Digital Signatures

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Lecture 13

Claude Shannon

Alan Turing

Merkle, Hellman & Diffie

Shamir , Rivest & Adleman
Turing Award ‘02

Manuel Blum
Turing Award ‘95

Andrew Yao
Turing Award ‘00

Goldwasser & Micali
Turing Award ‘12

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Lamport's One-Time Signature

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    - This can then be used to build a full-fledged signature scheme starting from one-time signatures (skipped)

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  - "Standard schemes" like RSA-PSS are based on this

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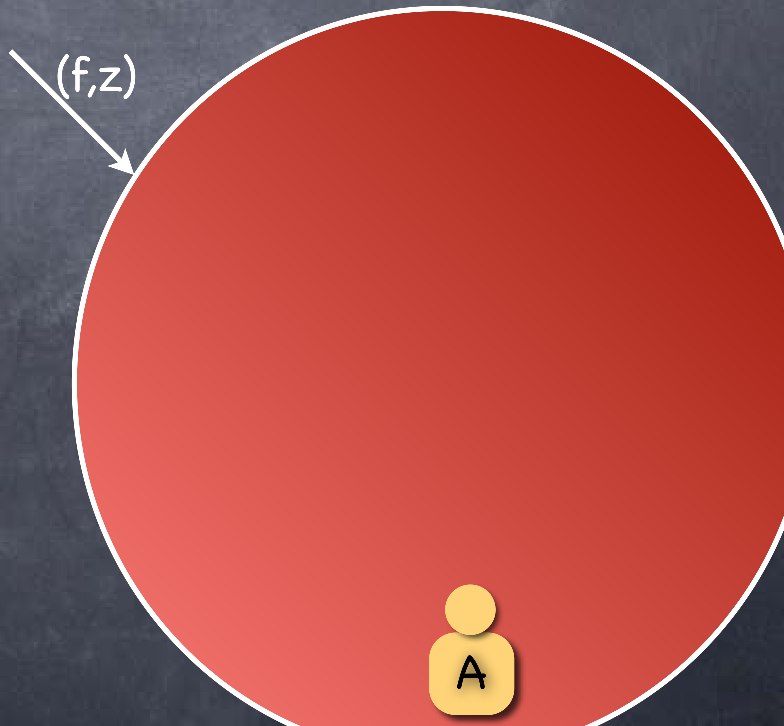

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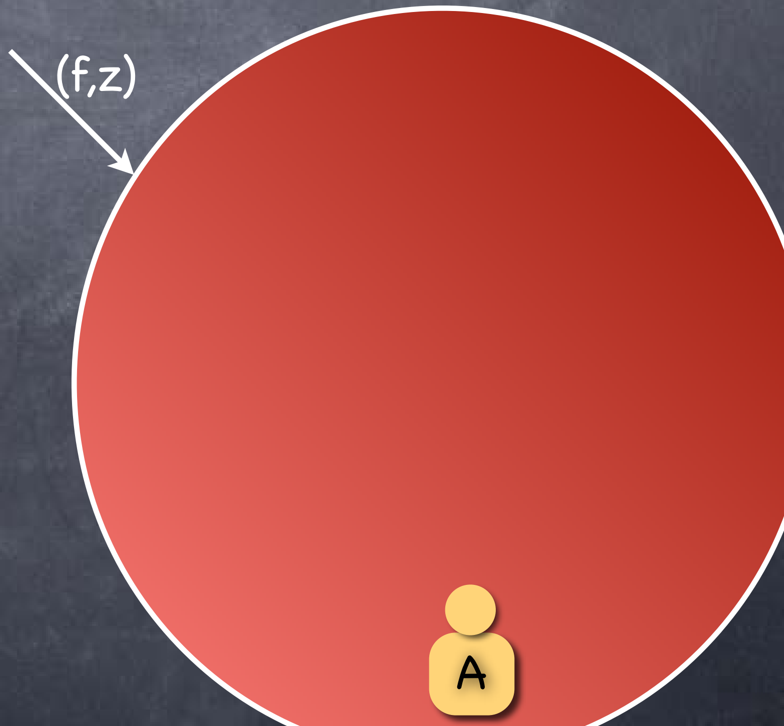

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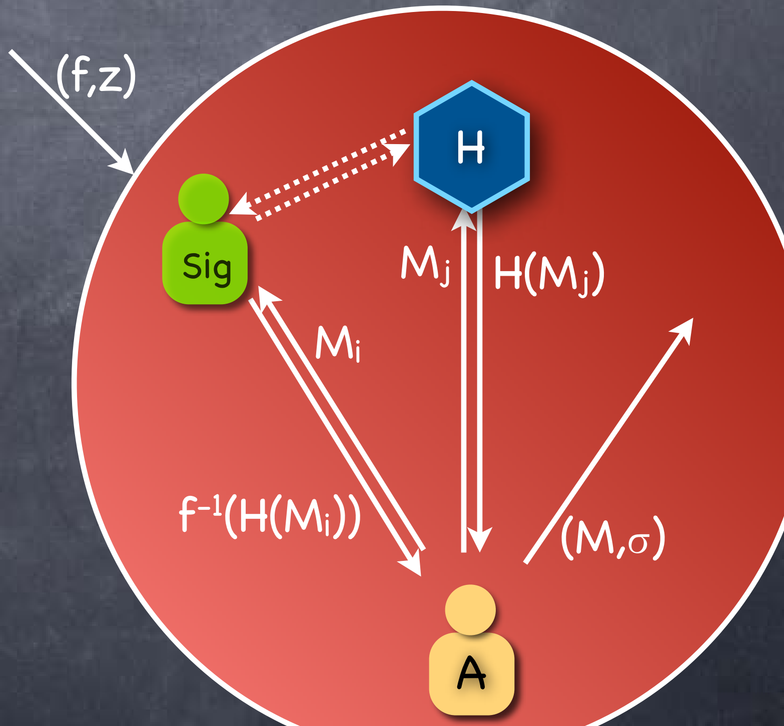

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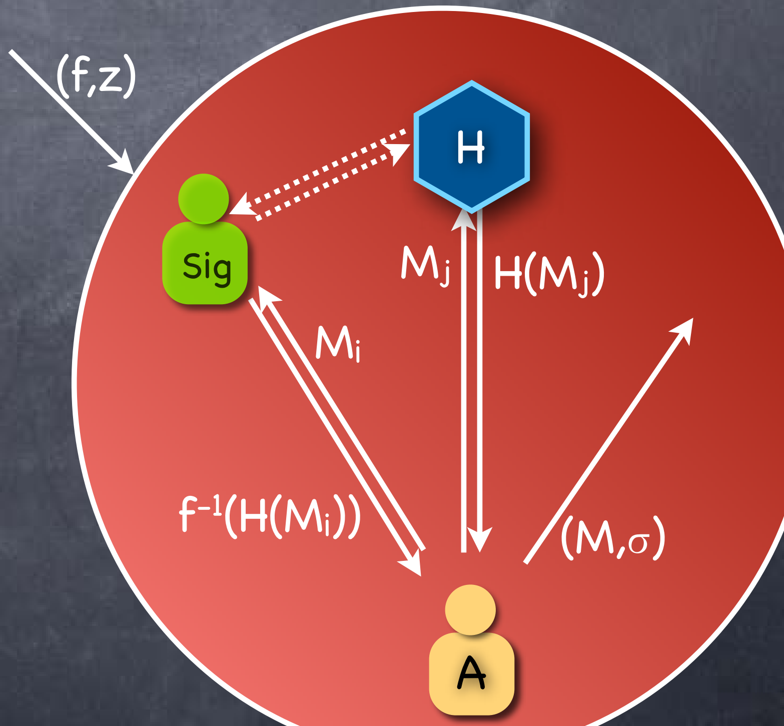

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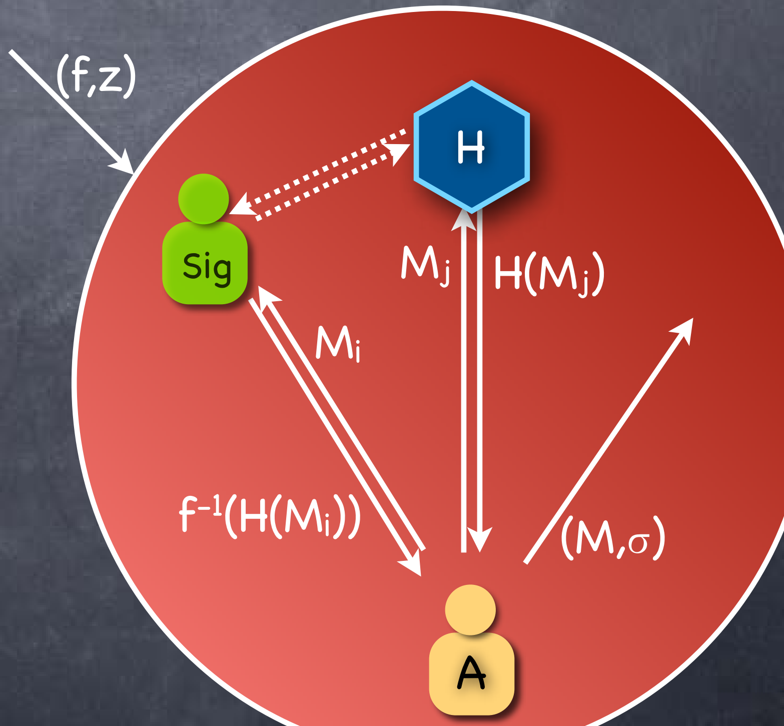

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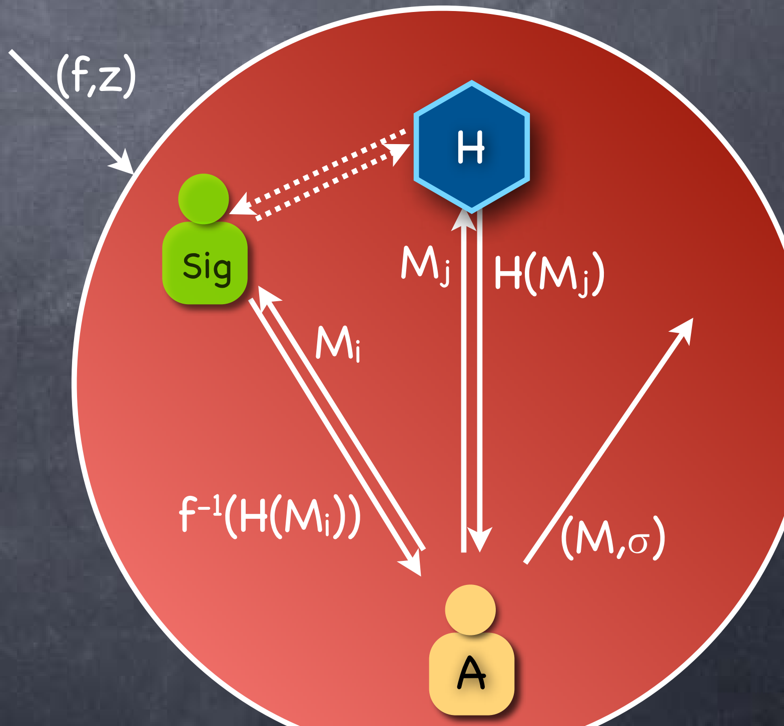

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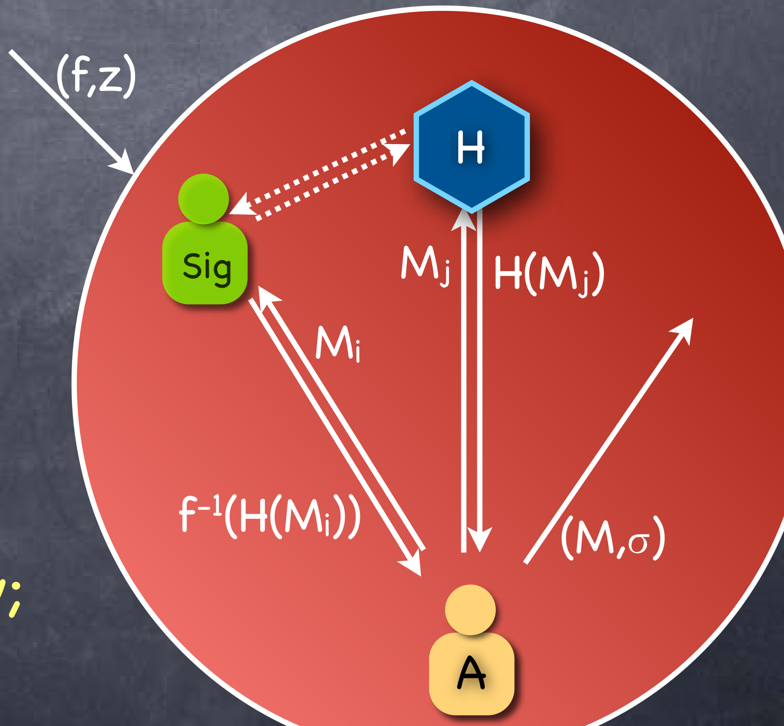
    - A* picks H(M) as x=f(y) for random y; then Sign(M) = $f^{-1}(x)$ = y



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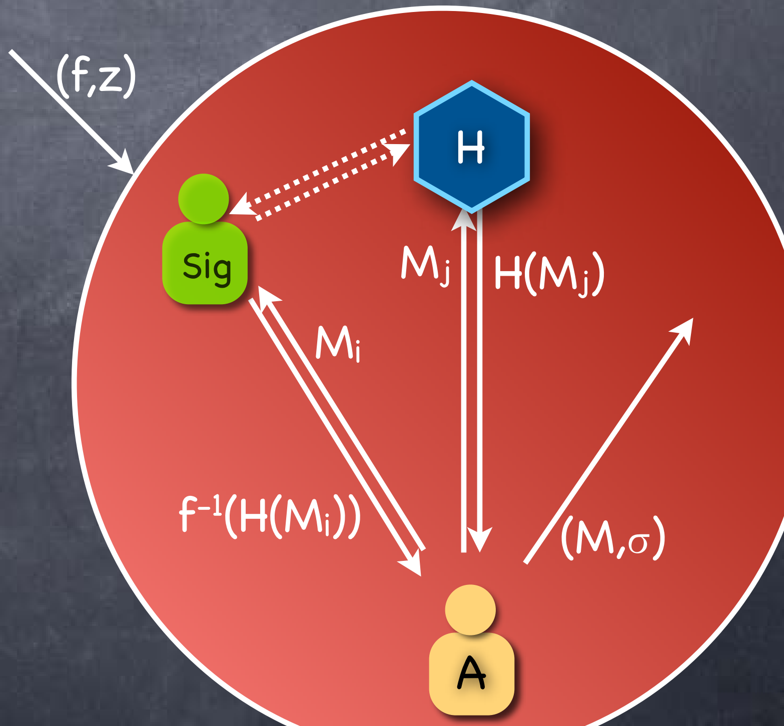

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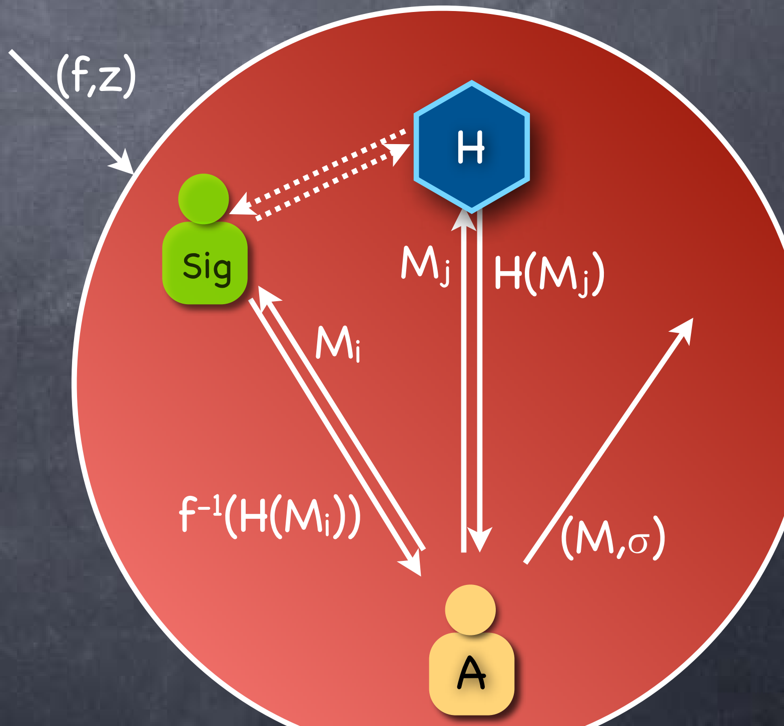

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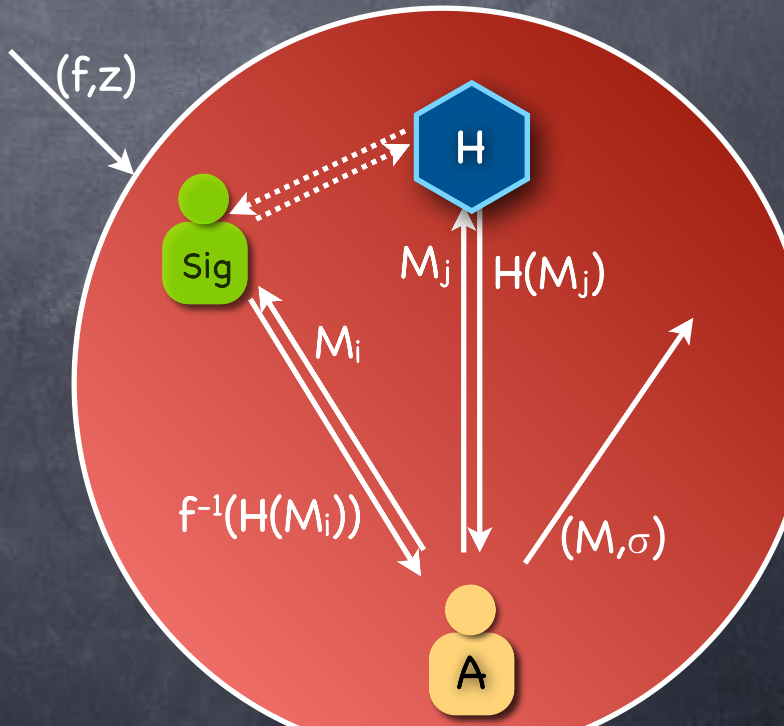

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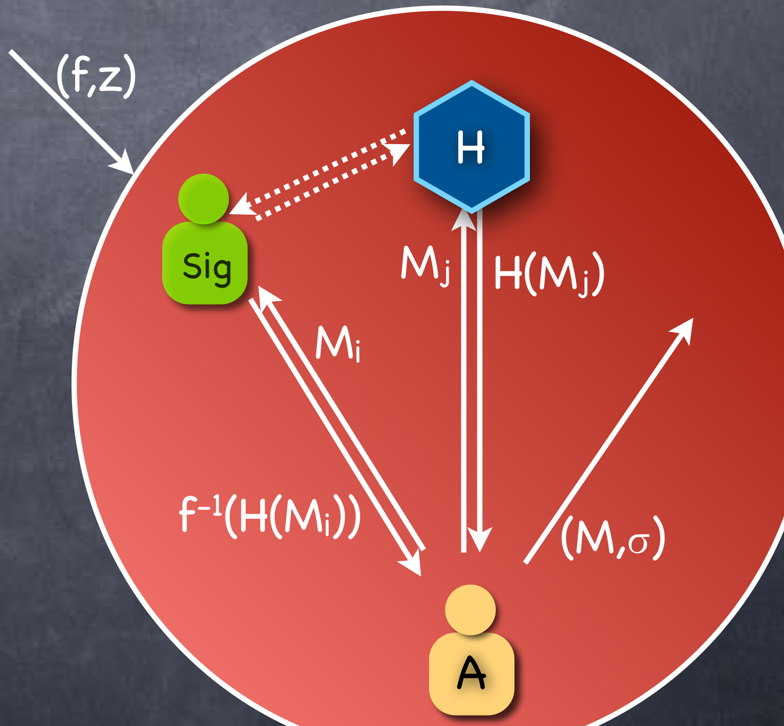

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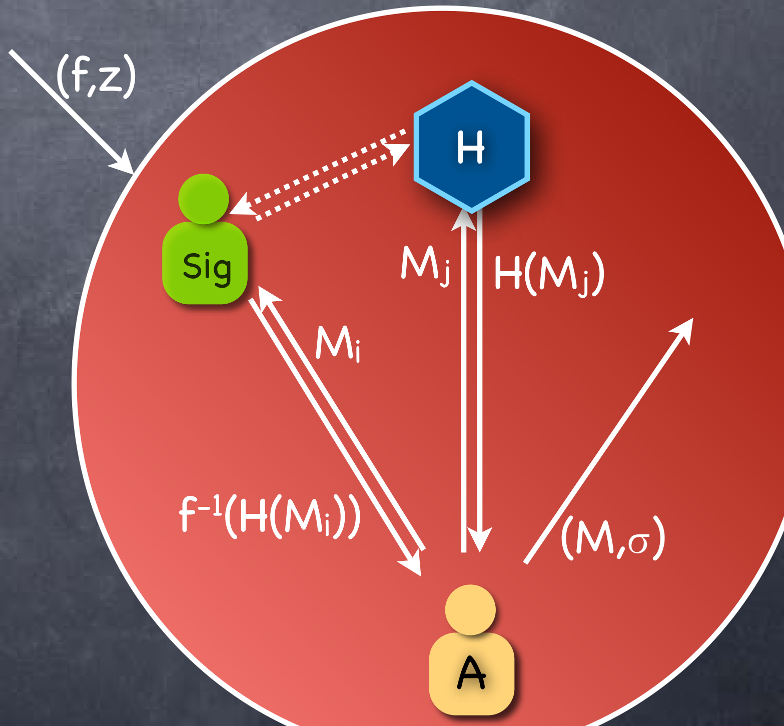

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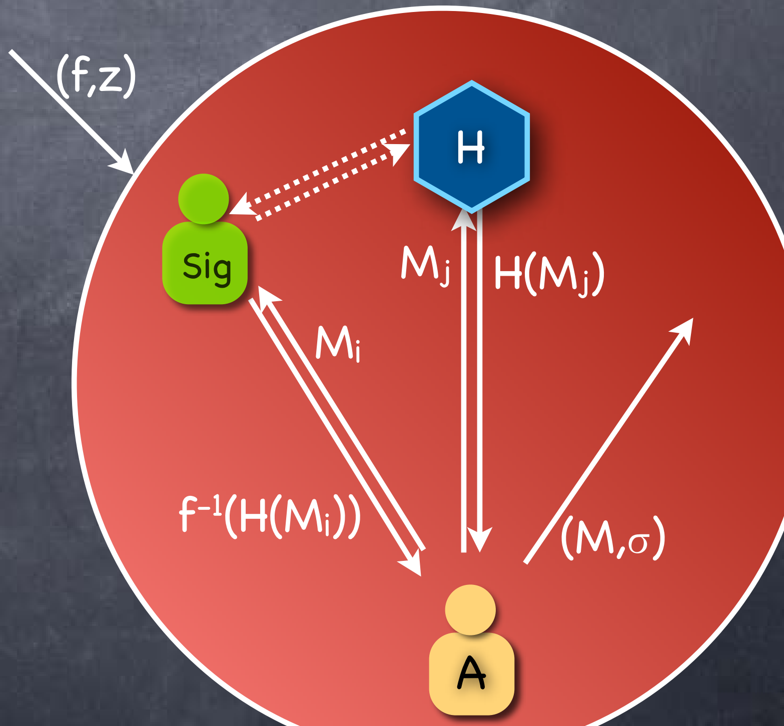

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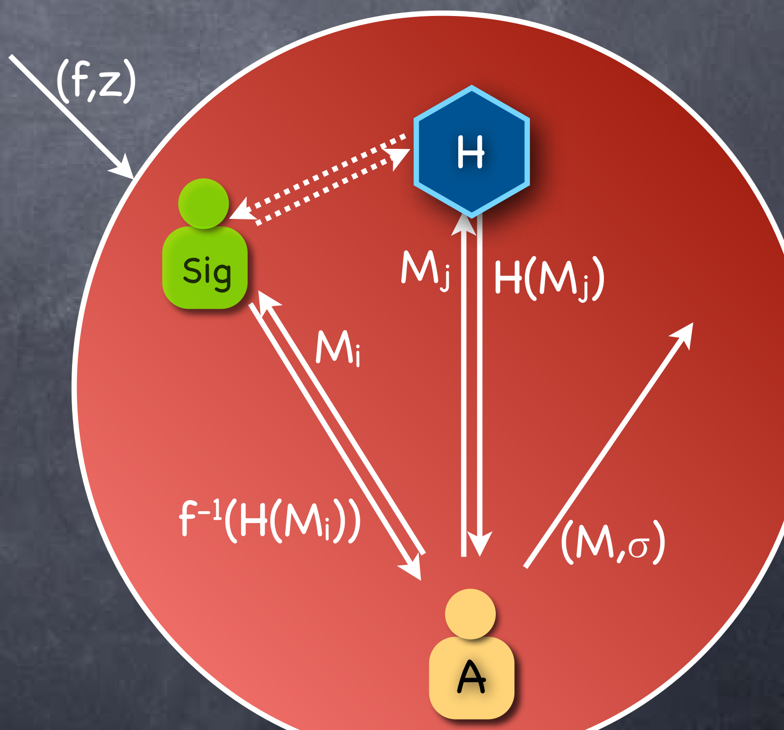

Proving Security in ROM

- A^* such that if A forges signature, then A^* can break T-OWP
  - A^* implements H and Sign: For each new M queried to H or Sign, A^* sets H(M)=f(y) for random y; then Sign(M) = y
  - But A^* should force A to invert z
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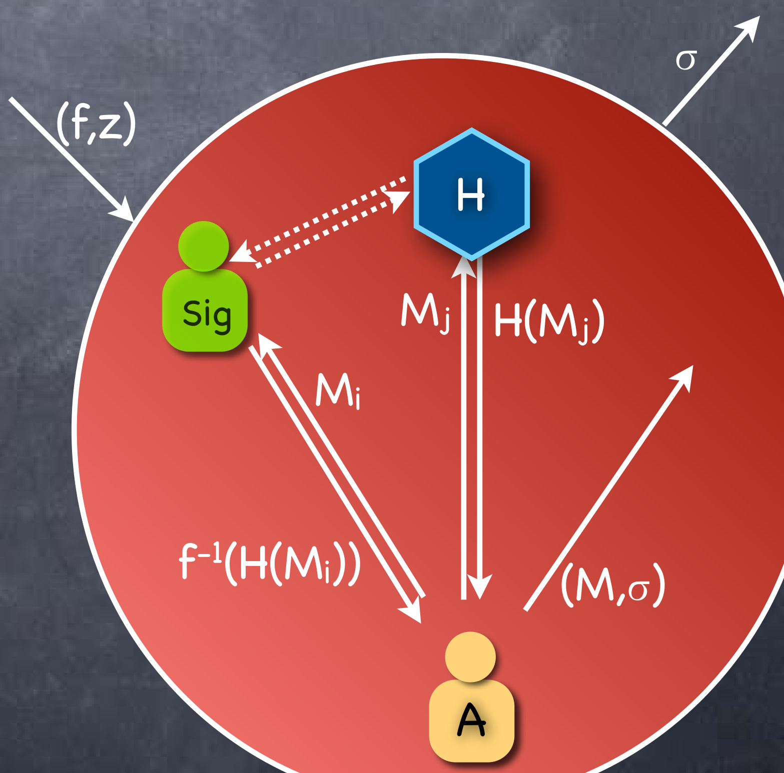
  - In that case forgery $\Rightarrow \sigma = f^{-1}(z)$



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- A general tool for purifying randomness: Randomness Extractor

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  - Key derivation: Alice and Bob extract a new key, which is pseudorandom (i.e., indistinguishable from a uniform bit string)