# Symmetric-Key Encryption: constructions

Lecture 5

PRG from One-Way Permutations

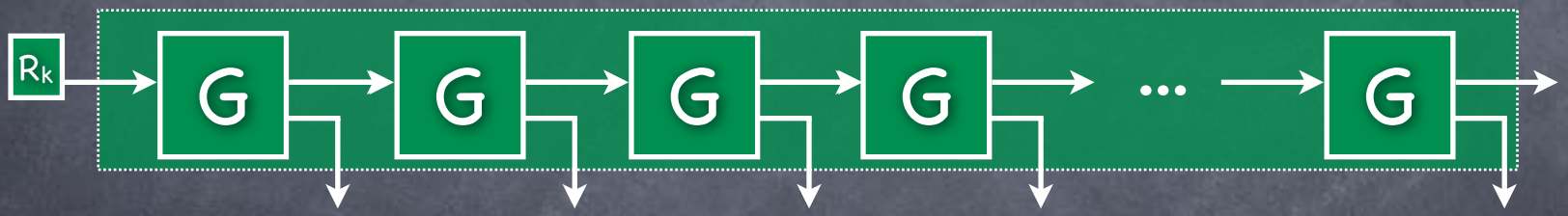
PRF, Block Cipher

RECALL

# PRG

- One-bit stretch PRG, $G_k$: $\{0,1\}^k \rightarrow \{0,1\}^{k+1}$


- Increasing the stretch
  - Can use part of the PRG output as a new seed




  - If the intermediate seeds are never output, can keep stretching on demand (for any "polynomial length")
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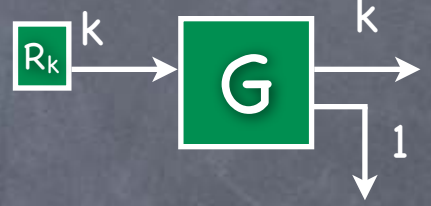


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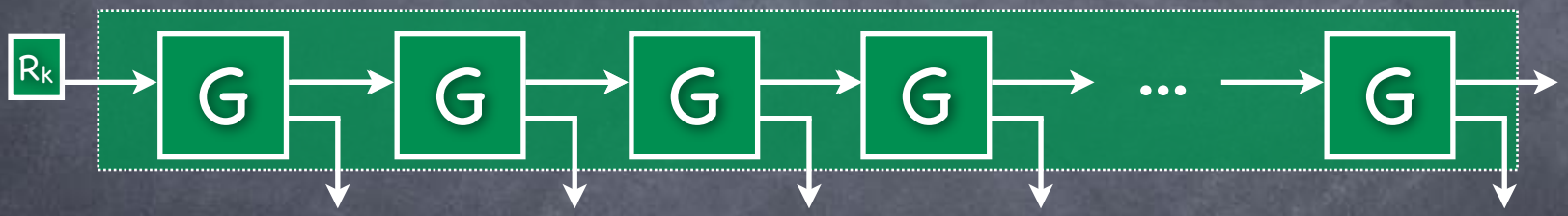


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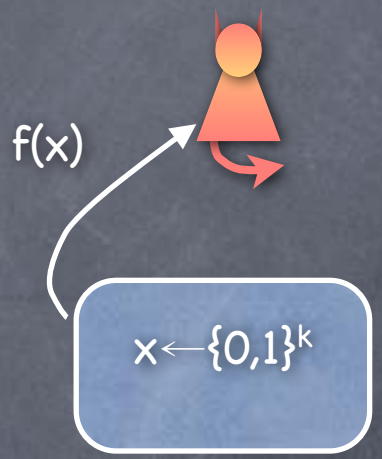

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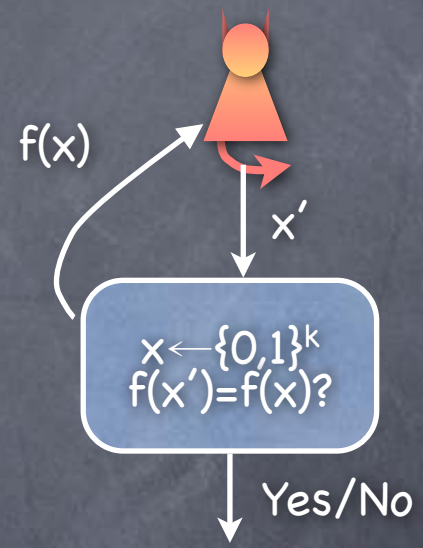

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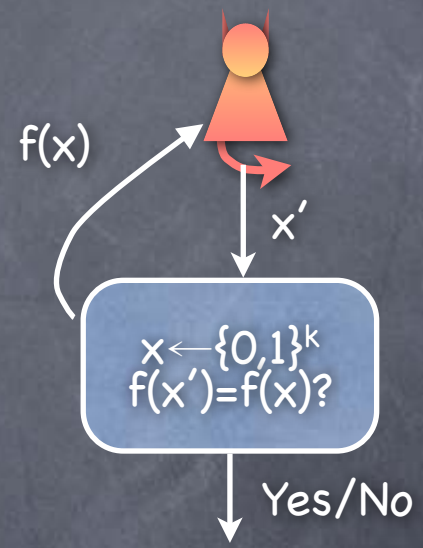

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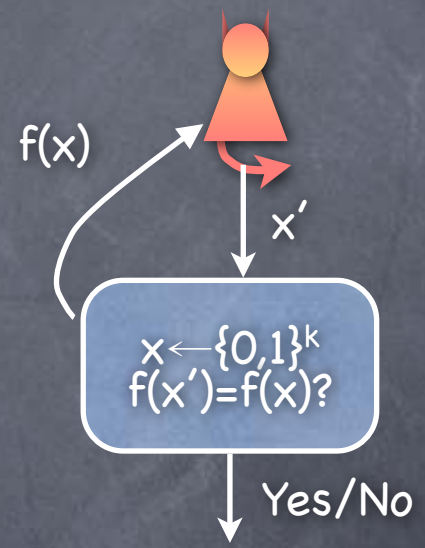

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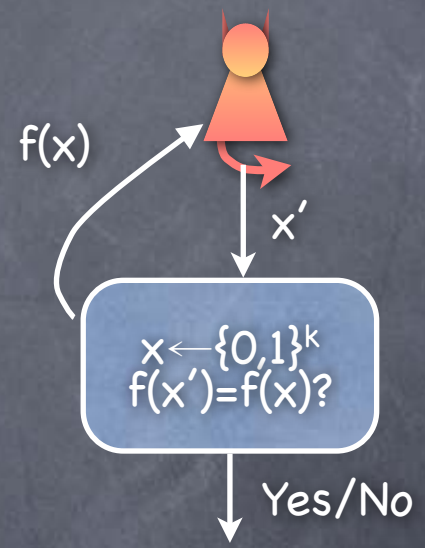

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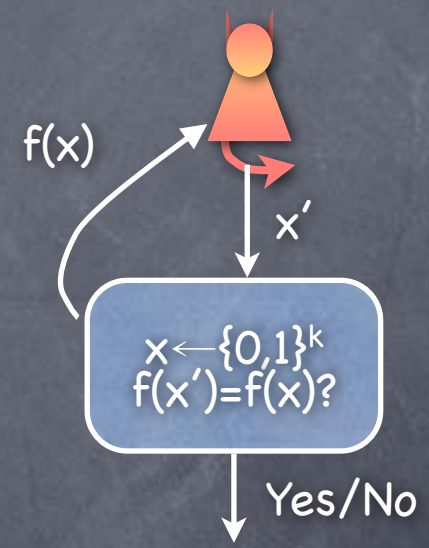

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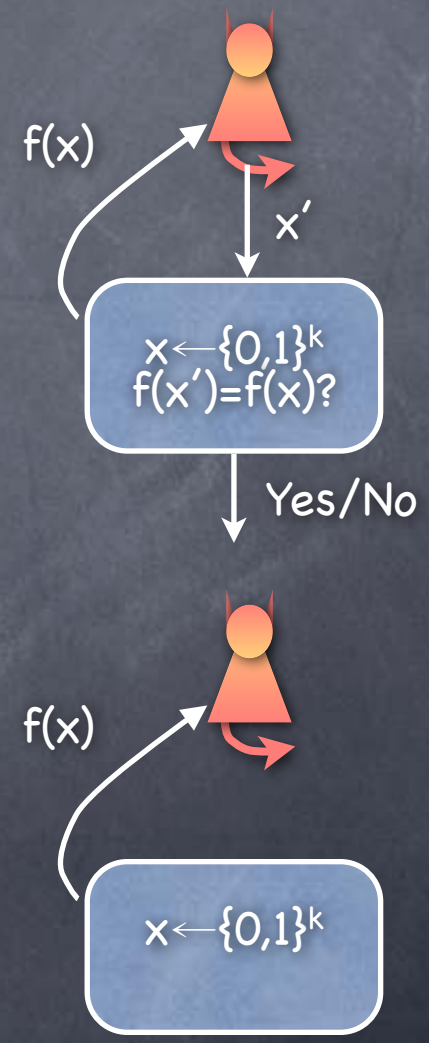

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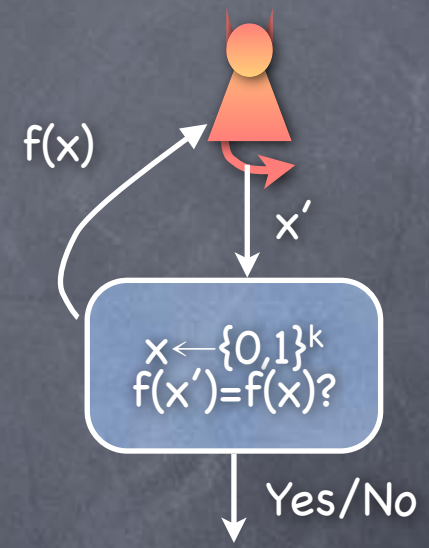


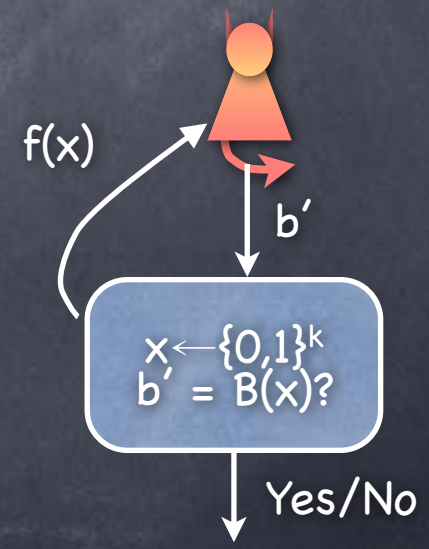

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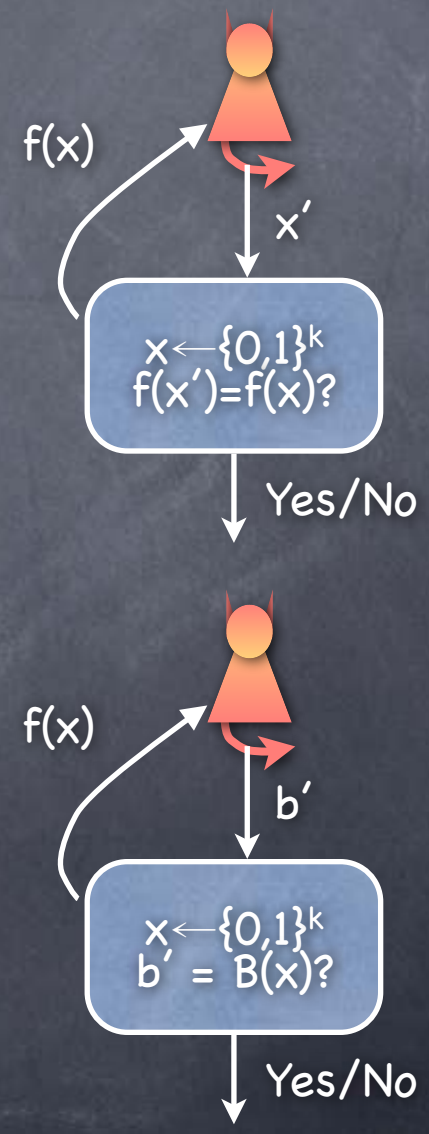

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    - Important that we require $|x|=|y|=k$, not just $|x \cdot y|=2k$ (otherwise, 2 is a valid factor of x.y with 3/4 probability)

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  - OWF Collection: A collection of subset sum problems, all with the same $(x_1, \ldots, x_k)$ (and independent S)

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    - **Reduction**: Given an algorithm for finding LSB(x) from $f_{Rabin}(x;n)$ for random x, one can use it to invert $f_{Rabin}$

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    - Predictor for B(x,r) is a "noisy channel" through which x, encoded as $(\langle x,0\rangle,\langle x,1\rangle \ldots \langle x,2^{|x|}-1\rangle)$ (Walsh-Hadamard code), is transmitted. Can recover x by error-correction (local list decoding)

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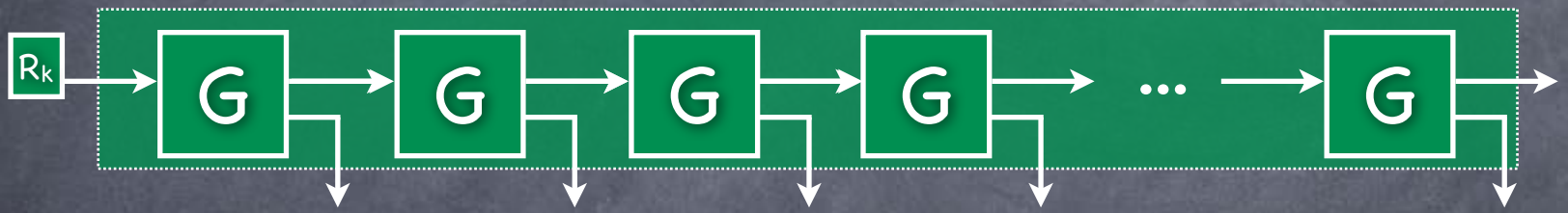

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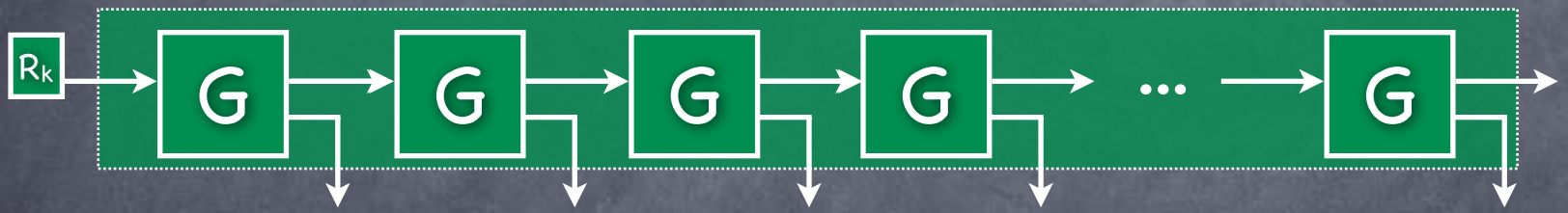


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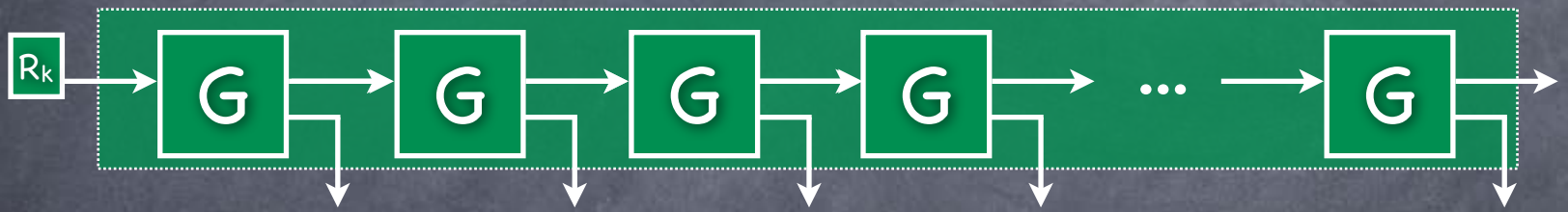


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  - Next: Constructing a proper (multi-message) SKE scheme

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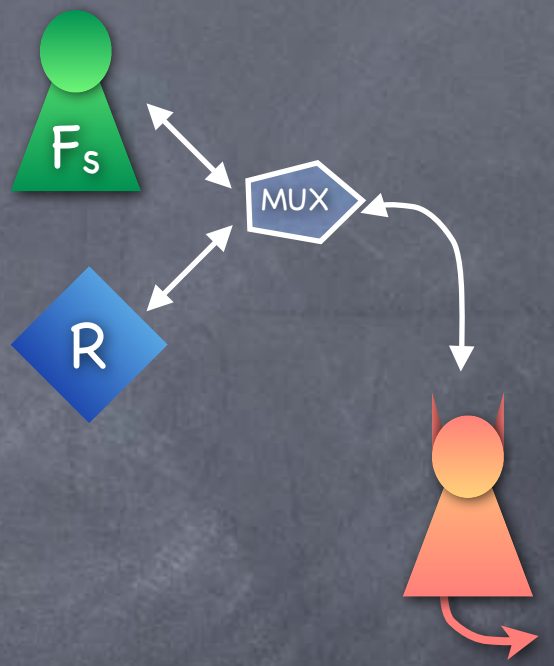

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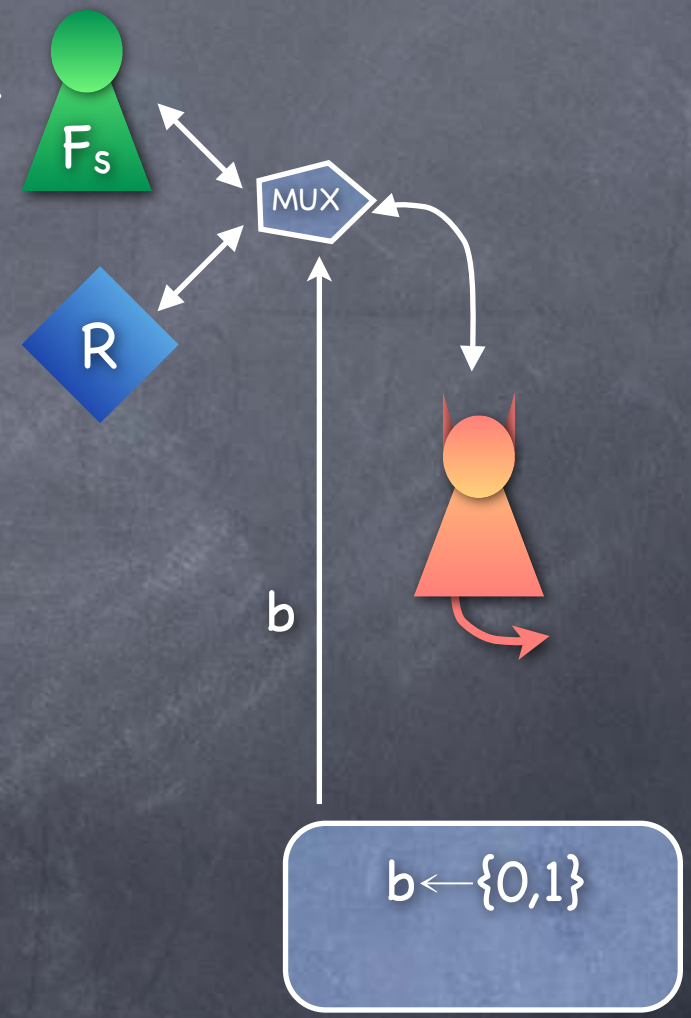

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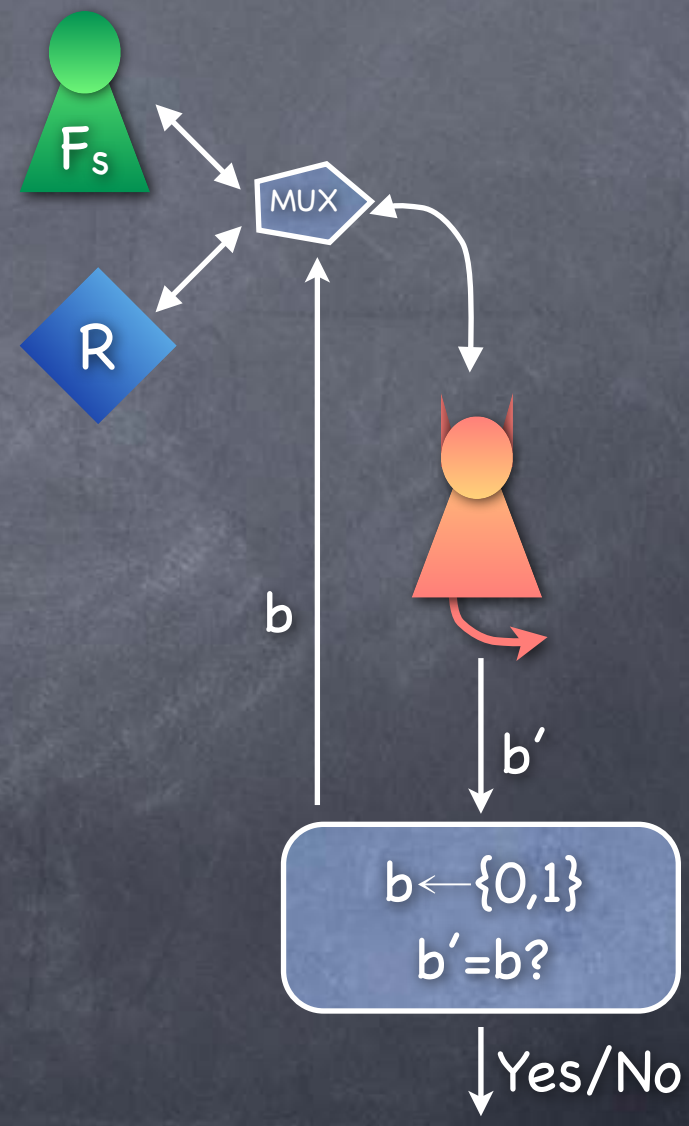

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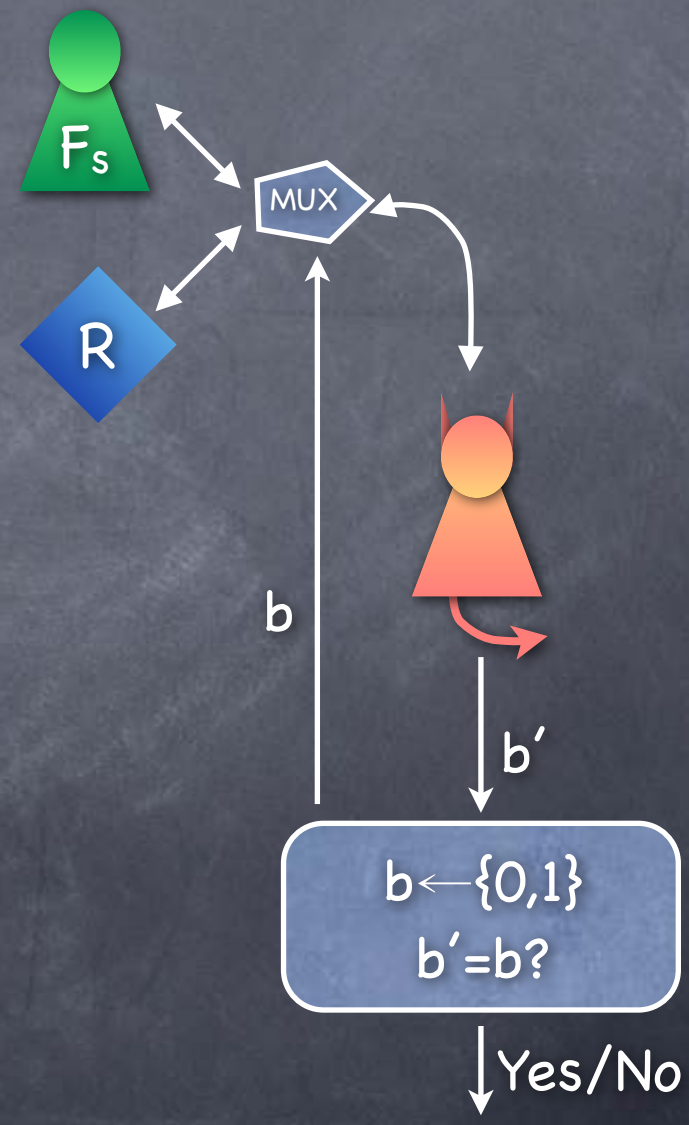

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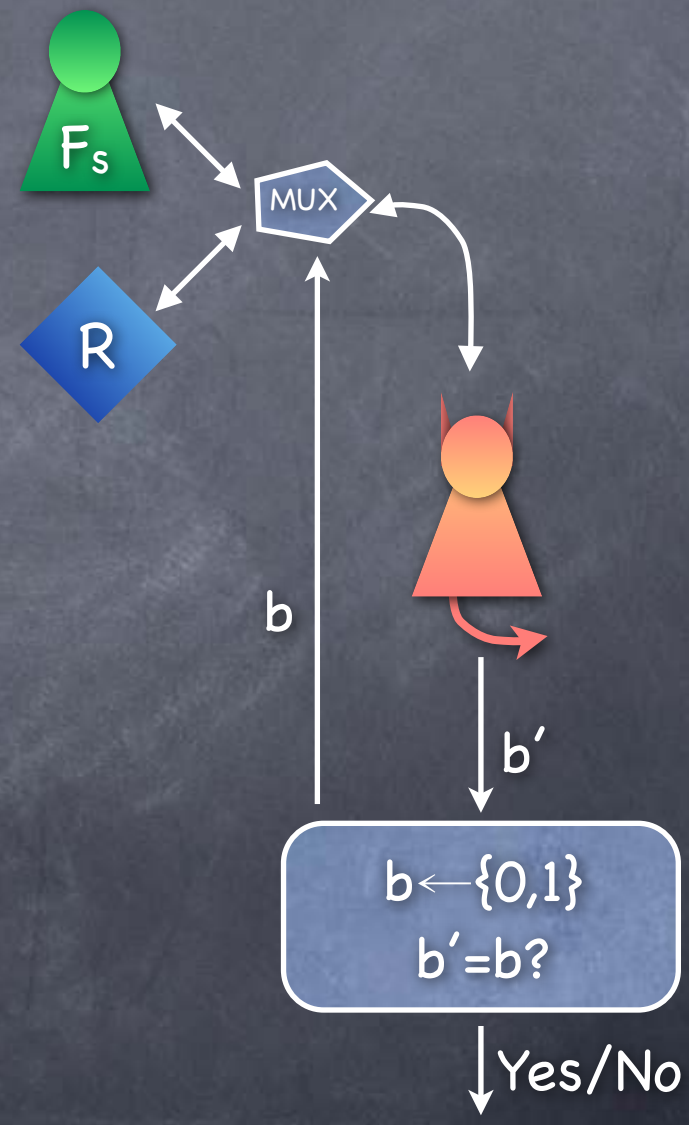

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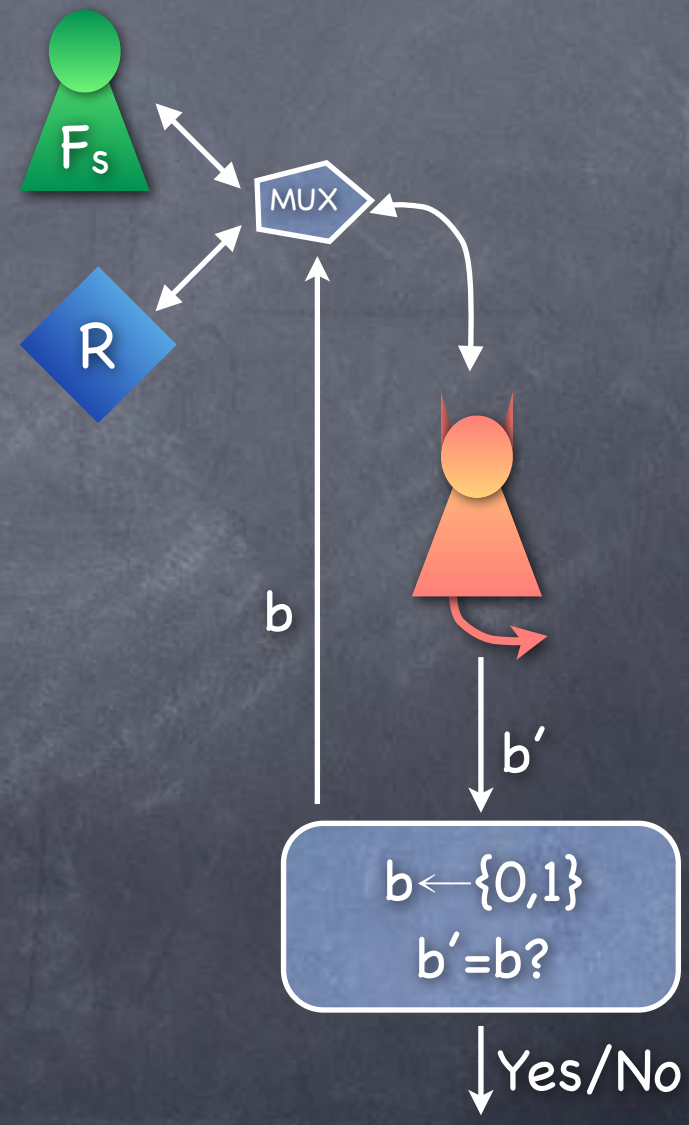

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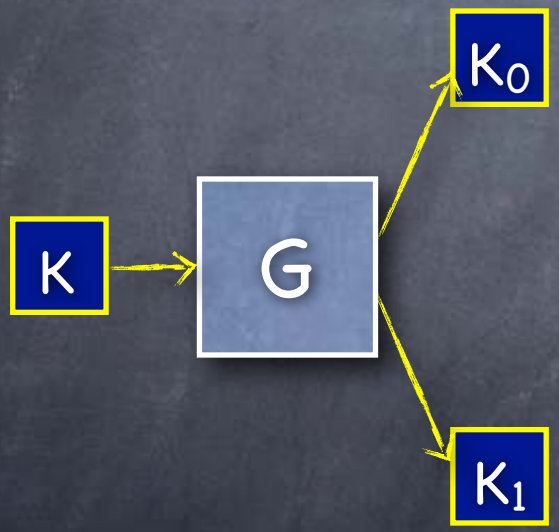

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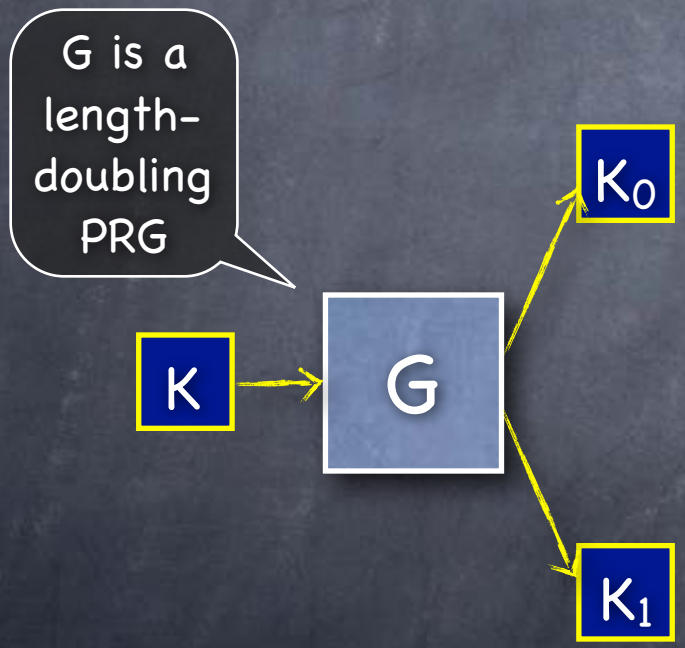

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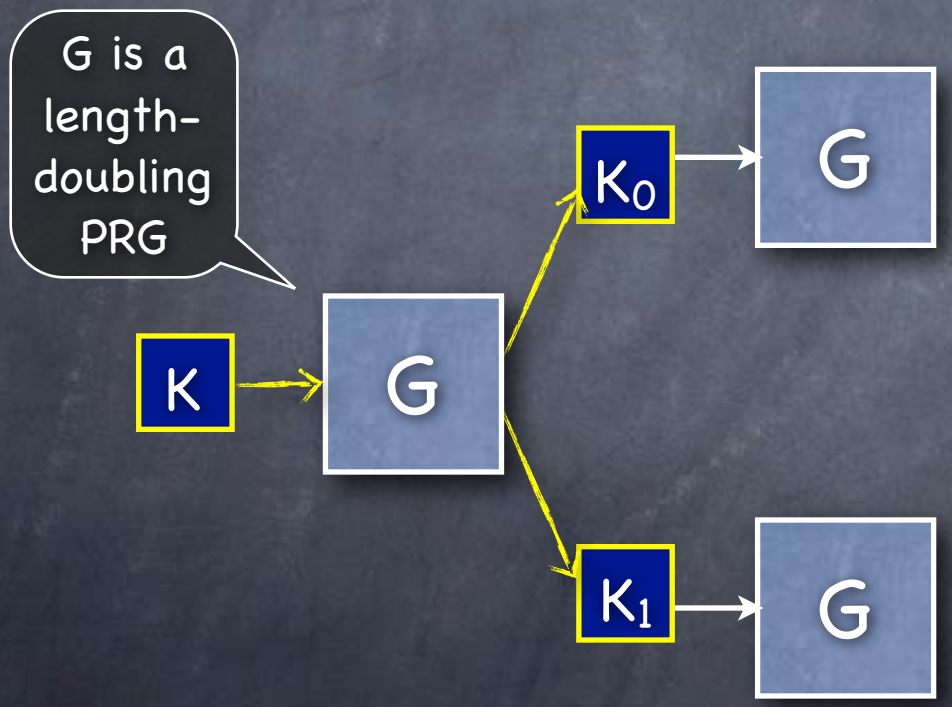

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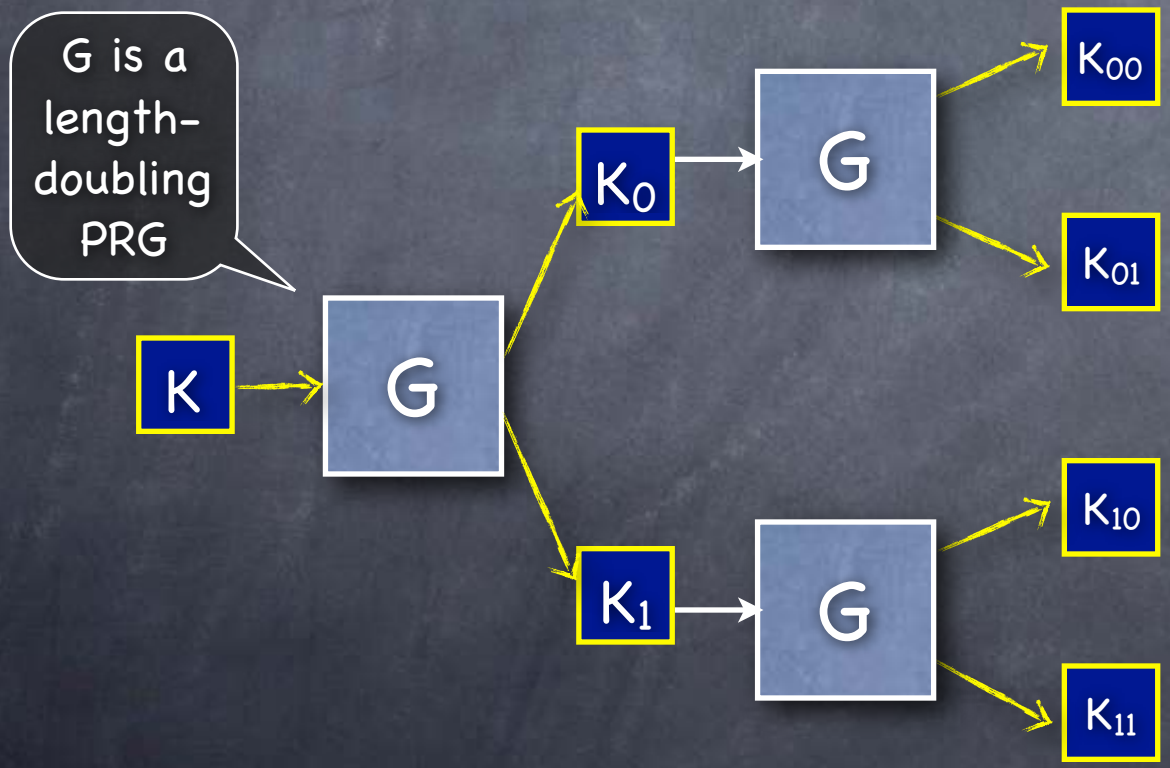

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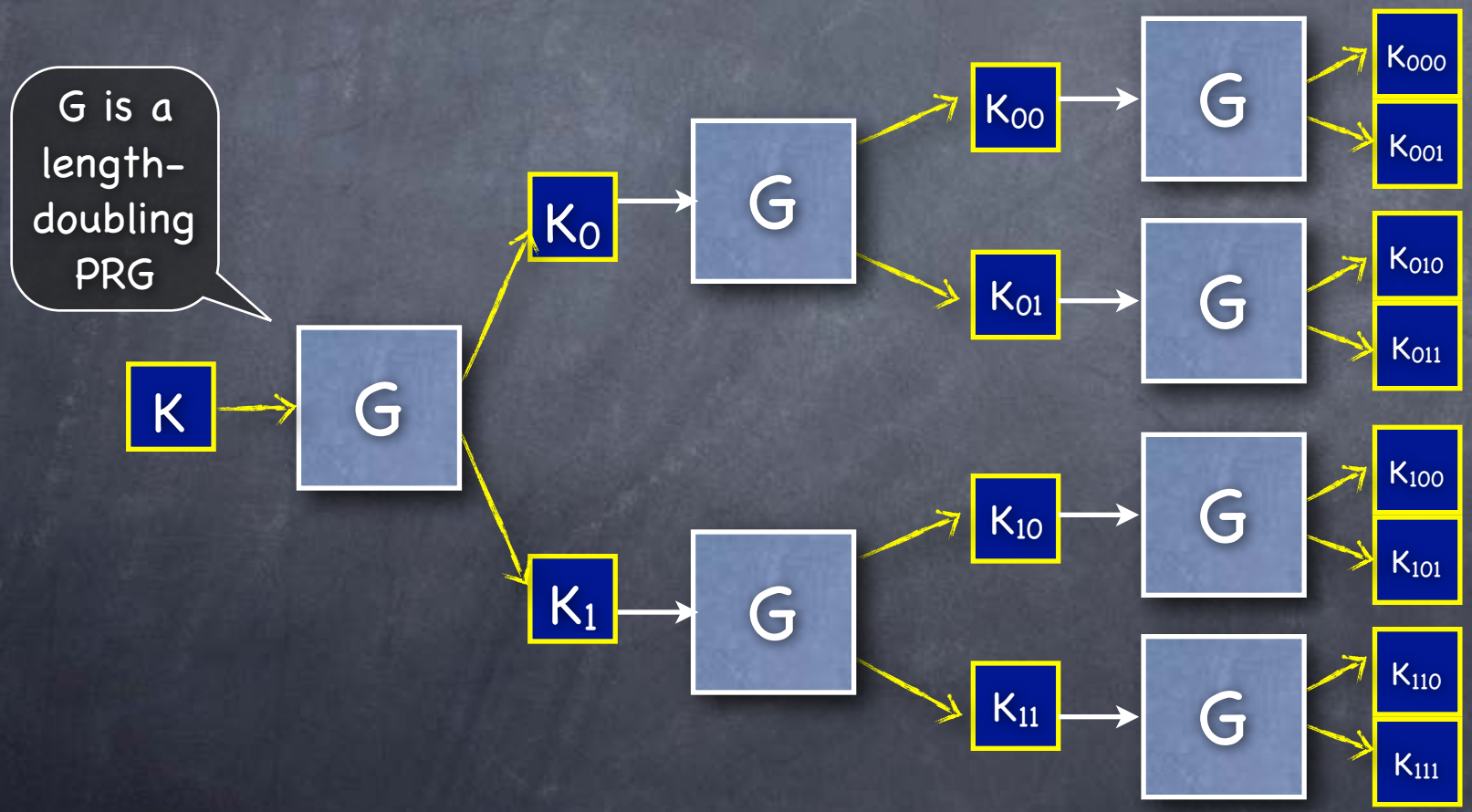

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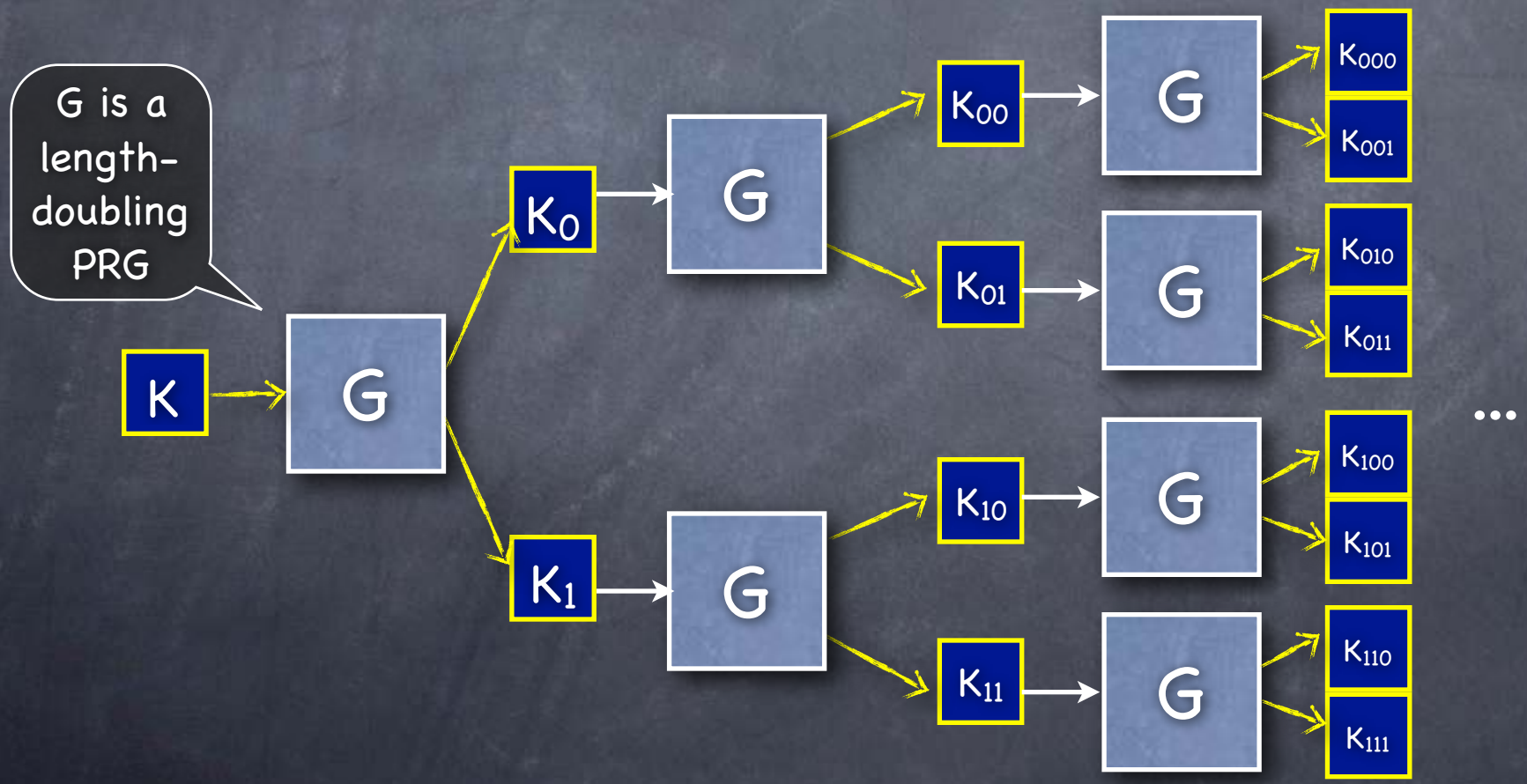

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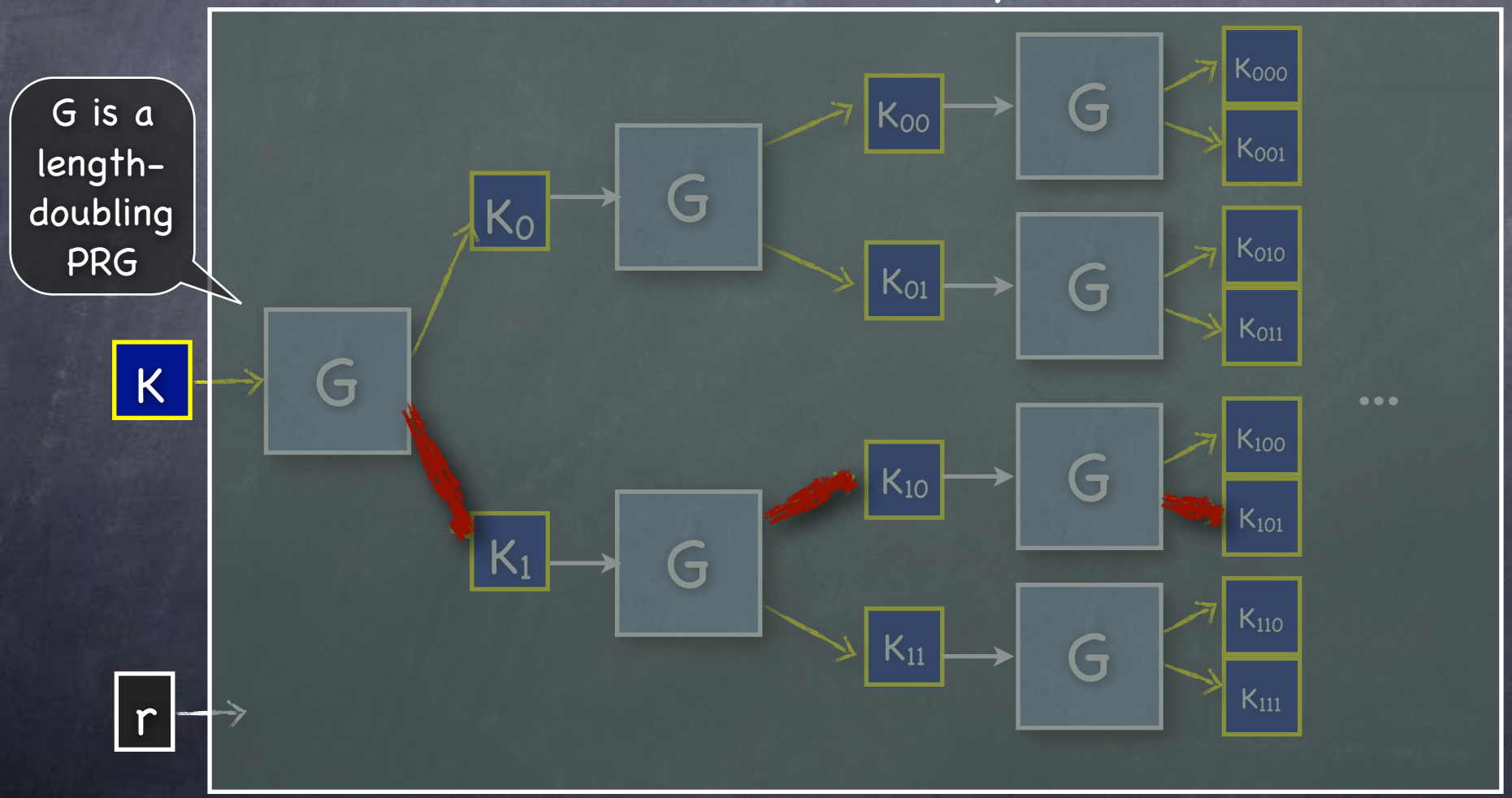

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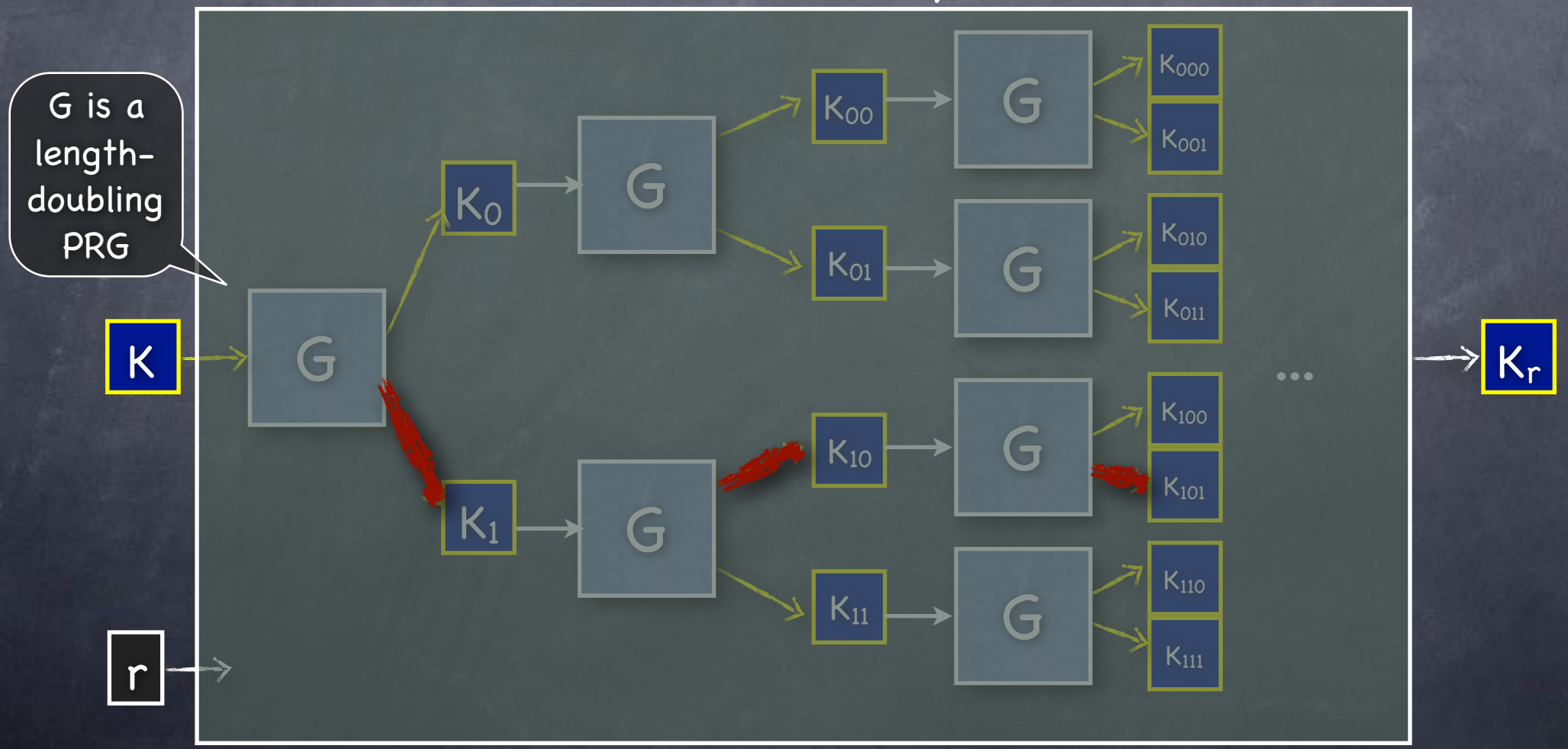

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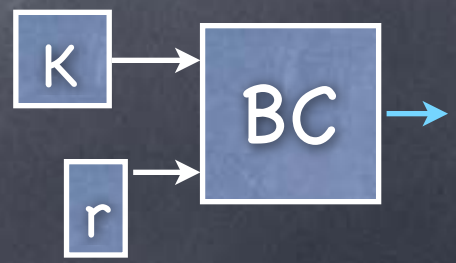

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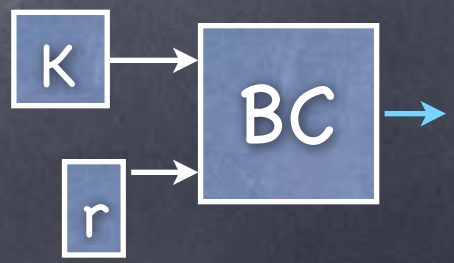

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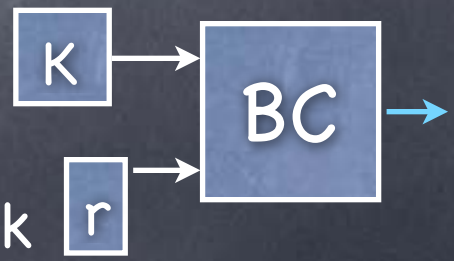

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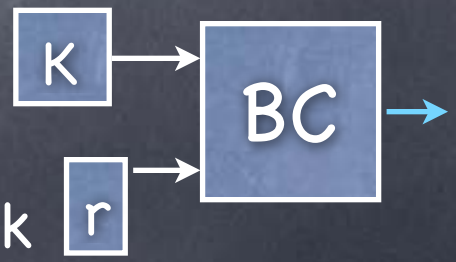

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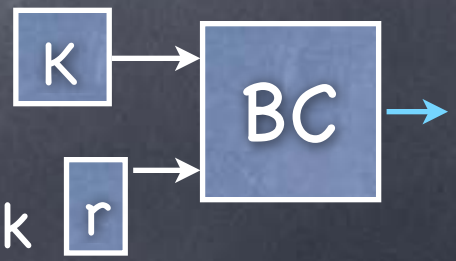

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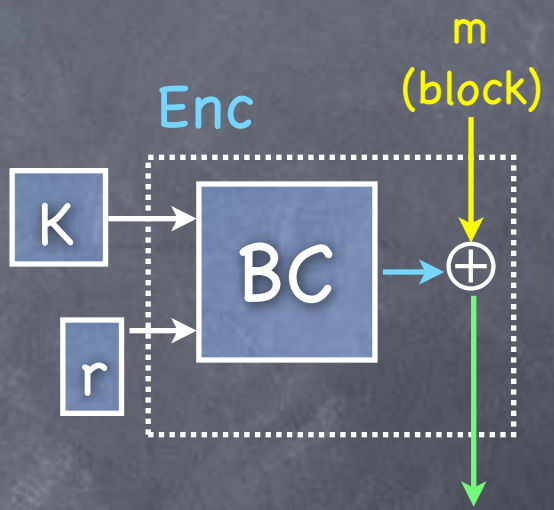

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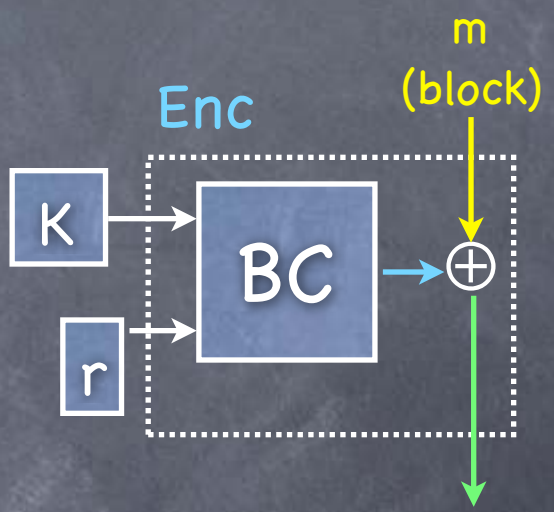

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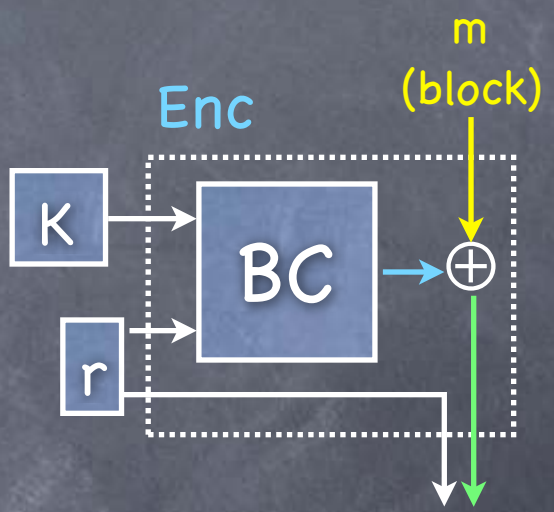

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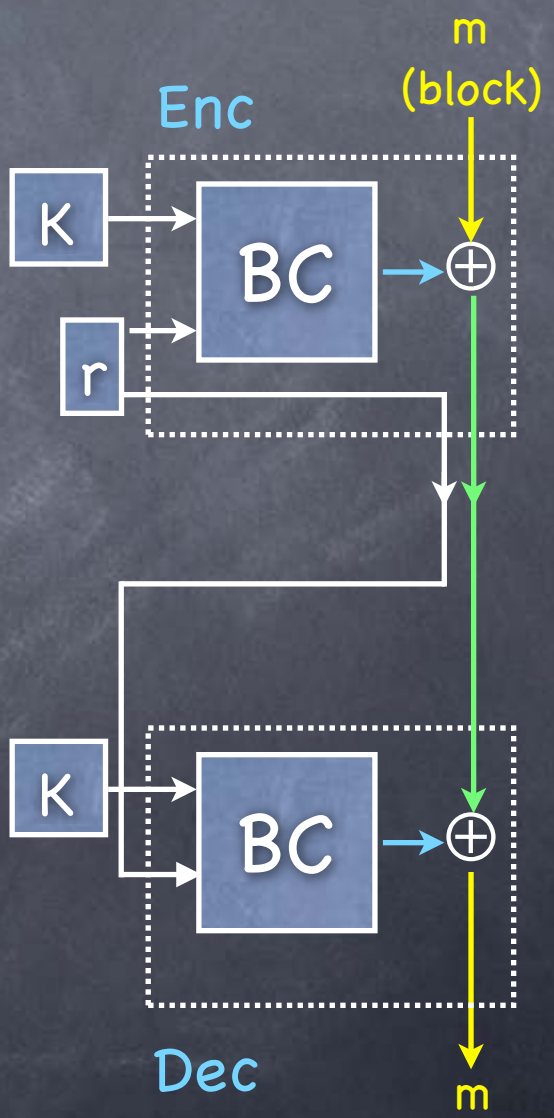

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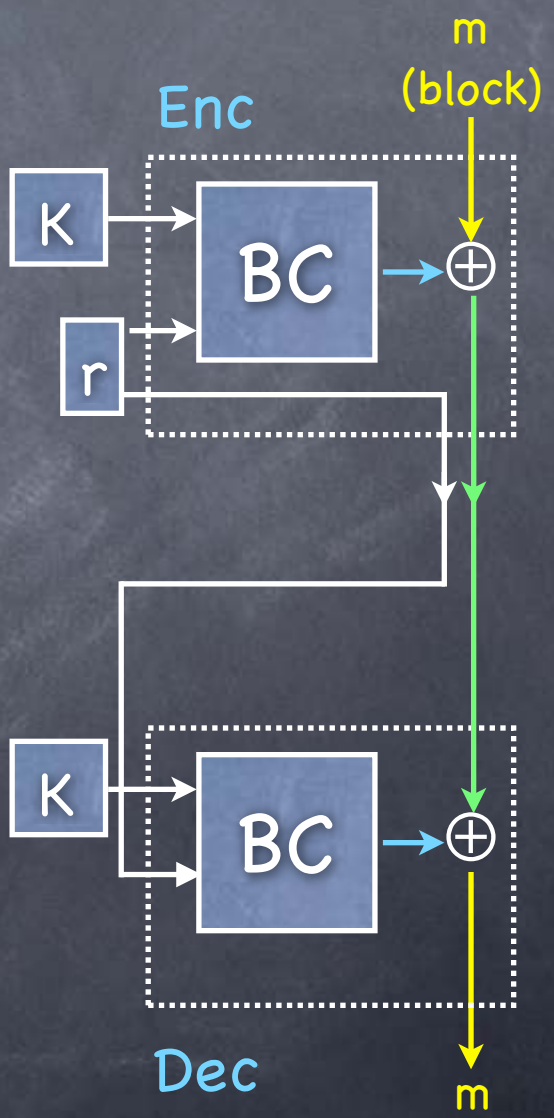

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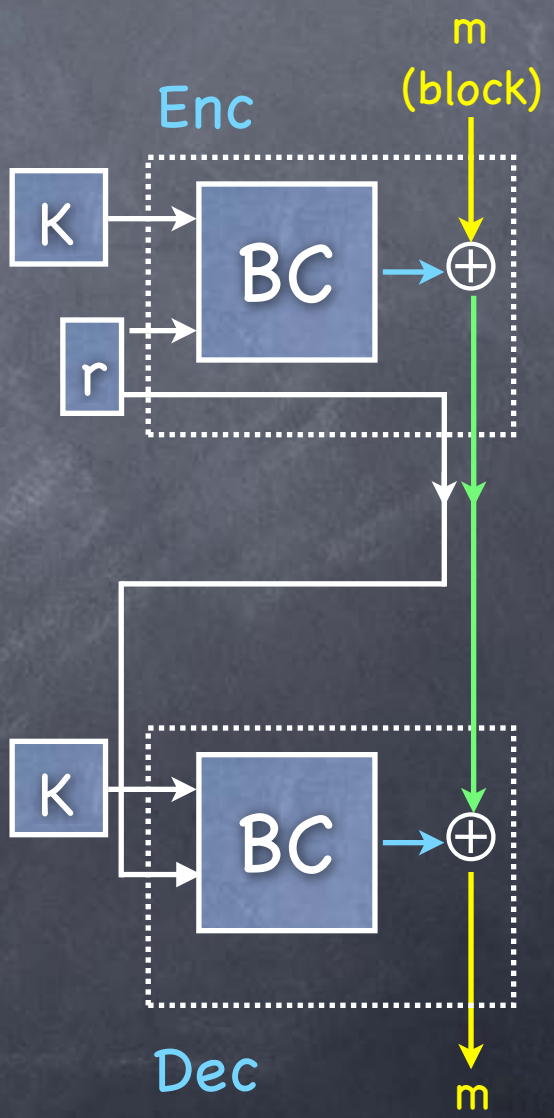

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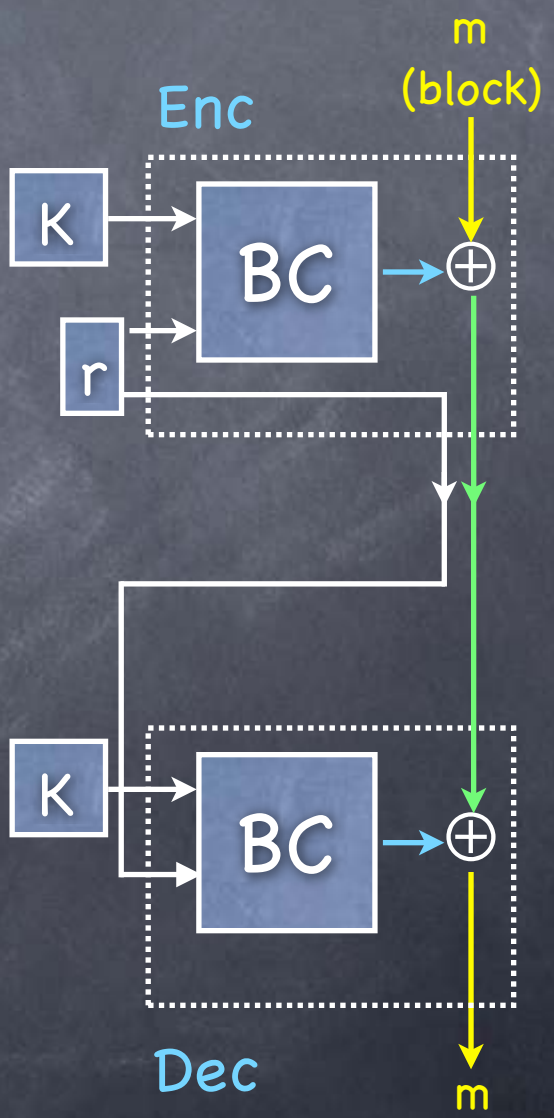

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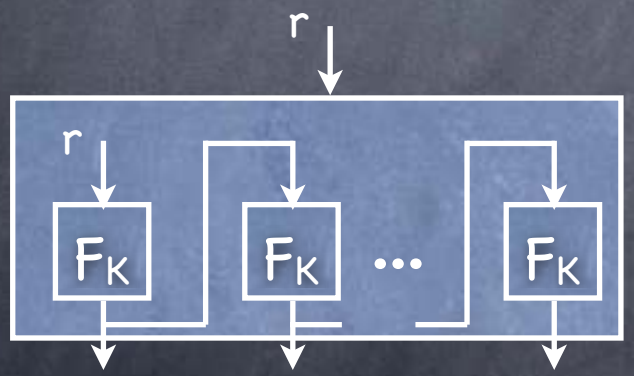

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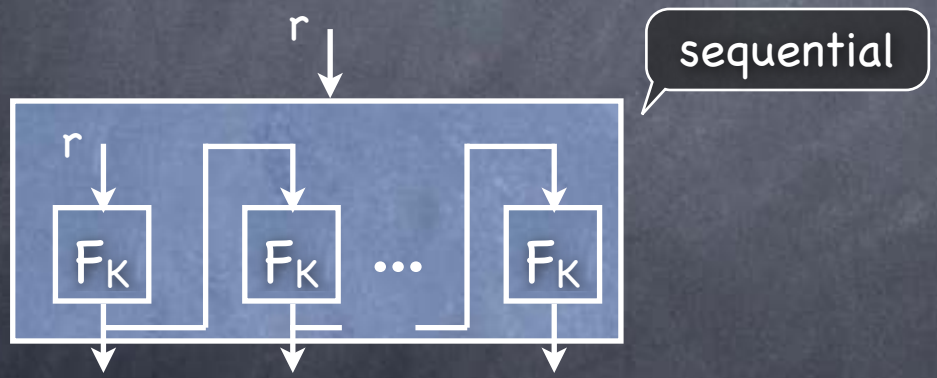

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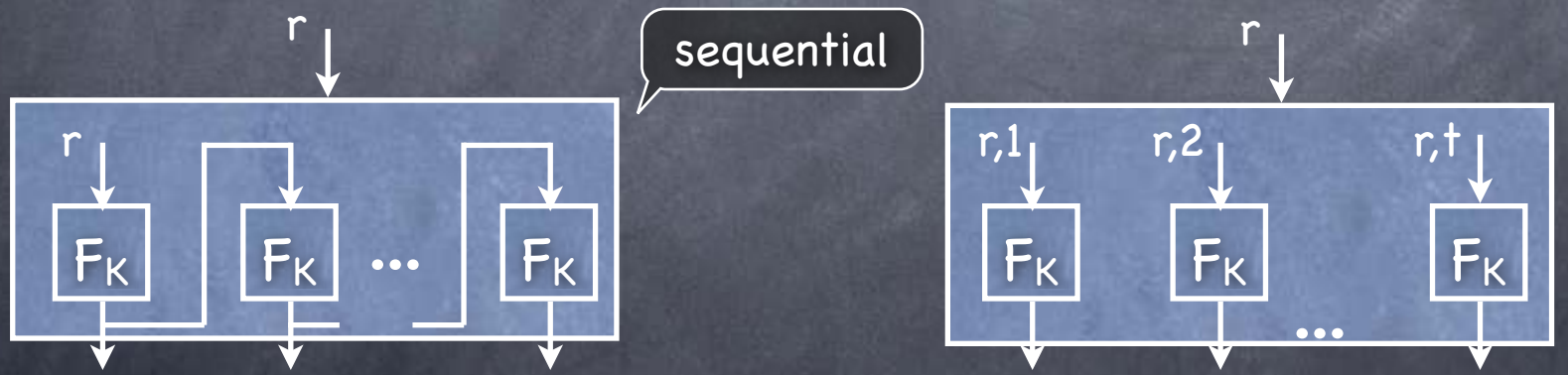

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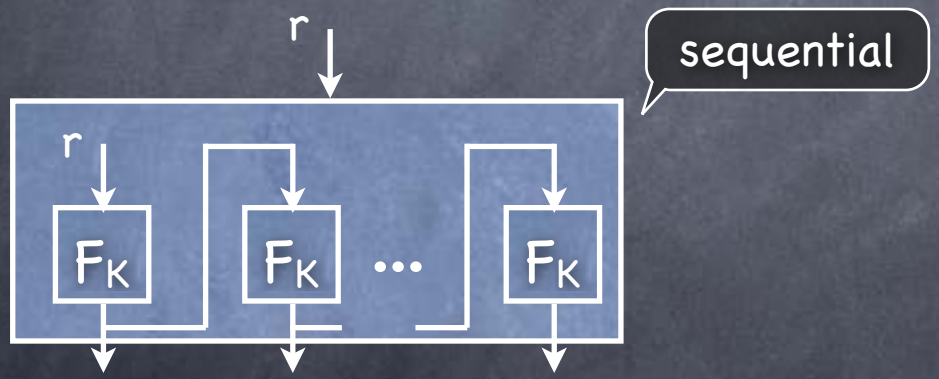
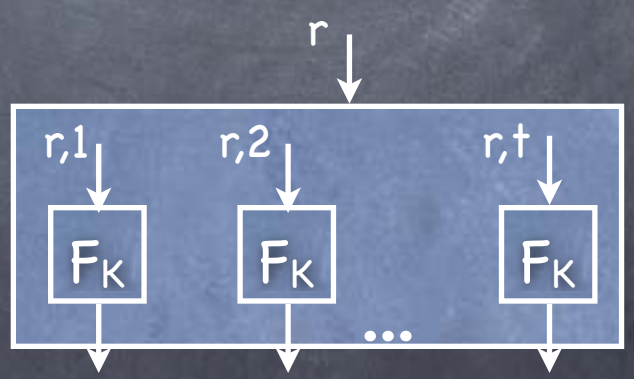



sequential

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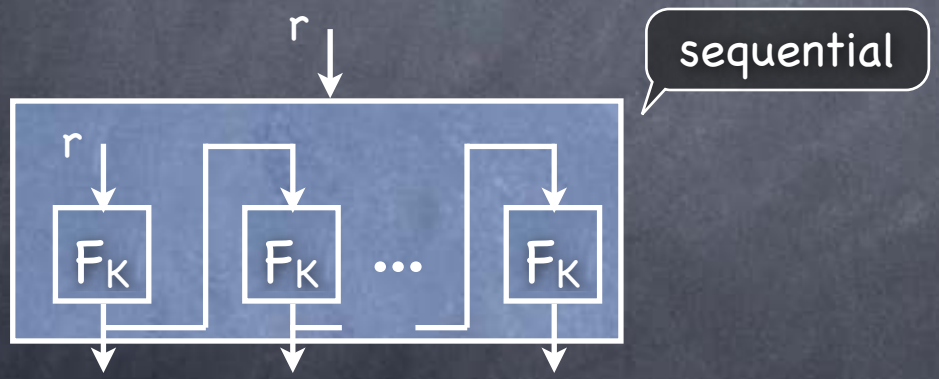
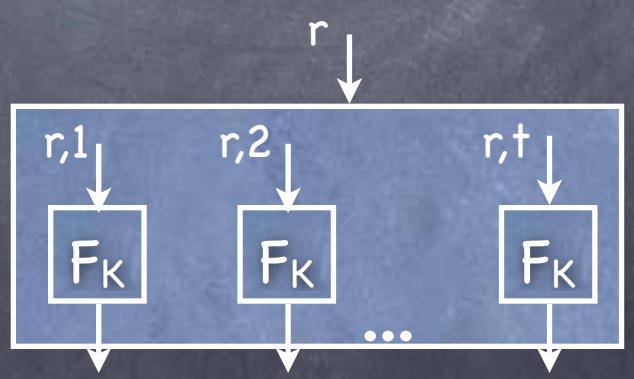



sequential




input length slightly decreased, based on an a priori limit on t

- Output is indistinguishable from t random blocks (even if input to $F_K$ known/chosen)

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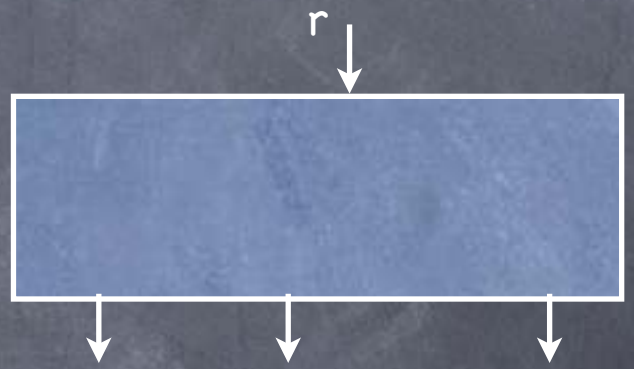

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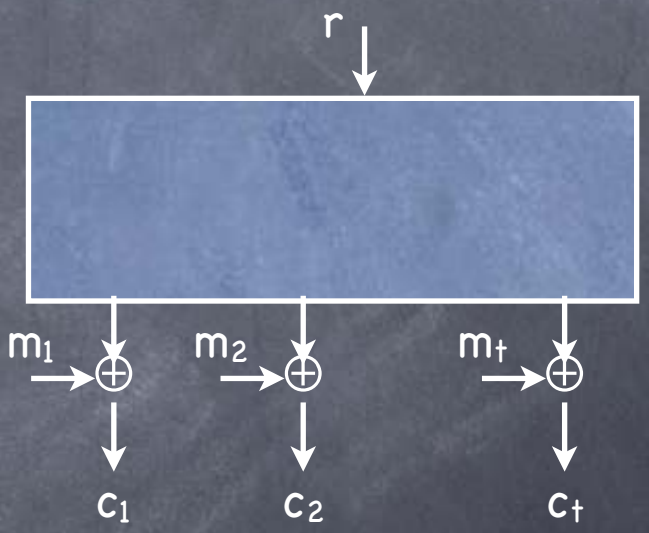

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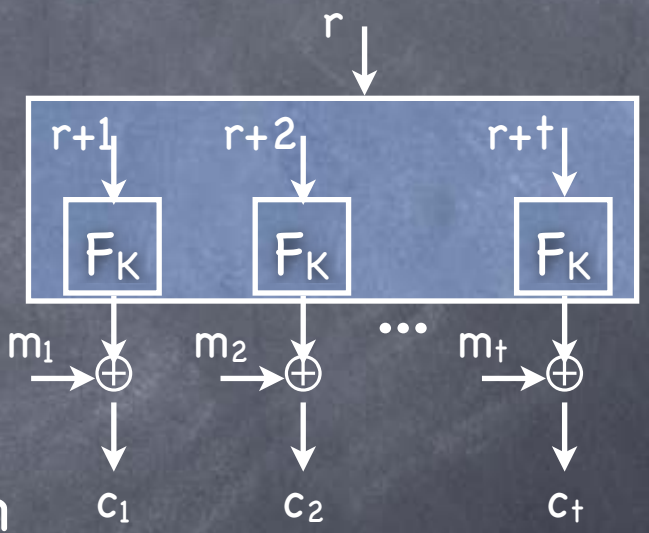

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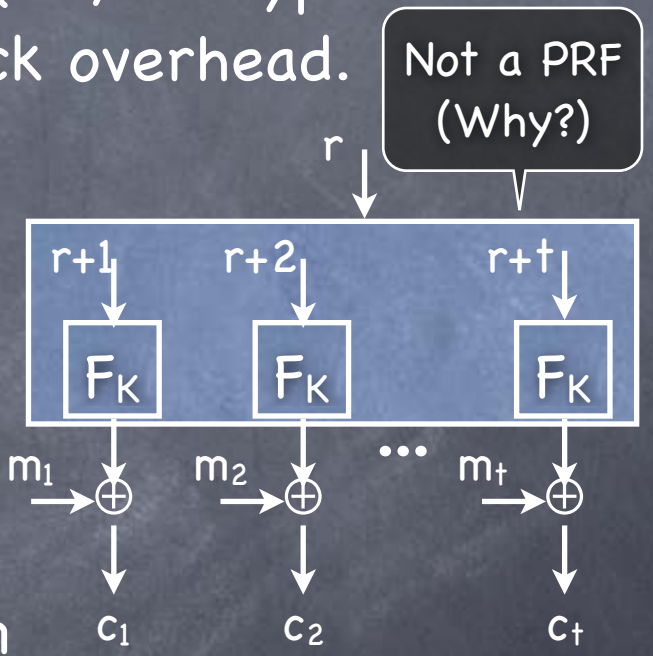

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  - **Cipher Block Chaining (CBC) mode:** Sequential encryption. Decryption uses $F_K^{-1}$. Ciphertext an integral number of blocks.