Public-Key Cryptography Lecture 9

Lecture 9 El Gamal Encryption

Lecture 9 El Gamal Encryption Public-Key Encryption from Trapdoor OWP

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Based on DH key-exchange

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 Alice, Bob generate a key using DH key-exchange

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Random x

X=g[×]

K=Y×

C=MK

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Random y

Y=a^y

K=X^y

M=CK⁻¹

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- KeyGen uses GroupGen to get (G,g)
 x, y uniform from [|G|]
- Message encoded into group element, and decoded



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• But sets $PK=(G,g,g^{\gamma})$ and $Enc(M_b)=(g^{\chi},M_bg^z)$

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A*(G,g; g^x,g^y,g^z) (where (G,g) ← GroupGen, x,y random and z=xy or random) plays the IND-CPA experiment with A:

• But sets $PK=(G,g,g^{y})$ and $Enc(M_{b})=(g^{x},M_{b}g^{z})$

Outputs 1 if experiment outputs 1 (i.e. if b=b')

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When z=xy, exactly IND-CPA experiment: A* outputs 1 with probability = 1/2 + advantage of A.



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Enough for an IND-CPA secure PKE scheme (e.g., Security of El Gamal)



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- Is there a similar construction for TPRG from OWP?
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 OWP does not suffice
 - Will start with "Trapdoor OWP"



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(PK,SK)←KeyGen X←{0,1}^k X′ = X?

∫Yes/No

f_{PK}(x),PK

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- Hardcore predicate:

B_{PK} s.t. (PK, f_{PK}(x), B_{PK}(x)) ≈ (PK, f_{PK}(x), r)



Yes/No

b

f_{PK}(x),PK



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KeyGen

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G

Ζ

SK

R

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• RSA function: $f_{RSA}(x; N,e) = x^e \mod N$ where N=PQ, P,Q k-bit primes, e s.t. $gcd(e,\varphi(N)) = 1$ (and x uniform from {0...N})
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© CPA-secure PKE



- CPA-secure PKE
- OH Key-exchange, El Gamal and DDH assumption



- CPA-secure PKE
- OH Key-exchange, El Gamal and DDH assumption
- Trapdoor PRG



- CPA-secure PKE
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- Next: CCA secure PKE

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Bob would accept only messages from Alice
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Only if it is indeed Eve's own message: she should know her own message!

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A subtle e-mail attack

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A subtle e-mail attack

Alice → Bob: Enc(m)

Suppose Enc SIM-CPA secure

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Alice → Bob: Enc(m) Eve: Hack(Enc(m)) = Enc(m*)

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A subtle e-mail attack

Alice → Bob: Enc(m) Eve: Hack(Enc(m)) = Enc(m*) (where m* = Reverse of m)

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 Suppose encrypts a character at a time (still secure)

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> I look around for your eyes shining I seek you in everything...

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> ...gnihtyreve ni uoy kees I gninihs seye ruoy rof dnuora kool I

Suppose Enc SIM-CPA secure

 Suppose encrypts a character at a time (still secure)

Alice → Bob: Enc(m) Eve: Hack(Enc(m)) = Enc(m*) (where m* = Reverse of m) Eve → Bob: Enc(m*) Bob → Eve: "what's this: m*?"

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A subtle e-mail attack

Hey Eve,

What's this that you sent me?

...gnihtyreve niuoy kees lgninihs seye ruoy rof

> dnuora kool l

I look around

for your eyes shining

in everything ...

Suppose Enc SIM-CPA secure

Suppose encrypts a character at a time (still secure)

Alice \rightarrow Bob: Enc(m) I seek you **Eve:** Hack(Enc(m)) = Enc(m*) (where m^{*} = Reverse of m) **Eve** \rightarrow **Bob:** Enc(m*) Bob → Eve: "what's this: m*?" **Eve: Reverse m* to_find m!**

> I look around for your eyes shining l seek vou in everything...

A subtle e-mail attack

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More subtly, the 1 bit – valid or invalid – may leak information on message or SK

















SIM-CCA Security (PKE)











Possible from generic assumptions

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- Significant efficiency gain using "Hybrid Encryption"

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Relatively low overhead on top of the (fast) SKE encryption

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 - Easy to prove using "composition" properties of the SIM definition
- Less security sufficient: KEM used to transfer a random key;
 DEM uses a new key every time.







Sel Gamal Encryption



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TPRG and TOWP



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CCA secure PKE



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Next: Constructions for CCA secure PKE