e-Cash

Lecture 26
Requirements
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- Users should not be able to cheat honest merchants. In particular, users should not be able to double-spend
An approach
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- At a merchant’s, the user gives the signed coin
- Merchant contacts the Bank (online) who ensures that the coin with that serial number has not been used before (i.e., no double spending) and the signature is valid. If so adds the coin to the spent-coin list
Blind Signatures
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Unforgeability: After t sessions, User cannot output signatures on t+1 distinct messages
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Correctness of \(C\): there exists \(c, \sigma, r_{PKE}, r_{Commit}\) such that \(c = \text{Commit}(m; r_{\text{commit}}), C = \text{Enc}_{PK}(c, \sigma; r_{PKE})\) and \(\text{Verify}_{VK}(c, \sigma)\) holds
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Blindness, because signer sees only \( \text{Commit}(m) \). Unlinkability from encryption. Unforgeability from soundness of NIZK, efficient decryption of PKE, and unforgeability of the signature scheme
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Efficient variants (under suitable assumptions) using Groth–Sahai NIZK (or NIWI) scheme and compatible primitives
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Idea: verification in two sessions of the spending protocol with the same coin exposes the user’s identity.
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- Spending: reveal \((s,d)\) where \(d := ID+Rt\), for a random challenge \(R\) from the merchant, along with a PoK of signature on \((ID’,s,t’)\) for some \(ID’,t’\) s.t. \(ID’+Rt’ = d\)
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- Merchant needs to transfer the User's proof to Bank (i.e.,
  Bank should be convinced that the merchant didn't fake)
Signatures with Proofs:
CL Signatures
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Camenisch–Lysyanskaya signatures: Uses Pedersen commitments; security under DDH and Strong RSA assumptions
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- Verification is interactive (but can be made transferable using Fiat-Shamir heuristics in the RO model)
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- e.g. (Weak) Boneh-Boyen signature: \( \text{Sign}_{SK}(x) = g^{1/(SK+x)} \)
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  - Trees with small denomination coins at the leaves; can spend any node (root of a subtree); spending a node and a descendent will reveal ID
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- Compact e-Cash: Remove linking multiple spending
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Recall previous (non-compact) scheme: get signature on (ID,s,t) during withdrawal and reveal (s,d) where d := ID+Rt for a challenge R, when spending the coin.
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Instead, let s, t be seeds of a PRF
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On spending for the \(i^{th}\) time, reveal \((S, D)\) where 
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S = \text{PRF}_s(i) \quad \text{and} \quad D = ID + RT, \quad \text{where} \quad T = \text{PRF}_t(i)
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\[S = PRF_s(i)\] and 
\[D = ID + RT, \text{ where } T = PRF_t(i)\]

Prove that \(ID,s,t,i,\)signature exist as claimed. Optionally, that \(i\) is in the range \([1,L]\) for some upper-bound \(L\).
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Need a PRF that supports efficient proofs
A PRF for compact e-Cash
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\[ F_{g,s}(x) = g^{1/(s+x+1)} \] where s is the seed \((g\) can be public) [DY05]
A PRF for compact e-Cash

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- Alternately, working in groups with bilinear pairings, can use Groth-Sahai NIZK (under appropriate assumptions)
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  - e.g. schemes not depending on Random Oracles, but on relatively untested assumptions
- Security/Efficiency/Usability issues: need to cancel stolen electronic wallet; need to recharge electronic wallet (cellphone?) online, but protect it from malware; efficient multiple denomination coins; allow transferability; tracing may not deter double-spending
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- And they cannot link together multiple proofs coming from the same user
Anonymous Credentials from P-Signatures
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User Alice has a public-key, $\text{PK}_A$, and a secret key $\text{SK}_A$. 
Anonymous Credentials from P-Signatures

- User Alice has a public-key, $PK_A$, and a secret key $SK_A$
- Alice needs pseudonyms with Bob and Carol, say $A_B$ and $A_C$
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- $A_B$ and $A_C$ will be (independent) commitments to $SK_A$ (using the commitment supported by the P-Signature).
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- **Obtaining credential:** Carol signs \( SK_A \) using the P-Signature scheme using \( A_C \) (without learning \( SK_A \)). If Carol is a root authority, she requires a proof that \( A_C \) is a valid commitment of \( SK_A \) that corresponds to \( PK_A \) (not anonymous). Else Carol verifies that \( A_C \) has a credential from the root authority (as below).
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**Obtaining credential**: Carol signs $SK_A$ using the P-Signature scheme using $A_C$ (without learning $SK_A$). If Carol is a root authority, she requires a proof that $A_C$ is a valid commitment of $SK_A$ that corresponds to $PK_A$ (not anonymous). Else Carol verifies that $A_C$ has a credential from the root authority (as below).

**Proving**: Alice wants to prove to Carol that owner of $A_C$ has a credential from Bob. She commits $SK_A$ again to get $A'$ and shows that she has a signature from Bob on the message in $A'$. She also proves that $A'$ and $A_C$ have the same message.
Today
Today

e-Cash
Today

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- Anonymous, offline validation and compact
e-Cash
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Relies on signatures, PRFs and NIZK
e-Cash

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Signatures with associated protocols (P-signatures, CL signatures, (partially) Blind signatures)
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