Functional Encryption

Lecture 27

Functional Encryption

- Plain encryption: for secure communication. Does not allow modifying encrypted data.
- Homomorphic Encryption: allows computation on encrypted data, but result remains encrypted
- Functional Encryption: allows computation so that results are available in the clear
  - Many interesting applications
  - Active/evolving area of research
    - Will sample a few results

Functional Encryption

- Ciphertext: Enc(Msg). Msg is fully or partially hidden
  - e.g., Msg = (T,M) where T is a public tag (a.k.a index)
- Key: KeyGen(f). Function f could be fully/partly hidden or not.
- "Decryption" Dec(Enc(Msg), KeyGen(f)) → f(Msg)
  - Public-index FE: $f(T,M) = \perp$ if $g(T)=0$; $f'(M)$ if $g(T)=1$
- Should reveal nothing else
  - Can formulate different levels of security
- Can be public-key (anyone can encrypt) or not
- KeyGen requires a master secret-key. If public-key, encryption needs only master public-key, else needs master secret-key.

Functional Encryption

- Trivial Example: when the family of functions is small
  - Keys will be issued only for $f \in \{f_1,\ldots,f_N\}$ for a small N
  - Can pre-compute all the functions, and encrypt the results!
    - $Enc(Msg) = (c_1,\ldots,c_N)$, where $c_i = E_{PKi}(f_i(Msg))$ using a PKE encryption scheme (with N independent keys)
    - $KeyGen(f_i) = (i, SK_i)$
  - Not function-hiding
    - If not public-key, can make it function-hiding by numbering f's randomly

Examples: IBE & ABE

- A public-index FE, where the index is the ID
- Functions f_{ID}: $f_{ID}(ID',M) = M$ if $ID=ID'$; $\perp$ otherwise
- Fuzzy IBE: $f_{ID}(ID',M) = M$ if ID "close to" ID'; $\perp$ otherwise
- Attribute-Based Encryption: if the index/key is not just a single ID, but a vector of "attributes" and a "policy" as to which attribute combinations allow revealing the message
  - Ciphertext-Policy ABE: Index is a policy (from a simple class); the function in the key gives a set of attributes
  - Key-Policy ABE: Index is a set of attributes; the function in the key gives a policy

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  - Fuzzy IBE
  - Audit log inspection: grant the auditor the authority to read only messages with certain attributes

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  - For efficiency need a small matrix

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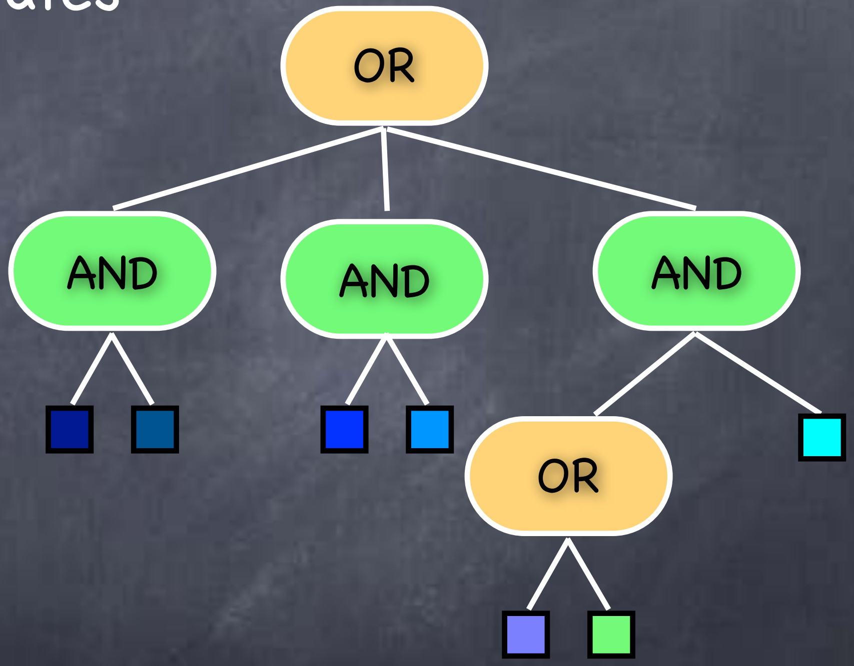

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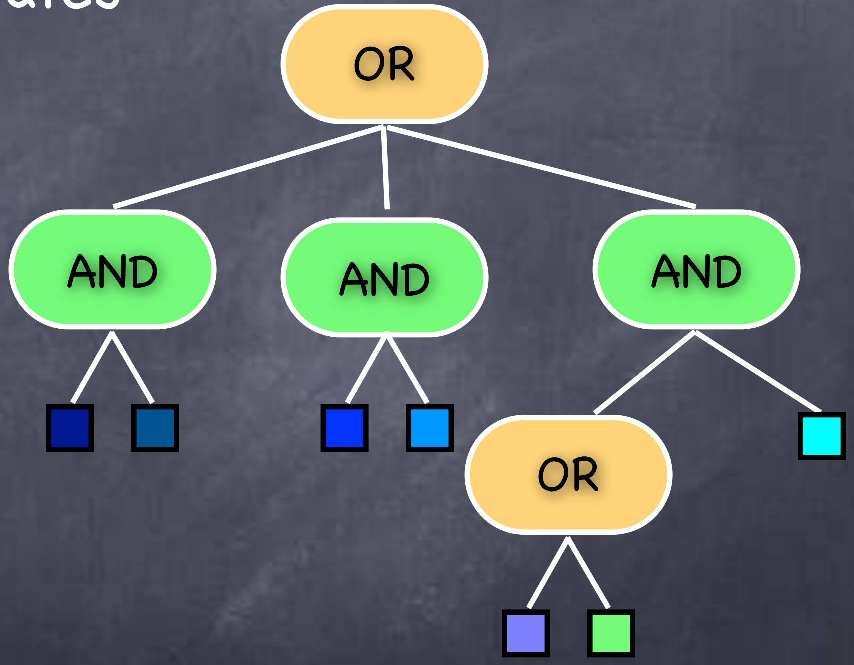

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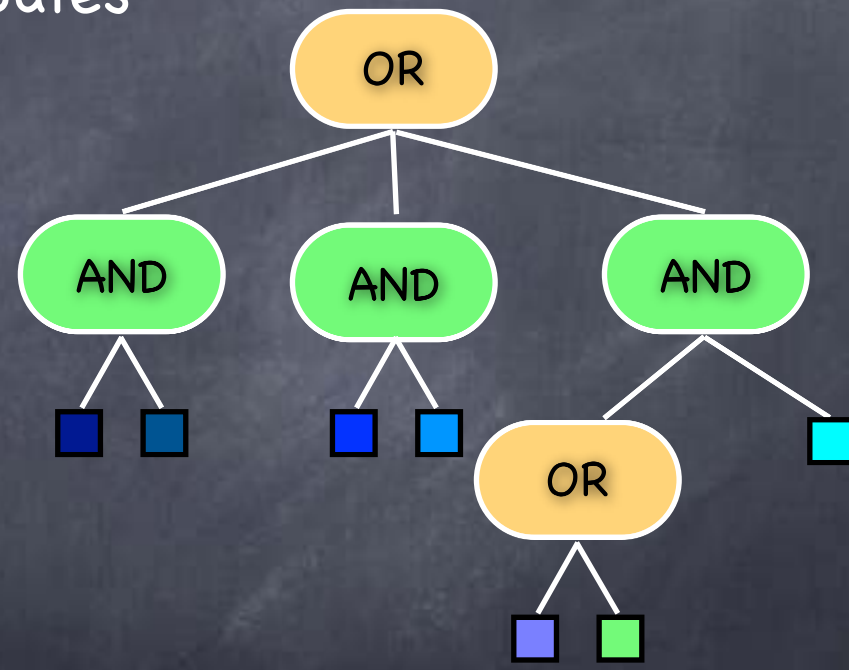

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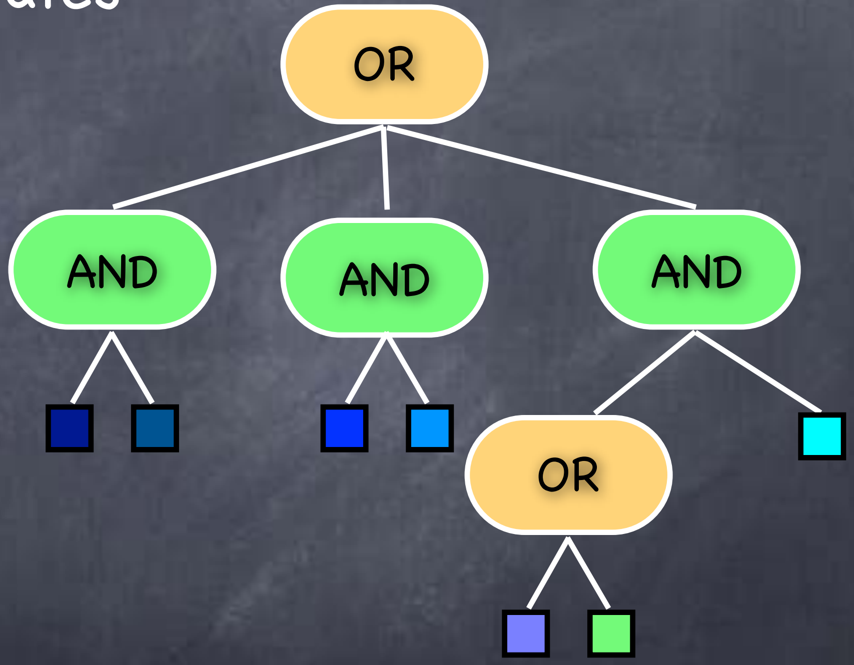

- Can generalize AND/OR to threshold gates

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- CPA security based on Decisional-BDH
  - Choosing a random vector u for each key helps in preventing collusion

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- Application: End-to-End privacy in Attribute-Based Messaging

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    - (f,g,h,f^x,g^y,h^{x+y}) and (f,g,h,f^x,g^y,h^z) indistinguishable for random f, g, h, x, y, z.

Single-Key FE

- In which key for only one function will be ever be released
  - Function is not known when ciphertexts are created (otherwise trivial [Why?])
- A single-key FE scheme supporting arbitrary functions (with circuits of a priori bounded size)
  - Encryption of m is a Garbled circuit encoding the universal function: $F(x,f) = f(x)$, with x set to m
  - Plus, 2n encrypted wire labels for the n input wires of f (using 2n public-keys in the master public-key)
  - Key for f: n secret-keys corresponding to the n bits of f
  - Can decrypt the labels of f → can evaluate $F(x,f)$

No Unbounded Sim-FE

- Suppose we require simulation-based security for FE
- Then there are function families which have no FE scheme that supports releasing an unbounded number of keys
- e.g., The message is the seed of the PRF. The function evaluates the PRF on an input (i.e., one key for each input)
  - Even suppose that the simulator knows a priori the set of inputs for which the adversary will obtain keys
  - { $PRF_s(x_i)$ | i=1 to N } are N k-bit pseudorandom strings
  - Simulation should encode them into an L-bit string (i.e., the simulated ciphertext)
    - If Nk >> L, not possible for truly random strings, and hence for pseudorandom strings too

Unbounded FE from Obfuscation

- Indistinguishability based definition for FE
- Indistinguishability Obfuscation (iO) suffices
- Simpler if we have a slightly stronger obfuscation:
  - KeyGen(f) = (f,$sign_{SK}$(f)), where SK is the signing key corresponding to a VK in the master public-key
  - Enc(msg) = Obfuscation of the following program:
    - Accept (f, σ). If $Verify_{VK}$(f, σ), then output f(msg)
  - Dec(C, K) : run C (which is a program) on input K=(f, σ)

Multi-Input FE

- Consider implementing an encrypted database: all values are kept encrypted, but insertion, deletion, look-up etc. should be possible publicly
- Need to compare pairs of ciphertexts. Not a ciphertext and a key
- More generally, compute $f(x_1,\ldots,x_d)$ given independently generated ciphertexts of x_i's (for a fixed f, or a family of f's)
- Public-key or private-key setting
    - Or a mix: some arguments to f can be publicly encrypted, and others cannot be
- IND security: cannot learn a challenge bit from keys/ciphertexts, if it cannot be learned in an IDEAL model

Multi-Input FE

- Can be constructed using obfuscation
- Enc(x,i), i.e., encrypt x as i^{th} argument: $E_{PK_i}(x)$, where E is the encryption algorithm in a CCA-secure PKE scheme. PK_i's in master PK
- KeyGen(f) : Obfuscate the following program:
  - Accept d ciphertexts $c_1,...,c_d$. $x_i \leftarrow D_{SK_i}(c_i)$ for all i.

    If all decryptions valid, output $f(x_1,...,x_d)$
- CCA-security needed to prevent the adversary from evaluating f on inputs related to encrypted messages
  - To use "realizable" obfuscation (involving only one hidden bit): instead of CCA security, use (c,c',π), where π is a "proof" that c and c' encrypt the same message under two keys.

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    - Not yet practical