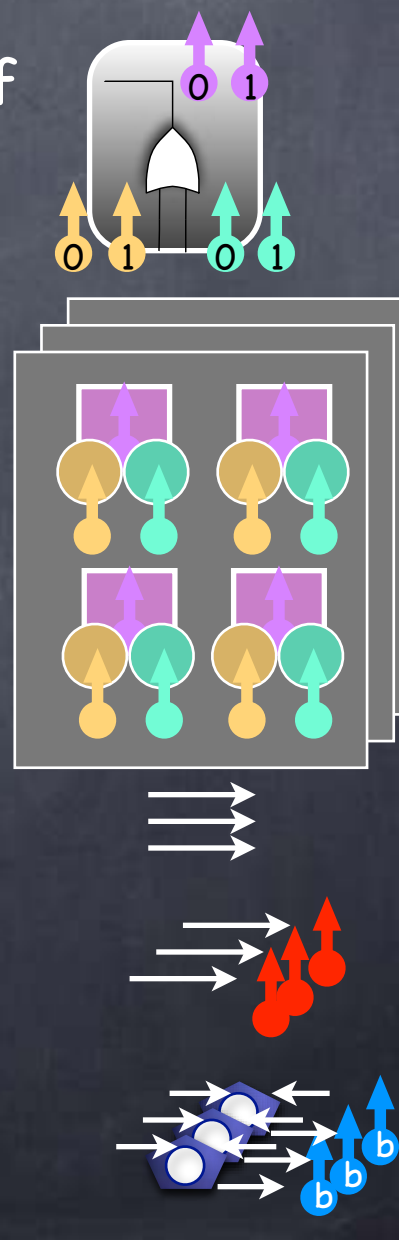

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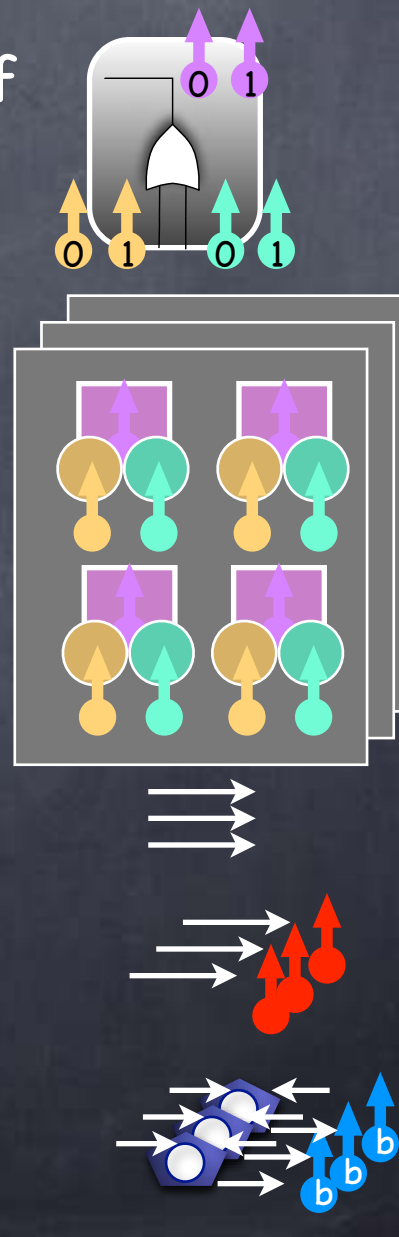

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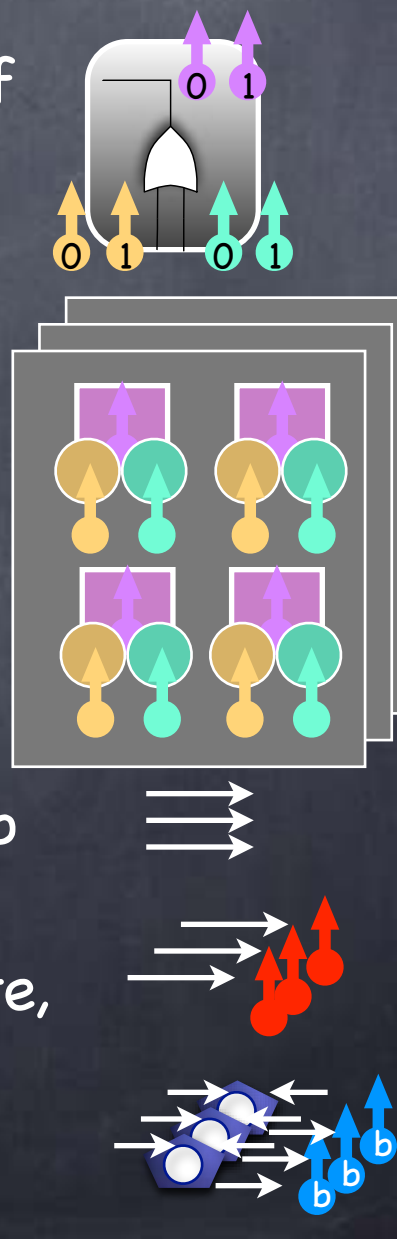

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- Much more efficient than the proof of concept protocol, but relies on one-way functions (PRG) in addition to OT

Today

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  - Yao's Garbled Circuit
  - Using OT and IND-CPA encryption
    - OT using TOWP
  - Composition (implicitly)
- Coming up: More protocols. More composition.